Name: ANSWER GUIDE

Collaborators:

Section Day/Time:

"It is an inherent property of intelligence that it can jump out of the task which it is performing, and survey what it has done; it is always looking for, and often finding, patterns."
-Douglas R. Hofstadter

Derivatives: Product and Quotient Rules

1. Determine the values of x for which $h(x) = x^3 + 9x^2 - 48x + 2$ is not changing.
(*Hint: The quadratic formula might be helpful here.*)

$$h'(x) = 3x^2 + 18x - 48 = 0$$
$$= 3(x+8)(x-2) = 0$$
$$\Rightarrow x = -8, 2$$

The Product Rule

$$(fg)'(x) = fg' + gf'$$

2. Calculate the following derivatives **using only the product rule** (and power/exponent rules):

a) $h(x) = \underbrace{(e^x + 1)}_{f}\underbrace{x^{-3}}_{g}$

$$h'(x) = (e^x+1)(-3x^{-4}) + (x^{-3})(e^x)$$

b) $f(t) = e^t + \underbrace{(2e^t + t)}_{f}\underbrace{(t^{2e} - e)}_{g}$

$$f'(t) = (2e^t + t)(2et^{2e-1}) + (t^{2e} - e)(2e^t + 1)$$

c) $p(x) = \underbrace{(1 + 2x + 3x^2)}_{f}\underbrace{(5x + 8x^2 - x^3)}_{g}$

$$p'(x) = (1 + 2x + 3x^2)(5 + 16x - 3x^2) + (5x + 8x^2 - x^3)(2 + 6x)$$

Discuss: How would this process have been different if you had used FOIL first?

TA: Kyle Hansen (kylehansen@math.ucsb.edu)
Office: South Hall. 6431 V
OH: M 11-12pm / ML: Th 1-3pm

3A: Week 4
Derivatives: Product and Quotient Rules

The Quotient Rule

$$\left[\frac{f(x)}{g(x)}\right]' = \frac{gf' - fg'}{(g)^2}$$

3. Calculate the following derivatives **using the quotient rule:**

a) $h(x) = \frac{\overbrace{e^x+1}^{f}}{\underbrace{x^3}_{g}}$ $\qquad h'(x) = \dfrac{(x^3)(e^x) - (e^x+1)(3x^2)}{x^6}$

Discuss: Does this problem look familiar? How does your answer here compare to 2.a)? They're the same!

b) $p(x) = \frac{\overbrace{3x^4 - 5x^2 + 2}^{f}}{\underbrace{x^2 - 1}_{g}}$ $\qquad p'(x) = \dfrac{(x^2-1)(12x^3-10x) - (3x^4-5x^2+2)(2x)}{(x^2-1)^2}$

Discuss: What would have happened if you had simplified the problem first?
(*Hint:* $3x^4 - 5x^2 + 2 = (x^2 - 1)(3x^2 - 2)$.)

except for at $x=1$, $p(x) = 3x^2 - 2$, so $p'(x) = 6x$

4. Calculate the derivative of $h(x) = \underbrace{(x^{-4} + 6\sqrt[5]{x^2})}_{a}\left(\frac{\overbrace{(x^9+2)}^{b}\overbrace{(x^2e^x)}^{c}}{\underbrace{x+1}_{d}}\right)$. Use the back of the next page to do your work if needed.

$$h(x) = a \cdot \frac{b \cdot c}{d} \Rightarrow h'(x) = a \cdot \left(\frac{b \cdot c}{d}\right)' + a' \cdot \frac{b \cdot c}{d}$$

$$= a \cdot \left[\frac{d(bc)' - (b \cdot c)d'}{d^2}\right] + a' \cdot \frac{b \cdot c}{d}$$

$$= a \cdot \left[\frac{d(b'c + c'b) - (bc)d'}{d^2}\right] + a' \cdot \frac{b \cdot c}{d}$$

$\boxed{a' = -4x^{-5} + \frac{12}{5}x^{-3/5}}$ $\boxed{b' = 9x^8}$ $\boxed{c' = 2xe^x + x^2e^x}$

$\boxed{d' = 1}$ Then plug in. (see last page)

Challenge Problems

The following problems will introduce you to the idea of the *chain rule*, which you will learn later this quarter. They are meant to challenge and strengthen your understanding of the product rule.

5. Use the **product rule** along with the following outline to make a guess for the derivative of $f(x) = e^{kx}$, when k is any integer.

a) Find the derivative of $a(x) = e^{2x} = (e^x)^2$.

$a'(x) = 2e^{2x}$

b) Use part a) to find the derivative of $b(x) = e^{3x}$. Use this to find the derivative of $c(x) = e^{4x}$.

$b(x) = e^{2x} \cdot e^x$

$b'(x) = 3e^{3x}$

c) Continue this process of adding 1 to the exponent and calculating that new function's derivative using the previous answer, until you notice a pattern. ***Discuss:*** What would you guess is the derivative of $f(x) = e^{kx}$, when k is any integer?

↳ come see me & discuss :)

6. Use the **product rule, and nothing else** to determine the derivative of $[f(x)]^{2048}$ with the following outline. *(Note: $f(x)$ represents any differentiable function, not a specific one.)*

a) Calculate the derivative of $[f(x)]^2$.

$\Rightarrow 2f(x) \cdot f'(x)$

$= [f(x)]^2 \cdot [f(x)]^2$

b) Using part a), calculate the derivative of $[f(x)]^4$. Next, calculate the derivative of $[f(x)]^8$.

$\Rightarrow 4[f(x)]^3 \cdot f'(x)$ $\Rightarrow 8[f(x)]^7 \cdot f'(x)$

c) Continue this process, doubling the exponent, until you see a pattern. ***Discuss*** the following questions: **(1)** What do you think the derivative of $[f(x)]^{2048}$ would be? **(2)** What about of $[f(x)]^{2049}$? **(3)** Of $[f(x)]^k$, where k is any integer? **(4)** How does this relate to the product rule, and how is it different?

Come see me & discuss :)

$$h'(x) = \left(x^{-4} + 6x^{2/5}\right) \cdot \left[\frac{(x+1)\left[(9x^8)(x^2e^x) + (2xe^x + x^2e^x)(x^9+2)\right] - (x^9+2)(x^2e^x) \cdot 1}{(x+1)^2} \right]$$

$$+ \left(-4x^{-5} + \frac{12}{5}x^{-3/5}\right) \cdot \frac{(x^9+2)(x^2e^x)}{x+1}$$