3A: Week 10
L'Hôpital's Rule and Sketching Curves

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Group Work

L'Hôpital's Rule

If $\lim_{x\to a} \frac{f(x)}{g(x)}$ has the form ∞/∞ or $0/0$, **L'Hôpital's Rule** says that $\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$

1. Calculate the following limits:

a) $\lim_{x\to\infty} \frac{\ln(3x)}{x^2} = \lim_{x\to\infty} \frac{\left(\frac{3}{3x}\right)}{(2x)} = 0$

$\approx \frac{\infty}{\infty}$ indeterminate form

$\approx \frac{0}{\infty}$ NOT indeterminate.

b) $\lim_{x\to 1^+}\left(\frac{1}{\ln(x)} - \frac{1}{x-1}\right) = \lim_{x\to 1^+}\left(\frac{(x-1)-\ln(x)}{\ln(x)(x-1)}\right) = \lim_{x\to 1^+}\frac{\left(1-\frac{1}{x}\right)}{\left[\left(\frac{1}{x}\right)(x-1)+1\ln(x)\right]}$

$\approx \infty - \infty$ indeterminate, but NOT L'Hôpital-able quite yet

$\approx \frac{0}{0}$

$\approx \frac{0}{0}$

$= \lim_{x\to 1^+}\frac{\left(\frac{1}{x^2}\right)}{\left(\frac{1}{x^2}\right)+\frac{1}{x}} = \frac{1}{2}$

c) $\lim_{x\to\infty}(e^x + x)^{1/x}$ $\Rightarrow$ consider... $\lim_{x\to\infty} \ln\left((e^x+x)^{1/x}\right) = \lim_{x\to\infty}\frac{1}{x}\cdot\ln(e^x+x)$

$\approx \infty^0$ indeterminate, not L'Hôpital.

$= \lim_{x\to\infty}\frac{\ln(e^x+x)}{x} = \lim_{x\to\infty}\frac{\left(\frac{e^x+1}{e^x+x}\right)}{1} = \lim_{x\to\infty}\frac{e^x+1}{e^x+x}$

$\approx \frac{\infty}{\infty}$

$\approx \frac{\infty}{\infty}$

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$= \lim_{x\to\infty}\frac{e^x}{e^x+1} = \lim_{x\to\infty}\frac{e^x}{e^x} = 1$

$\approx \frac{\infty}{\infty}$

Now, recall how we changed the problem by introducing ln. after "$\Rightarrow$".
Undo this by raising e to the power of the answer $\Rightarrow e^1 = \boxed{e}$

2. Sketch the graph of $f(x) = \dfrac{x^2+x-2}{x^2}$, filling in the following guide taken from lecture notes. (Try not to look at your notes unless absolutely needed.)

a) Determine the domain of f.

$(-\infty, 0) \cup (0, \infty)$

b) Find the x- and y- intercepts of the graph.

x-intercepts: $0 = (x^2+x-2) = (x-1)(x+2)$
$\Rightarrow$ at $(1,0)$ and $(-2,0)$

y-intercept: None :(why not?)

c) Determine the symmetry (even/odd) of the graph (if it exists).

D.N.E.

d) Find the vertical asymptotes of f.

$\lim_{x\to 0^-} \dfrac{x^2+x-2}{x^2} = \lim_{x\to 0^-} 1 + \frac{1}{x} - \frac{2}{x^2} = -\infty$ — vertical asymptote at $x=0 \Rightarrow -\infty$.

$\lim_{x\to 0^+} \dfrac{x^2+x-2}{x^2} = -\infty$

e) Determine the behavior at the ends of the graph by looking at $\lim_{x\to\infty} f(x)$ and $\lim_{x\to -\infty} f(x)$

$\lim_{x\to\infty} \dfrac{x^2+x-2}{x^2} = 1$

$\lim_{x\to -\infty} \dfrac{x^2+x-2}{x^2} = 1$

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f) Draw number lines and use derivatives (f' and f'') to determine where f is ...

i) increasing ~~(0,4)~~
$(0,4)$

ii) decreasing
$(-\infty,0) \cup (4,\infty)$

iii) concave up
$(6,\infty)$

iv) concave down
$(-\infty,0) \cup (0,6)$

$f'(x) = \dfrac{4-x}{x^3} \Rightarrow$ critical values at $x=0, 4$

f': $- - - -$ (0) $+ + + +$ (4) $- - - -$

$f''(x) = 2\left(\dfrac{x-6}{x^4}\right) \Rightarrow$ critical values at $x=0, 6$

f'': $- - - -$ (0) $- - - -$ (6) $+ + + +$

Use this information to determine all local and absolute extrema.

Local max at $x=4 \Rightarrow \left(4, \frac{9}{8}\right)$ or $(4, 1.125)$

g) Evaluate f at the "interesting" places (i.e., at places such as those in b & parts of f

$(1,0)$
$(-2,0)$
$(4, 1.125)$
$(6, 1.111..)$

h) Sketch! the graph using all the information, and "connecting the dots".