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Group Work

Log and Exponent Rules

$\frac{d}{dy}e^x = e^x$ $\frac{d}{dy}a^x = \ln(a) \cdot a^x$

$\frac{d}{dy}\ln(x) = \frac{1}{x}$ $\frac{d}{dy}\log_b(x) = \frac{1}{x\ln(b)}$

1. Compute each derivative:

a) $f(x) = 3^x \log_3(x)$

$$f'(x) = 3^x \cdot \frac{1}{x \ln 3} + \log_3(x) \cdot \ln(3) \cdot 3^x$$

b) $g(x) = \ln(\sin(x)) - (x^4 - 3x)^{10}$

$$g'(x) = \frac{\cos(x)}{\sin(x)} - 10(x^4 - 3x)^9 \cdot (4x^3 - 3)$$

2. Find the equation of the tangent line to the graph of $f(x) = \ln(x)\log_2(x)$ at $x = 2$.

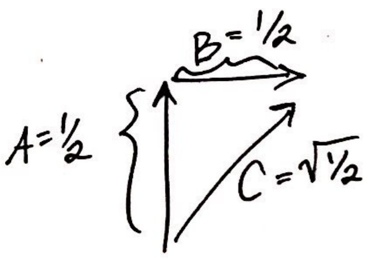
$$f'(x) = \ln(x) \cdot \frac{1}{x\ln(2)} + \log_2(x) \cdot \frac{1}{x}$$

$$f'(2) = \frac{1}{2} + \frac{1}{2} = \frac{1}{4}$$

$$y - (\ln(2) \cdot \log_2(2)) = \frac{1}{4}(x-2) \Rightarrow y = \frac{1}{4}x - \frac{1}{2} + \ln 2$$

3. **Expository information**: Radar guns measure the rate of change in distance between the gun and the object it is measuring. For instance, a reading of "55 mph" means the object is moving away from the gun at a rate of 55 mph, whereas a measurement of "-25 mph" would mean that the object is approaching the gun at a rate of 25 mph. If the radar gun is moving (say, attached to a police car) then radar readouts are only immediately understandable if the gun and the object are moving along the same line. If a police officer is traveling 60 mph and gets a readout of 15 mph, they knows that the car ahead of them is moving away at a rate of 15 mph, meaning the car is traveling 75 mph. (This straight–line principle is one reason officers park on the side of the highway and try to shoot straight back down the road. It gives the most accurate reading.)

Suppose an officer is driving due north at 30 mph and sees a car moving due east. Using her radar gun, she measures a reading of 20 mph. By using landmarks, she believes both she and the other car are about 1/2 mile from the intersection of their two roads. If the speed limit on the other road is 55 mph, is the other driver speeding? (*Hint:* first draw a diagram.)



$$A^2 + B^2 = C^2 \xrightarrow{\text{implicit diff.}} 2A\frac{dA}{dt} + 2B\frac{dB}{dt} = 2C\frac{dC}{dt}$$

$$\left.\begin{array}{ll} A = \frac{1}{2} & \frac{dA}{dt} = -30 \\ B = \frac{1}{2} & \\ C = \sqrt{\frac{1}{2}} & \frac{dC}{dt} = 20 \end{array}\right\}$$

Plug in and rearrange to get $\frac{dB}{dt} \approx 58$ mph.

4. Differentiate the following function with respect to x:

$$y = \frac{\sqrt{5x+8} \cdot \sqrt[3]{1 - 9\cos(4x)}}{\sqrt[4]{x^2 + 10x}}.$$

$$\ln(y) = \ln\left(\frac{\sqrt{5x+8} \cdot \sqrt[3]{1-9\cos(4x)}}{\sqrt[4]{x^2+10x}}\right) = \frac{1}{2}\ln(5x+8) + \frac{1}{3}\ln(1-9\cos(4x)) - \frac{1}{4}\ln(x^2+10x)$$

$$\Rightarrow \frac{y'}{y} = \frac{1}{2}\cdot\frac{5}{5x+8} + \frac{1}{3}\cdot\frac{9\sin(4x)\cdot 4}{1-9\cos(4x)} - \frac{1}{4}\cdot\frac{2x+10}{x^2+10x}$$

$$\Rightarrow y' = \left[\frac{\sqrt{5x+8}\cdot\sqrt[3]{1-9\cos(4x)}}{\sqrt[4]{x^2+10x}}\right]\cdot\left[\frac{1}{2}\cdot\frac{5}{5x+8} + \frac{1}{3}\cdot\frac{36\sin(4x)}{1-9\cos(4x)} - \frac{1}{4}\cdot\frac{2x+10}{x^2+10x}\right]$$