Time independent shrodinger wave equation

▷ shrodinger wave equation

▷ $-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi = i\hbar\frac{\partial\psi}{\partial t}$ [general equation]

▷ $-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V\psi = i\hbar\frac{d\psi}{dt}$ [time independent equation]

[redacted]

$\Psi(x,t) = \psi(x)\,\phi(t)$

▷ $\frac{\partial}{\partial t}\Psi = \psi\frac{\partial\phi}{\partial t}$ 1) $\psi\frac{\partial\phi}{\partial t}$

▷ $\frac{\partial^2}{\partial x^2}\Psi = \frac{\partial^2\psi}{\partial \cancel{x}}\phi$ 2) $\frac{\partial^2\psi}{\partial x^2}\phi$

$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi = i\hbar\frac{\partial\psi}{\partial t}$

▷ $$\frac{-\frac{\hbar^2}{2m}\left[\frac{\partial^2\psi}{\partial x^2}\phi\right] + V\left[\psi(x)\,\phi(t)\right] = i\hbar\,\psi\frac{\partial\phi}{\partial t}}{\psi\phi}$$

▷

$-\frac{\hbar^2}{2m}\frac{1}{\phi}\frac{\partial^2\psi}{\partial x^2}\phi + V = i\hbar\frac{1}{\psi}\frac{\partial\phi}{\partial t}\psi$

▷ $$\frac{-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}\cancel{\phi}}{\psi\cancel{\phi}} + \frac{V\cancel{\psi\phi}}{\cancel{\psi\phi}} = \frac{i\hbar\frac{\partial\phi}{\partial t}\cancel{\psi}}{\cancel{\psi}\phi}$$

▷ $$\frac{-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}}{\psi} + V = \frac{i\hbar\frac{\partial\phi}{\partial t}}{\phi}$$

$$\underbrace{\qquad}_{}$$

$H\psi = E \qquad\qquad E\varphi = E$

1) $$\frac{-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}}{\psi} + V = \frac{i\hbar\frac{\partial\phi}{\partial t}}{\phi}$$

1) $$\frac{-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2}}{\psi} + V = E$$

2) $$\boxed{\frac{i\hbar\frac{d\phi}{dt}}{\phi} = E}$$

equation 2

$$i\hbar\frac{d\phi}{dt}\cdot\frac{1}{\phi} = E$$

$$\frac{d\phi}{dt} = \frac{E\phi}{i\hbar}, \quad \frac{-i}{-i\cdot i\hbar}$$

$$\frac{d\phi}{dt} = \frac{-E\phi\cancel{i}}{i\cdot(-i\cancel{)}\hbar)}$$

$$= \frac{-E\phi i}{-(-1)\hbar}$$

$$\frac{d\phi}{dt} = \frac{-E\phi i}{\hbar}$$

$$\int\frac{1}{\phi}\,d\phi = \int -\frac{Ei}{\hbar}\,dt$$

$$\log(\phi) = -\frac{Eit}{\hbar}$$

$$\boxed{\phi(t) = e^{-\frac{Eit}{\hbar}}}$$

$$\psi(x,t) = \psi(x)\, e^{-\frac{iEt}{\hbar}}$$

$$|\psi|^2 = (\psi)\cdot(\psi)^*$$

$$= \left[\psi(x)\, \cancel{e^{-\frac{iEt}{\hbar}}}\right]\cdot\left[\psi(x)^*\cdot \cancel{e^{\frac{iEt}{\hbar}}}\right]$$

$$|\psi|^2 = \left[\psi(x)(\psi(x))^*\right]$$

[redacted]

- They have definite energy state

$\underline{H\psi}$

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi \qquad \boxed{\hat{H} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V}$$

$\boxed{H}$

$$\boxed{H\psi = E\psi}$$

- $$\langle H\rangle = \int \psi^*\psi \langle H\psi\rangle\, dx$$

$$= E\psi \int \psi^*\psi$$

$$= E\int_{-\infty}^{+\infty} \psi^*\psi$$

$$\boxed{= E}$$

$$\langle H^2 \rangle = \int \psi^* \psi H^2 \psi \, dx$$

$$= \int \psi^* \psi [H \cdot H \cdot \psi] \, dx$$

$$= \int \psi^* \psi [H (H\psi) \, dx]$$

$$= \int \psi^* \psi [E (E\psi) \, dx]$$

$$= \int \psi^* \psi [E^2 \psi] dx$$

$$= E^2 \psi \int \psi^* \psi \, dx$$

$$= E^2.$$

$$\sigma = \langle E^2 \rangle - \langle E \rangle^2$$

$$= 0$$

[redacted]