Name:

Collaborators:

Section Day/Time:

"It is an inherent property of intelligence that it can jump out of the task which it is performing, and survey what it has done; it is always looking for, and often finding, patterns."
-Douglas R. Hofstadter

Derivatives: Product and Quotient Rules

1. Determine the values of x for which $h(x) = x^3 + 9x^2 - 48x + 2$ is not changing.
(*Hint: The quadratic formula might be helpful here.*)

The Product Rule

$$(fg)'(x) =$$

2. Calculate the following derivatives **using only the product rule** (and power/exponent rules):

a) $h(x) = (e^x + 1)x^{-3}$

b) $f(t) = e^t + (2e^t + t)(t^{2e} - e)$

c) $p(x) = (1 + 2x + 3x^2)(5x + 8x^2 - x^3)$

Discuss: How would this process have been different if you had used FOIL first?

The Quotient Rule

$$\left[\frac{f(x)}{g(x)}\right]' =$$

3. Calculate the following derivatives **using the quotient rule**:

a) $h(x) = \frac{e^x+1}{x^3}$

Discuss: Does this problem look familiar? How does your answer here compare to 2.a)?

b) $p(x) = \dfrac{3x^4 - 5x^2 + 2}{x^2 - 1}$

Discuss: What would have happened if you had simplified the problem first?
(*Hint:* $3x^4 - 5x^2 + 2 = (x^2 - 1)(3x^2 - 2)$.)

4. Calculate the derivative of $h(x) = (x^{-4} + 6\sqrt[5]{x^2})\left(\dfrac{(x^9+2)(x^2e^x)}{x+1}\right)$. Use the back of the next page to do your work if needed.

Challenge Problems

The following problems will introduce you to the idea of the *chain rule*, which you will learn later this quarter. They are meant to challenge and strengthen your understanding of the product rule.

5. Use the **product rule** along with the following outline to make a guess for the derivative of $f(x) = e^{kx}$, when k is any integer.

a) Find the derivative of $a(x) = e^{2x} = (e^x)^2$.

b) Use part a) to find the derivative of $b(x) = e^{3x}$. Use this to find the derivative of $c(x) = e^{4x}$.

c) Continue this process of adding 1 to the exponent and calculating that new function's derivative using the previous answer, until you notice a pattern. ***Discuss:*** What would you guess is the derivative of $f(x) = e^{kx}$, when k is any integer?

6. Use the **product rule, and nothing else** to determine the derivative of $[f(x)]^{2048}$ with the following outline. *(Note: $f(x)$ represents **any** differentiable function, not a specific one.)*

a) Calculate the derivative of $[f(x)]^2$.

b) Using part a), calculate the derivative of $[f(x)]^4$. Next, calculate the derivative of $[f(x)]^8$.

c) Continue this process, doubling the exponent, until you see a pattern. ***Discuss*** the following questions: **(1)** What do you think the derivative of $[f(x)]^{2048}$ would be? **(2)** What about of $[f(x)]^{2049}$? **(3)** Of $[f(x)]^k$, where k is any integer? **(4)** How does this relate to the product rule, and how is it different?