Name:

Collaborators:

Section Day/Time:

Limits and Continuity

Limit Laws

$\lim_{x\to a}[f(x)+g(x)] =$ $\lim_{x\to a} f(x) + \lim_{x\to a} g(x)$

$\lim_{x\to a}[c\cdot g(x)] =$ $c\cdot \lim_{x\to a} g(x)$

$\lim_{x\to a}[f(x)\cdot g(x)] =$ $\lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x)$

$\lim_{x\to a}\left[\frac{f(x)}{g(x)}\right] =$ $\frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$

as long as $g(x)\neq 0$

$\lim_{x\to a}[f(g(x))] =$ $f\left(\lim_{x\to a} g(x)\right)$

as long as f is continuous at $\lim_{x\to a} g(x)$

(i.e. $\lim_{x\to H} f(x) = f(H)$ for $H=\lim_{x\to a} g(x)$)

If $f(x)=g(x)$ for $x\neq a$, then, provided the limit exists,

$\lim_{x\to a} f(x) =$ $\lim_{x\to a} g(x)$

1. Compute the following limits:

a) $\lim_{z\to 3}\left(z^2e^z+\frac{z}{2e^z}\right)$

$3^2e^3 + \frac{3}{2e^3} = 9e^3 + \frac{3}{2e^3}$

b) $\lim_{b\to 0}\frac{\cos(b)-\sin(b)}{b^2+1}$ $= \frac{1-0}{1^2+1}$

$= \frac{1}{2}$

c) $\lim_{x\to 1}\cancel{\frac{x-1}{x^2-2x+1}}$ $\quad \frac{x-1}{x^2-1}$

$= \lim_{x\to 1}\frac{x-1}{(x-1)(x+1)}$

$= \lim_{x\to 1}\frac{1}{x+1} = \frac{1}{2}$

d) Suppose $\lim_{y\to 2} g(y)=3$ and $\lim_{y\to 4} g(y)=2$. Find $\lim_{y\to 2}[g(y^2)\cdot g(y)^2]$ $= \lim_{y\to 2} g(y^2)\cdot \lim_{y\to 2} g(y)^2$

$= \lim_{y\to 2^2} g(y)\cdot\left(\lim_{y\to 2} g(y)\right)^2$

$= 2\cdot 3^2$

$= 18$

e) $\lim_{a\to 1^-}\frac{\ln(\cancel{a}\,0.5 - 1)}{1-a}$ *(Does* $\lim_{a\to 1}\frac{\ln(\cancel{a}\,0.5 - 1)}{1-a}$ *exist? Discuss.)*

$\lim_{a\to 1^-}\frac{\ln(0.5)}{1-a} = -\infty$

f) Use the Squeeze Theorem to determine $\lim_{x\to 0} x^4\sin(\pi/x)$.

$-x^4 \le x^4\sin\left(\frac{\pi}{x}\right) \le x^4$

So $\lim_{x\to 0} x^4\sin\left(\frac{\pi}{x}\right) \le \lim_{x\to 0} x^4 = 0$

and $0 = \lim_{x\to 0} -x^4 \le \lim_{x\to 0} x^4\sin(\pi/x)$.

So $\lim_{x\to 0} x^4\sin\left(\frac{\pi}{x}\right) = 0$.

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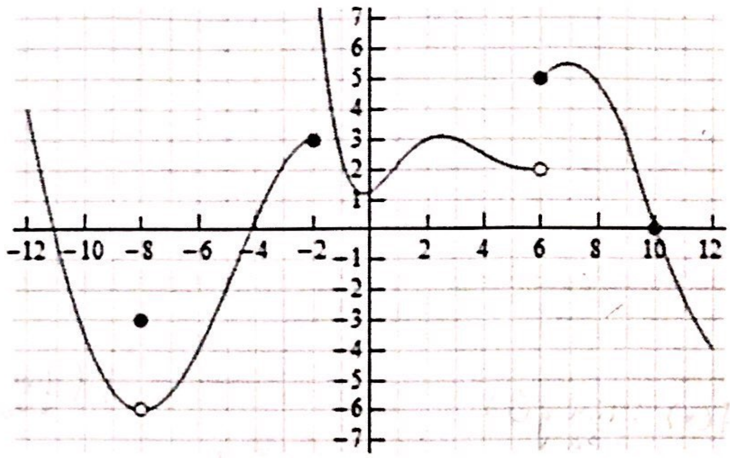


2. Determine where $f(x)$, graphed above, is discontinuous. Classify each type of discontinuity, and calculate $\lim_{x\to -8} f(f(x))$.

At $x=-8$, removable discontinuity
$x=-2$, vertical asymptote
$x=6$, jump discontinuity

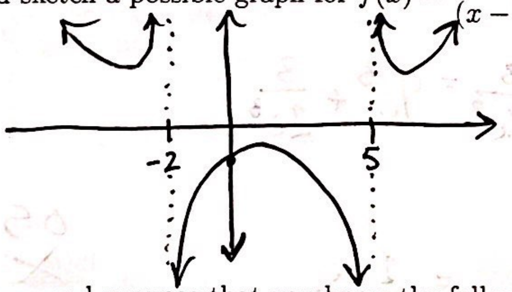
$$\lim_{x\to -8} f(f(x)) = f\left(\lim_{x\to -8} f(x)\right) = f(-6) = -4$$

Asymptotes
Let $a \neq \pm\infty$. A function f has a at $x=a$ if $\lim_{x\to a^{+/-}} f(x) = \pm\infty$.

Discuss: By looking at a function of the form $f(x) = \frac{g(x)}{h(x)}$ where $p(x)$ and $q(x)$ are polynomials, how do you think we can find where the vertical asymptotes are?

3. Find all vertical asymptotes, and sketch a possible graph for $f(x) = \dfrac{x^{2020} - 2019}{(x-5)(x+2)}$

Vertical asymptotes at $x=5$ and $x=-2$



NOTE: THIS IS A SKETCH, AND IS NOT MEANT TO BE ENTIRELY ACCURATE.

4. Suppose a function g is continuous, and suppose that you know the following about g:

1. $g(-3) = -4$
2. $g(3) = \pi$
3. $g(5) < -2$

What is the least number of roots (zeros) that g must have? Where must they be located?

By the I.V.T., g must have at least 2 roots: one within the interval from $(-3, 3)$, and one in $(3, 5)$.