CS 430 Introduction to Algorithms **Fall Semester, 2016**

Homework 6

Assigned: Nov 16 *Due: Nov. 30*

Note: Section 01: the hard copy must be submitted no later than the start of the final exam.

Please respect the following guidelines for writing pseudocode:

1. C instructions are fine. But do not write object-oriented additions. Do not declare or use any class. Declare only procedures (if necessary) and explain in words what each procedure does, and what is the use of each parameter.
2. One instruction per line
3. Match the brackets with a horizontal line
4. Number your lines
5. Write down if your array is indexed $0 \ldots n-1$ or $1 \ldots n$.

Problem 1 Present full pseudocode of a variant of Prim's algorithm that runs in time $O(|V|^2)$ for a graph $G = (V, E)$ given by an adjacency matrix A. Analyze the running time.

Problem 2 Give an example of a weighted directed graph $\vec{G}$ with negative-weight edges, but no negative-weight cycle, such that Dijkstra's algorithm incorrectly computes the shortest-path distances from some start vertex v. Use the algorithm version from the handout.

A four-vertex example is possible. Draw the graph, mention the start vertex, show the result of Dijkstra's algorithm, and point out for which vertex the result is incorrect.

(This was on a previous final exam)

Problem 3 A *complete digraph* has exactly one directed edge (also called *arc*) from every vertex u to every vertex v other than itself. Let G be a complete digraph with non-negative arc weights. Let the *capacity* of a path be the *minimum* arc weight along it, and let the *capacity* of a pair of nodes (u, v) be the *maximum* capacity of a path from u to v. Find a Dijkstra-like algorithm to find, for all $v \neq s$, the capacity of (s, v). (Node s is a fixed source.)

Present the pseudocode, analyze the running time, and prove correctness.

Problem 4 Exercise 25.2-1 on page 699 of the textbook. It has the same number (but different page) in the second edition of the book.