

Name:

"The moving power of mathematical invention is not reasoning, but imagination."
-Augustus De Morgan

Collaborators:

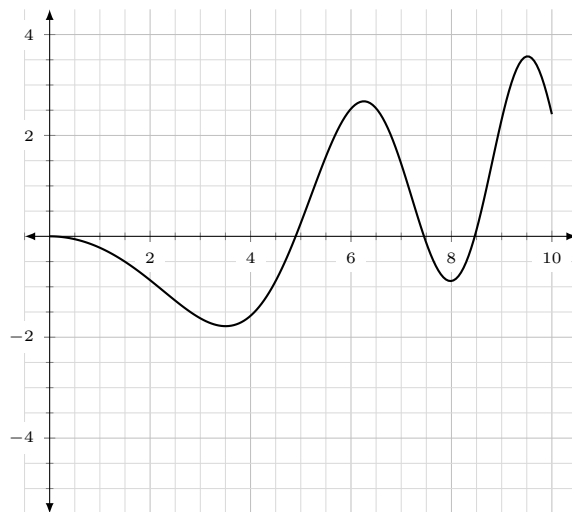
Section Day/Time:

Tangents, Velocity, and Limits

Equation of a Tangent Line

A line tangent to curve $f(x)$ intersecting the y -axis at point b and having slope m can be expressed in *slope-intercept form*: $y = mx + b$.

1. Draw the lines tangent to the corresponding values of x , and complete the chart on the right with appropriate estimates.



x	slope at $f(x)$	equation of line tangent to $f(x)$
2		
4		
6		
8		

2. A kite's height in meters above the ground at time t is modeled by the function $f(t) = (2 - t) \sin(t) + 8$, where t is measured in seconds. Use the table provided to estimate the vertical velocity of the kite at the given points using points close to t . Be sure to include units in the final column!

t	$f(t + 0.001)$	$f(t - 0.001)$	estimated velocity of kite at time t
2			
4			
6			
8			

3. Compute the following limits:

a) $\lim_{z \rightarrow 3} \frac{z^2}{(z-2)^2}$

b) $\lim_{x \rightarrow 1} x^3 - \frac{x+2}{x^2+x-2}$

c) $\lim_{a \rightarrow 1^-} \frac{\ln(a-1)}{1-a}$

Discuss: Does $\lim_{a \rightarrow 1} \frac{\ln(a-1)}{1-a}$ exist? Why or why not?

d) Suppose $\lim_{y \rightarrow 2} g(y) = 3$ and $\lim_{y \rightarrow 4} g(y) = 2$. Find $\lim_{y \rightarrow 2} [g(y^2) \cdot g(y)^2]$

Asymptotes

Let $a \neq \pm\infty$. A function f has a *vertical asymptote* at $x = a$ if $\lim_{x \rightarrow a^{+/-}} f(x) = \quad$.

Discuss: By looking at a function of the form $f(x) = \frac{g(x)}{h(x)}$ where $p(x)$ and $q(x)$ are polynomials, how do you think we can find where the vertical asymptotes are?

4. Find all vertical asymptotes, and sketch a possible graph for each of the following functions:

a) $f(x) = \frac{x^{2020} - 2019}{(x-5)(x+2)}$

b) $g(x) = \csc(x) = \frac{1}{\sin(x)}$

Discuss: Give our definition of a vertical asymptote, how do you think a *horizontal asymptote* would be defined? Can you think of any functions with a horizontal asymptote?