Name: ANSWER GUIDE

"The only constant is change."
-Hericletus

Collaborators:

Section Day/Time:

Limits & Derivatives

1. Evaluate each limit, showing all your work.

a) $\lim_{x\to\infty} 4x^7 - 18x^3 + 9$

$$= \lim_{x\to\infty} x^7\left(4 - 18/x^4 + 9/x^7\right)$$
$$= \left(\lim_{x\to\infty} x^7\right)\cdot\left(\lim_{x\to\infty} 4 - \frac{18}{x^4} + \frac{9}{x^7}\right)$$
$$= \infty \cdot 4 = \boxed{\infty}$$

b) $\lim_{x\to-\infty} 4x^7 - 18x^3 + 9$

: same work

$$= \left(\lim_{x\to-\infty} x^7\right)\cdot\left(\lim_{x\to-\infty} 4 - \frac{18}{x^4} + \frac{9}{x^7}\right)$$
$$= -\infty \cdot 4 = \boxed{-\infty}$$

c) $\lim_{x\to\infty} \frac{8-4x^2}{9x^2+5x} \cdot \frac{1/x^2}{1/x^2}$

$$= \lim_{x\to\infty} \frac{8/x^2 - 4}{9 + 5/x}$$
$$= \frac{\lim_{x\to\infty}\left(8/x^2 - 4\right)}{\lim_{x\to\infty}\left(9 + 5/x\right)} = \boxed{\frac{-4}{9}}$$

d) $\lim_{x\to-\infty} \frac{8-4x^2}{9x^2+5x}$

: SAME WORK

$$= \frac{\lim_{x\to-\infty}\left(8/x^2 - 4\right)}{\lim_{x\to-\infty}\left(9 + 5/x\right)} = \boxed{\frac{-4}{9}}$$

Derivative Definition(s)
The derivative of a function $f(x)$ at the number $x = a$ is $f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$
The derivative for all $y = f(x)$ is $f'(x) = \lim_{x_0\to x} \frac{f(x_0)-f(x)}{x_0 - x}$

2. Use the definition of the derivative to find the derivative of $f(x) = \frac{x+1}{x+4}$. Show all your work.

$$f'(x) = \lim_{x_0\to x} \frac{\left(\frac{x_0+1}{x_0+4}\right) - \left(\frac{x+1}{x+4}\right)}{x_0 - x} = \lim_{x_0\to x} \frac{\left[\frac{(x_0+1)(x+4) - (x+1)(x_0+4)}{(x_0+4)(x+4)}\right]}{x_0 - x}$$

$$= \lim_{x_0\to x} \frac{\left[\frac{(x_0x + x + 4x_0 + 4) - (x_0x + 4x + x_0 + 4)}{(x_0+4)(x+4)}\right]}{x_0 - x}$$

$$= \lim_{x_0\to x} \frac{\left[\frac{3x_0 - 3x}{(x_0+4)(x+4)}\right]}{x_0 - x} = \lim_{x_0\to x} \frac{(x_0 - x)\cdot\frac{3}{(x_0+4)(x+4)}}{x_0 - x}$$

$$= \lim_{x_0\to x} \frac{3}{(x_0+4)(x+4)} = \frac{3}{(x+4)(x+4)} = \frac{3}{x^2+8x+16}$$

3. Find the equation of the tangent line to the curve $f(x) = -x^2 + 4x$ at the point $(2, 4)$ by using the definition of the derivative. Sketch a graph to check your answer.

If you have time, use the derivative definition to find $f'(x)$ for all x, and sketch part of its graph.

$$f'(2) = \lim_{h\to 0} \frac{[-(2+h)^2+4(2+h)] - 4}{h}$$

$$= \lim_{h\to 0} \frac{[-(4+4h+h^2)+(8+4h)]-4}{h}$$

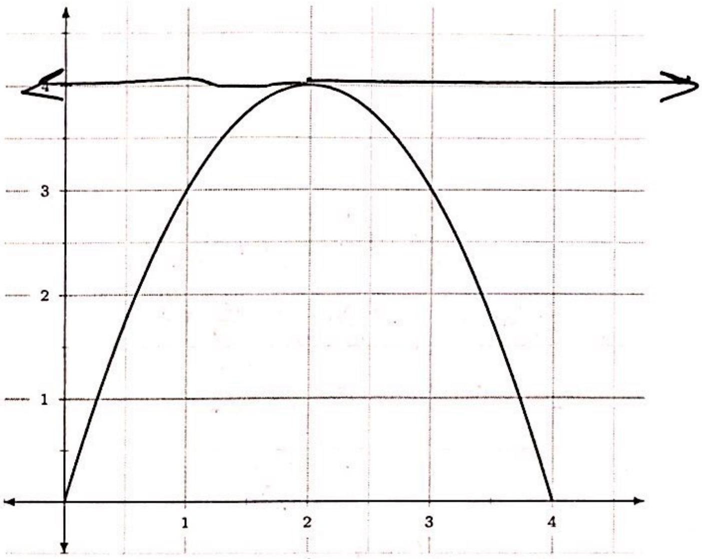
$$= \lim_{h\to 0} \frac{8-4-4+4h-4h-h^2}{h}$$

$$= \lim_{h\to 0} \frac{-h^2}{h} = \lim_{h\to 0} \frac{h}{1}$$

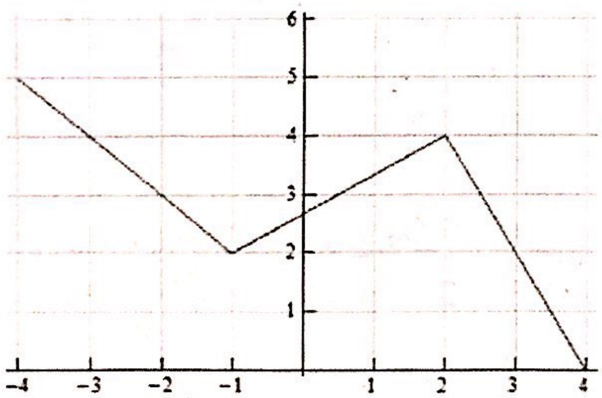
$$= 0$$

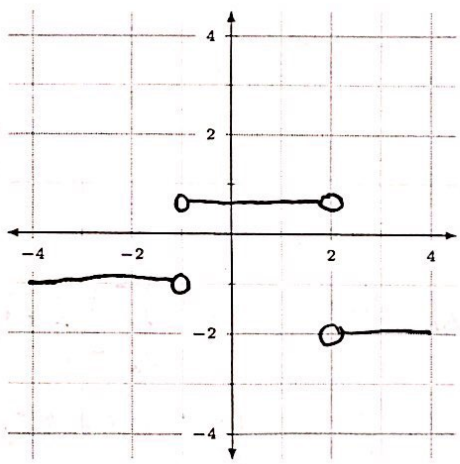
So, $m = \text{slope} = 0$

equation of tangent line: $y-4=0(x-2) \Rightarrow \boxed{y=4}$



4. Sketch the derivative of the graph of $f(x)$ depicted below.





Discuss: If $f(x)$ above is the derivative of some function $g(x)$ so that $g'(x) = f(x)$, what might the graph of $g(x)$ look like?