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OH: M 11-12pm / **ML:** Th 1-3pm

Name:

Collaborators:

"It is an inherent property of intelligence that it can jump out of the task which it is performing, and survey what it has done; it is always looking for, and often finding, patterns."
-Douglas R. Hofstadter

Section Day/Time:

Derivatives: Power, Exponent, and Product Rules

The Power Rule

If $f(x) = x^a$ for any number $a \neq 0$, then $f'(x) =$

The Exponent Rule

The derivative of $f(x) = e^x$ is $f'(x) =$

1. Calculate the following derivatives **using only the power rule and the exponent rule** (and, of course, the constant & coefficient & summation rules).

a) $f(x) = -4x^8 - 9x^2 + 25 + x^{-11} + \dfrac{x^6}{3x} - \sqrt[5]{x^2}$

b) $g(t) = e^t + (2t^2 + t)(t^{2e} + e)$

2. Determine the values of x for which $h(x) = x^3 + 9x^2 - 48x + 2$ is not changing.
(*Hint: The quadratic formula might be helpful here.*)

The Product Rule

If $f(x)$ and $g(x)$ are both differentiable functions, then $[f(x)g(x)]' =$

3. Calculate the following derivatives **using the product rule *before* using the power or exponent rule**. On (a) and (c), do **not** use FOIL, and do **not** simplify your answer. On (b), **do** simplify.

a) $p(x) = (1 + 2x + 3x^2)(5x + 8x^2 - x^3)$

Discuss: How would this process have been different if you had used FOIL first?

b) $f(x) = (e^x)^2$

Discuss: What do you think the derivative of $(e^x)^3$ would be? What about $(e^x)^n$, where n is any positive integer?

4. Use the **product rule, and nothing else** to determine the derivative of $[f(x)]^{2048}$ with the following outline. *(Note: $f(x)$ represents **any** differentiable function, not a specific one.)*

a) Calculate the derivative of $[f(x)]^2$.

b) Using part a), calculate the derivative of $[f(x)]^4$. Next, calculate the derivative of $[f(x)]^8$.

c) Continue this process, doubling the exponent, until you see a pattern. ***Discuss*** the following questions: **(1)** What do you think the derivative of $[f(x)]^{2048}$ would be? **(2)** What about of $[f(x)]^{2049}$? **(3)** Of $[f(x)]^k$, where k is any integer? **(4)** How does this relate to the product rule, and how is it different? *(This is an example of **the chain rule**, which will appear later.)*