Deutsch Jotza Algorithm

- We input certain queries, $f:\{0,1\}^n$ that is acknowledg by the oracle and gives its output

$$f\{0,1\}^n \rightarrow \{0,1\}^m \Rightarrow$$

Oracle will gather up all the solutions or output and will output a final numerical value that will determine whether the output is constant or balanced.

0	0	0	0		1
0	0	1	0		1
0	1	0	0		1
0	1	1	0		1
1	1	0	1	or,	0
	⋮		1		0
			1		0
1	1	1	1		0

$\{0\}^m \rightarrow \boxed{1}$

$\{1\}^m \rightarrow \boxed{1}$

$\boxed{\text{constant}}$

$(-1)^{f(x)} + (-1)^{f(x)}$

$(-1)^0 + (-1)^1$

$\rightarrow$ balanced $= 0$

[redacted]

Deutsch Jotza in classical solution

$\underbrace{\frac{N}{2}\boxed{+1}}_{\text{input}} = \boxed{2^{n-1}}\boxed{+1}$ outputs

$n :-$ # number of queries

$n > 3$

$n \leq 3$ → constant, undeclared

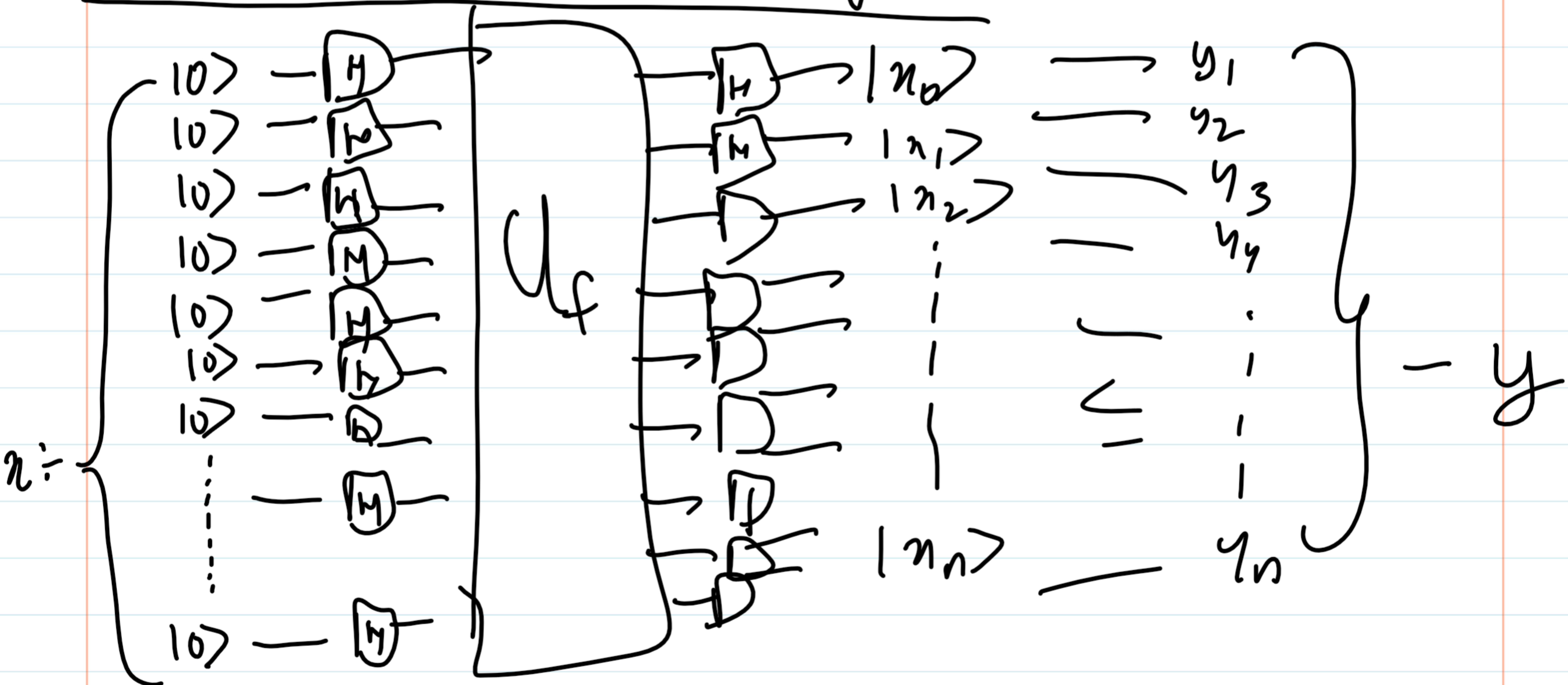
0
0
1
0
1

+1 → constant ☑, balanced ☑, undeclared ☑

0	0	0
0	0	0
0	0	0
1	0	1
1	0	1
1	0	0

[redacted]

Circuit for Deutsch Jotza Algorithm.



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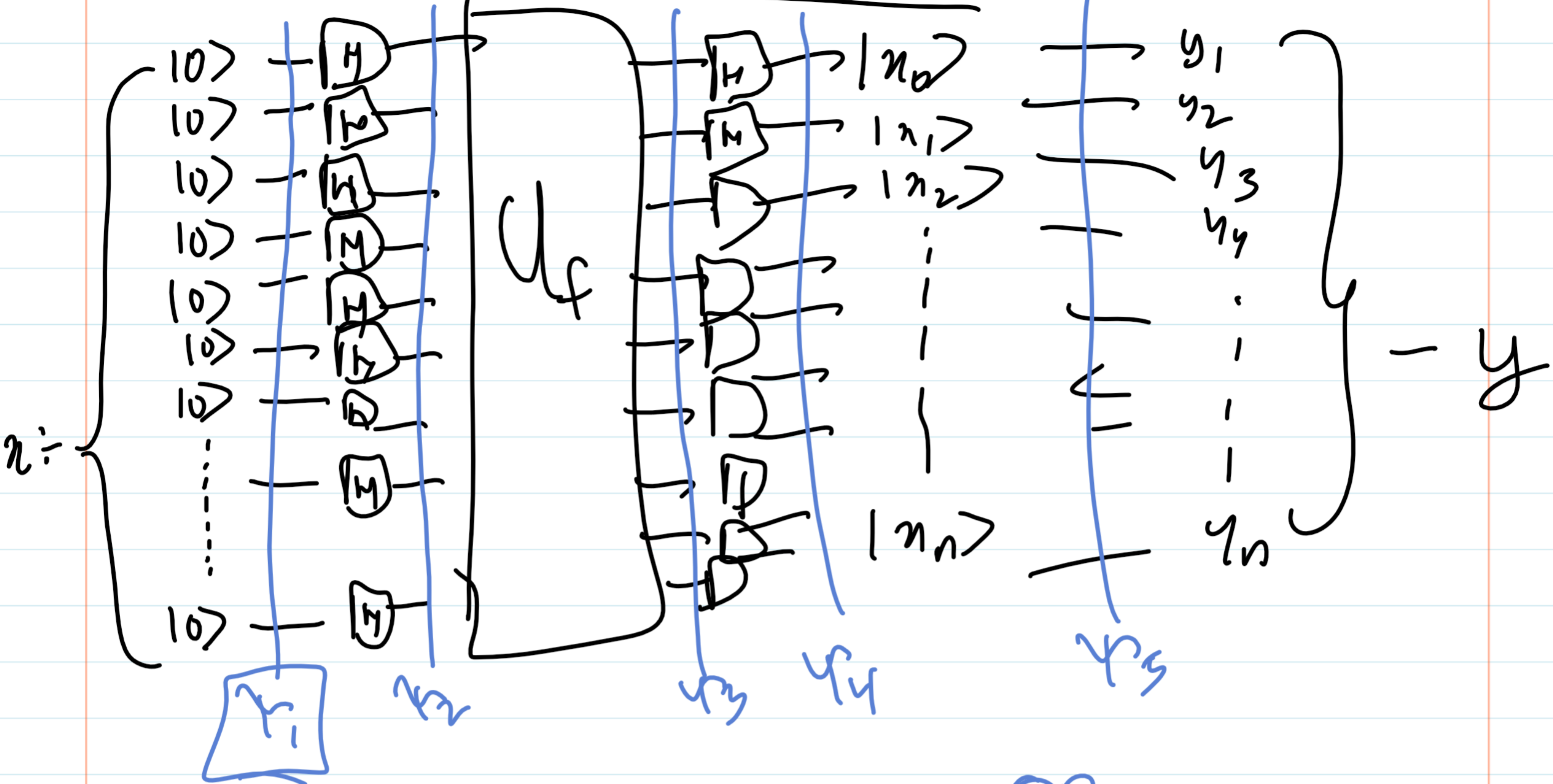

$$\psi_1 : |00000000\ldots\ldots0\rangle = |0\rangle^{\otimes n}$$

$$\psi_2 : H^{\otimes n}|0\rangle^{\otimes n}$$

$$H : \frac{1}{\sqrt{2}}\left[|0\rangle \pm |1\rangle\right]$$

$$= \sum_{x:\{0,1\}^n} \frac{1}{\sqrt{2^n}} (-1)^{f(x)\cdot k} \cdot |k\rangle$$

$$= \frac{1}{\sqrt{2}}\left[(-1)^{f(x)\cdot k} \cdot |k\rangle\right]$$

$$= \frac{1}{\sqrt{2^n}} \cdot |k\rangle$$

$$\psi_3 \doteq U_f \cdot H^{\otimes n} \cdot |0\rangle^{\otimes n}$$

$$\doteq \left[(-1)^{f(x)} |x\rangle \right] \cdot \left[\frac{1}{\sqrt{2^n}} (-1)^{f(x) \cdot k} |k\rangle \right]$$

$$\doteq \left[(-1)^{f(x)} |x\rangle \right] \cdot \left[\frac{1}{\sqrt{2^n}} |k\rangle \right]$$

$$\psi_4 = H^{\otimes n} \cdot U_f \cdot H^{\otimes n} \cdot |0\rangle^{\otimes n}$$

$$= \frac{1}{\sqrt{2^n}} \left[(-1)^{f(x) \cdot k} |k\rangle \right] \cdot \left[(-1)^{f(x)} |x\rangle \right] \cdot \left[\frac{1}{\sqrt{2^n}} (-1)^{f(x) \cdot k} |k\rangle \right]$$

$$\cdot |0\rangle^{\otimes n}$$

$$= \underbrace{\frac{1}{2^n} \left[(-1)^{f(x) \cdot k} |k\rangle \right]}_{C_n} (-1)^{f(x)} |x\rangle \cdot |0\rangle^{\otimes n}$$