

Definition of combinatorial problem

► combinatorial optimization is about finding the best combination of items from the given options to achieve a goal

► combinatorial optimization is best when you want to maximize or minimize like cost, time or value

example

number of items = 3
max-weight = 4 kg

main mission

1) take the items that can maximize your total weight but won't exceed it

item 1 = 1 kg
item 2 = 2 kg
item 3 = 3 kg

maximum weight
that it can withstand

4 kg

item 3 & item 1

1) they maximize cost value
2) they reach maximum weight without exceeding it.

the example shown is an example of knapsack problem

General mathematical formula for Knapsack problem

general equation for a Knapsack problem

$$\max_n f(n) = \sum_{i=1}^n v_i x_i$$

• v_i can represent any value, (it can also denote as the weight value or the value that distinguishes how important that specific item is)

• x_i binary values so either you can have one or zero. If it is 1, we include the v_i value (whatever the item is, we include that) else we exclude it since it automatically ends up zero.

constraint equation

$$\sum_{i=1}^n w_i x_i \leq W$$

Whatever the number of items that exist, the total number of items weight cannot exceed the maximum weight.

cargo item	v_i (value)	w_i (weight)
Medical Supplies	100 (2nd least)	5
Food Packs	50 (least)	3
Spare parts	200 (most)	8
Fuel Containers	150 (2nd most)	7

What are we given

maximum plane weight = 10 tons

Knapsack problem

maximize the total value

$$\max_{\mathbf{x}} f(\mathbf{x}) = 100x_1 + 50x_2 + 200x_3 + 150x_4$$

where $x_i \in \{0, 1\}$

objective function

$$100x_1 + 50x_2 + 200x_3 + 150x_4 = \max_{\mathbf{x}} f(\mathbf{x})$$

constraint

$$5x_1 + 3x_2 + 8x_3 + 7x_4 \leq 10$$

QUBO

$$\min_{\mathbf{x}} \mathbf{x}^T \mathbf{Q} \mathbf{x}$$

