



Synthetic Linking
Functions

Gorney (2021)

Background

Purpose

Method

Results &
Discussion

Differences in Test
Form Difficulties

Differences in Group
Abilities

Conclusion

References

When to Use Synthetic Linking Functions in Small-Sample Equating

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Random Equating Error

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- Whenever samples are used to estimate an equating relationship, **random equating error** is present
- Especially concerning in small-sample situations
- Minimize by using the **identity function**, which is analogous to not equating at all
 - The standard error of equating (SEE) is zero
 - Produces very biased results when test forms differ markedly in difficulty



Synthetic Linking Functions

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A **synthetic linking function** computes a weighted average of the identity function and a second equating function (Kim, von Davier, & Haberman, 2008).

$$syn_Y(x) = we_Y(x) + (1 - w)id_Y(x) \quad (1)$$

where w is a weight between 0 and 1, e is an equating function other than the identity function, and id is the identity function.



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The purpose of this study was to identify which, if any, small-sample situations are best handled by synthetic equating.

- Treated the difference in test form difficulties as a random effect
- Treated the difference in group abilities as a random effect
- Used weights that satisfy the symmetry property of equating (Holland & Strawderman, 2011)



Data

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- Simulated a 100-item test containing 30 anchor items
- Sample sizes: $N = 10, 25, 50, 100$
- 1,000 replications



Equating Design and Methods

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- Common-item nonequivalent groups design
- Equating methods
 - ➊ Nominal weights mean (NM)
 - ➋ Tucker mean (TM)
 - ➌ Tucker linear (TL)
 - ➍ Chained linear (CL)



Differences in Test Form Difficulties

Bias

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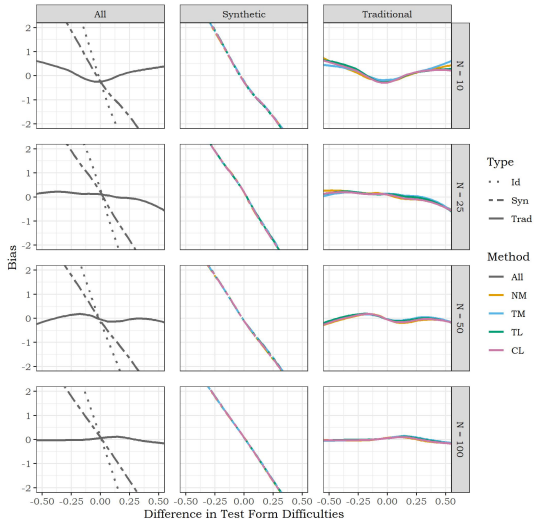
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- Identity and synthetic equating were very biased when the test forms differed even slightly in difficulty
- Traditional equating displayed small amounts of bias overall





Differences in Test Form Difficulties

Root Mean Squared Error (RMSE)

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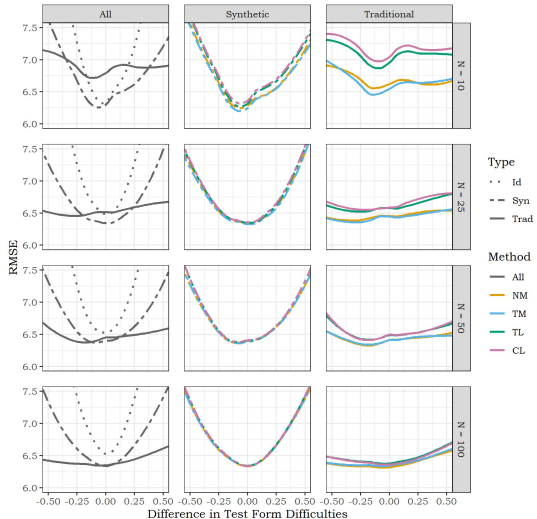
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- Synthetic equating preferred when the test forms were similar in difficulty and sample sizes were small
- Traditional equating preferred in all other cases
- NM and TM equating tended to yield the lowest RMSEs





Differences in Group Abilities

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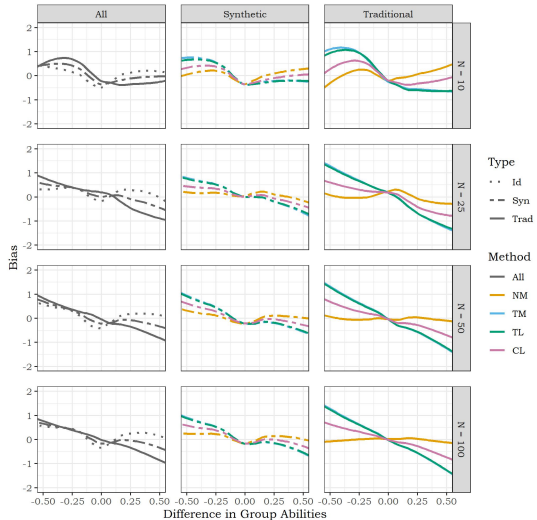
Differences in Test Form Difficulties

Differences in Group Abilities

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- When groups were similar in ability, all equating types yielded similar amounts of bias
- Traditional equating displayed more bias in the extreme cases
- NM equating was the least biased, on average





Differences in Group Abilities

Root Mean Squared Error (RMSE)

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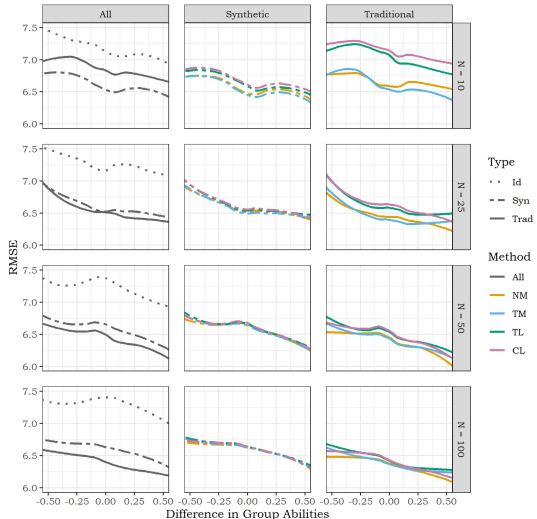
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- Synthetic equating preferred when the sample size was 10
- Traditional equating preferred for larger sample sizes
- NM and TM equating tended to yield the lowest RMSEs





Conclusion

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- Synthetic equating may be considered when the sample size is 25 or smaller and when it is known that the test forms do not differ markedly in difficulty
- In all other cases, use traditional equating functions
- For very small samples, NM and TM equating tended to be the most effective
- Future research: different test or group characteristics, weights, and equating methods



Thank you!

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For more information, please contact kyliengorney@gmail.com.



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- Kim, S., von Davier, A. A., & Haberman, S. (2008). Small-sample equating using a synthetic linking function. *Journal of Educational Measurement*, 45(4), 325–342.