

Group Work

L'Hôpital's Rule

If $\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$ has the form $\frac{\infty}{\infty}$ or $\frac{0}{0}$, L'Hôpital's Rule says that $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$

1. Calculate the following limits:

$$\begin{aligned} \text{a) } \lim_{x \rightarrow \infty} \frac{\ln(3x)}{x^2} &= \lim_{x \rightarrow \infty} \frac{\left(\frac{3}{3x}\right)}{(2x)} = 0 \\ &\approx \frac{\infty}{\infty} \quad \nearrow \quad \approx \frac{0}{\infty} \\ &\text{indeterminate form} \quad \text{NOT indeterminate.} \end{aligned}$$

$$\begin{aligned} \text{b) } \lim_{x \rightarrow 1^+} \left(\frac{1}{\ln(x)} - \frac{1}{x-1} \right) &= \lim_{x \rightarrow 1^+} \left(\frac{(x-1) - \ln(x)}{\ln(x)(x-1)} \right) = \lim_{x \rightarrow 1^+} \frac{\left(1 - \frac{1}{x}\right)}{\left[\left(\frac{1}{x}\right)(x-1) + 1 \ln(x)\right]} \\ &\approx \infty - \infty \quad \approx \frac{0}{0} \quad \approx \frac{0}{0} \\ &\text{indeterminate, but NOT L'Hôpital-able quite yet} \end{aligned}$$

$$\rightarrow = \lim_{x \rightarrow 1^+} \frac{\left(\frac{1}{x^2}\right)}{\left(\frac{1}{x^2}\right) + \frac{1}{x}} = \frac{1}{2}$$

$$\begin{aligned} \text{c) } \lim_{x \rightarrow \infty} (e^x + x)^{1/x} &\Rightarrow \text{consider... } \lim_{x \rightarrow \infty} \ln((e^x + x)^{1/x}) = \lim_{x \rightarrow \infty} \frac{1}{x} \cdot \ln(e^x + x) \\ &\approx \infty^0 \quad \text{indeterminate, not L'Hôpital.} \\ &= \lim_{x \rightarrow \infty} \frac{\ln(e^x + x)}{x} = \lim_{x \rightarrow \infty} \frac{\left(\frac{e^x + 1}{e^x + x}\right)}{1} = \lim_{x \rightarrow \infty} \frac{e^x + 1}{e^x + x} \\ &\approx \frac{\infty}{\infty} \quad \approx \frac{\infty}{\infty} \end{aligned}$$

$$\begin{aligned} \text{2} \rightarrow &= \lim_{x \rightarrow \infty} \frac{e^x}{e^x + 1} = \lim_{x \rightarrow \infty} \frac{e^x}{e^x} = 1 \\ &\approx \frac{\infty}{\infty} \end{aligned}$$

Now, recall how we changed the problem by introducing \ln . after \Rightarrow "Undo this by raising e to the power of the answer $\Rightarrow e^1 = [e]$ "

2. Sketch the graph of $f(x) = \frac{x^2 + x - 2}{x^2}$, filling in the following guide taken from lecture notes.
(Try not to look at your notes unless absolutely needed.)

a) Determine the domain of f .

$$(-\infty, 0) \cup (0, \infty)$$

b) Find the x - and y - intercepts of the graph.

$$\text{x-intercepts: } 0 = x^2 + x - 2 = (x-1)(x+2) \\ \Rightarrow \text{at } (1, 0) \text{ and } (-2, 0)$$

y-intercept: None :(why not?)

c) Determine the symmetry (even/odd) of the graph (if it exists).

D.N.E.

d) Find the vertical asymptotes of f .

$$\lim_{x \rightarrow 0^-} \frac{x^2 + x - 2}{x^2} = \lim_{x \rightarrow 0^-} 1 + \frac{1}{x} - \frac{2}{x^2} = -\infty$$

vertical asymptote
at $x=0 \Rightarrow -\infty$.

$$\lim_{x \rightarrow 0^+} \frac{x^2 + x - 2}{x^2} = -\infty$$

e) Determine the behavior at the ends of the graph by looking at $\lim_{x \rightarrow \infty} f(x)$ and $\lim_{x \rightarrow -\infty} f(x)$

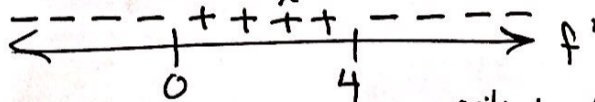
$$\lim_{x \rightarrow \infty} \frac{x^2 + x - 2}{x^2} = 1$$

$$\lim_{x \rightarrow -\infty} \frac{x^2 + x - 2}{x^2} = 1$$

f) Draw number lines and use derivatives (f' and f'') to determine where f is

i) increasing ~~(0, 4)~~
(0, 4)

$$f'(x) = \frac{4-x}{x^3} \Rightarrow \text{critical values at } x=0, 4$$

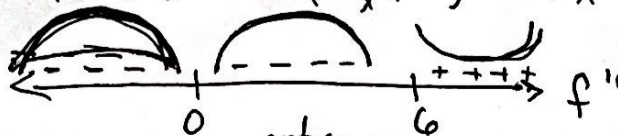


ii) decreasing

$(-\infty, 0) \cup (4, \infty)$

iii) concave up
(6, ∞)

$$f''(x) = 2 \left(\frac{x-6}{x^4} \right) \Rightarrow \text{critical values at } x=0, 6$$



iv) concave down
 $(-\infty, 0) \cup (0, 6)$

Use this information to determine all local and absolute extrema.

Local max at $x=4 \Rightarrow (4, \frac{9}{8})$ or $(4, 1.125)$

g) Evaluate f at the "interesting" places (i.e., at places such as those in b & ^{parts} c) ~~(0, 0)~~

$(1, 0)$

$(-2, 0)$

$(4, 1.125)$

$(6, 1.111\dots)$

h) Sketch! the graph using all the information, and "connecting the dots".

