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OH: M 11-12pm / **ML:** Th 1-3pm

3A: Week 3
Exam 1 Practice Worksheet

Name:

"A recall is worth a thousand repetitions"
-Dr. Wayne Iba

Collaborators:

Section Day/Time:

Exam 1 Practice Worksheet

1. Find an equation of the line that passes through the points on the graph $f(x) = x^2$ when $x = 3$ and $x = 5$. Write the equation in point-slope form.

$$m = \frac{f(3)-f(5)}{3-5} = \frac{9-25}{-2} = \frac{-16}{-2} = 8$$

$$\Rightarrow y - 3^2 = 8(x-3)$$

$$\boxed{\Rightarrow y - 9 = 8(x-3)}$$

OR

$$y - 5^2 = 8(x-5)$$

$$\boxed{\Rightarrow y - 25 = 8(x-5)}$$

2. The point $(5, 1)$ is on the graph of $f(x) = \sqrt{x-4}$.

a) Compute the slope of the secant line through $(5, 1)$ and the point that is on the graph of $f(x)$ at $x = 5.01$.

$$m = \frac{f(5.01) - f(5)}{5.01 - 5} = \frac{\sqrt{5.01-4} - 1}{0.01} = \boxed{0.4987}$$

b) Use this answer to guess the slope of the tangent line to $f(x)$ at the point $(5, 1)$.

Guess: $\boxed{m = 0.5}$

Check: How might you check this guess?

3. Evaluate the limit $\lim_{h\to 0} \frac{\sqrt{1+h}-1}{h}$.

$$\lim_{h\to 0} \frac{\sqrt{1+h}-1}{h} = \lim_{h\to 0} \frac{\sqrt{1+h}-1}{h} \cdot \frac{\sqrt{1+h}+1}{\sqrt{1+h}+1}$$

$$= \lim_{h\to 0} \frac{(1+h) - 1^2}{h(\sqrt{1+h}+1)} = \lim_{h\to 0} \frac{h}{h(\sqrt{1+h}+1)}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{1+h}+1} = \frac{1}{\sqrt{1}+1} = \boxed{\frac{1}{2}}$$

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4. Find and sketch all asymptotes (vertical and horizontal) of the graph $f(x) = \frac{3x^2+1}{x^2-4}$.

$f(x) = \frac{3x^2+1}{(x-2)(x+2)} \Rightarrow$ V.A. at $x=2$, $x=-2$ $\Rightarrow \lim_{x\to 2^-} f(x) \approx \frac{+}{-\cdot +} = -\infty$

$\Rightarrow \lim_{x\to 2^+} f(x) \approx \frac{+}{+\cdot +} = \infty$

$\Rightarrow \lim_{x\to -2^-} f(x) \approx \frac{+}{-\cdot -} = \infty$

$\Rightarrow \lim_{x\to -2^+} f(x) \approx \frac{+}{-\cdot +} = -\infty$

See sketch below

H.A.: $\lim_{x\to\infty} \frac{3x^2+1}{x^2-4} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \lim_{x\to\infty} \frac{3+\frac{1}{x^2}}{1-\frac{4}{x^2}} = 3$

$\lim_{x\to-\infty} \frac{3x^2+1}{x^2-4} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \lim_{x\to-\infty} \frac{3+\frac{1}{x^2}}{1-\frac{4}{x^2}} = 3$

5. Use the Intermediate Value Theorem to show that the function $f(x) = x^4 + x - 9$ has a root on the open interval $(1, 2)$. *(Note: You do not need to actually find a root.)*

$f(1) = 1^4 + 1 - 9 = -7 < 0$

$f(2) = 2^4 + 2 - 9 = 16 + 2 - 9 = 9 > 0$

So $f(1) < 0 < f(2)$, and since f is a polynomial, by I.V.T. $f(x) = 0$ for some $1 < x < 2$.

6. Let $f(x) = \sqrt{5-x}$. Find $f'(x)$ using the limit definition of a derivative.

$\lim_{x_0\to x} \frac{\sqrt{5-x_0} - \sqrt{5-x}}{x_0 - x} = \lim_{x_0\to x} \frac{\sqrt{5-x_0} - \sqrt{5-x}}{x_0 - x} \cdot \frac{\sqrt{5-x_0} + \sqrt{5-x}}{\sqrt{5-x_0} + \sqrt{5-x}}$

$= \lim_{x_0\to x} \frac{(5-x_0)-(5-x)}{(x_0-x)\left[\sqrt{5-x_0} + \sqrt{5-x}\right]} = \lim_{x_0\to x} \frac{x-x_0}{x_0-x} \cdot \frac{1}{\sqrt{5-x_0} + \sqrt{5-x}}$

$= -1 \cdot \frac{1}{2\sqrt{5-x}} = \boxed{-\frac{1}{2\sqrt{5-x}}}$

7. Use the given graph of the function $f(x)$ below to sketch the graph of $f'(x)$ on the same set of axes.