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Bi 195 Set 1

1.

a) h = heart c = club s = spade d = diamond $A, 2, \ldots, K = 1, 2, \ldots, 13$

$$\mathcal{X} = \left\{ \begin{matrix} h1, h2, \ldots, h13, c1, c2, \ldots, c13, \\ s1, s2, \ldots, s13, d1, d2, \ldots, d13 \end{matrix} \right\}$$ A through K for each suit

$P(x) = 1/52$ for $x \in \mathcal{X}$

b) r = red b = black

$\mathcal{X} = \{r, b\}$ $P(X = x) = 1/2$ $P(r) = 1/2$ $P(b) = 1/2$

2.

a) 0 = does not fire 1 = fires once

single neuron $\mathcal{X} = \{0, 1\}$ $|\mathcal{X}| = 2$

$0, 1, \ldots, m$ = number of neurons that fire

m neuron network $\mathcal{X} = \{0, 1, \ldots, m\}$ $|\mathcal{X}| = m+1$

$|\mathcal{X}|$ is the number of neurons we are looking at plus 1

b)

for a single neuron, we know that we select a time interval such that it fires max once. However, because neurons fire with some probability, we will just have some probability $P(x) = p$ that the neuron will fire in the time interval.

for a network of m neurons, we have each neuron having some probability to fire, giving us a binomial-looking distribution of neurons fired. Assuming equal neuron probabilities, we would get $P(x) = \binom{m}{x} p^x (1-p)^m$

3.

a) m trials for m heads

$$P_m = \binom{m}{m} 0.5^m \, 0.5^{m-m} = 0.5^m$$

b) 10 trials for 8 heads

$$P_8 = \binom{10}{8} 0.5^8 \, 0.5^2 = 45 \cdot 0.5^{10} = 0.0439$$

c) 50 trials for 8 heads

$$P_8 = \binom{50}{8} 0.5^8 \, 0.5^{42} = \binom{50}{8} 0.5^{50} = 4.768 \cdot 10^{-7}$$

4.

coefficient of variation $\frac{\sigma}{\mu}$, Fano factor $\frac{\sigma^2}{\mu}$

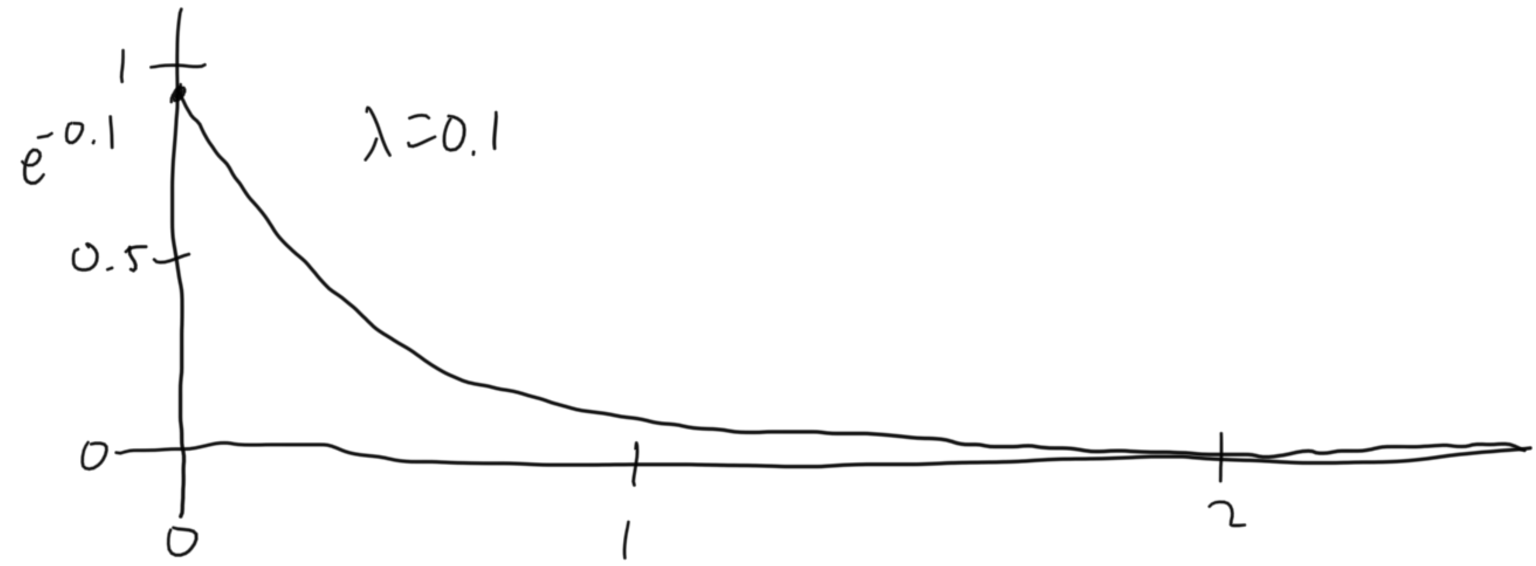
$\mu = np \quad \sigma^2 = np(1-p) \quad \sigma = \sqrt{np(1-p)}$

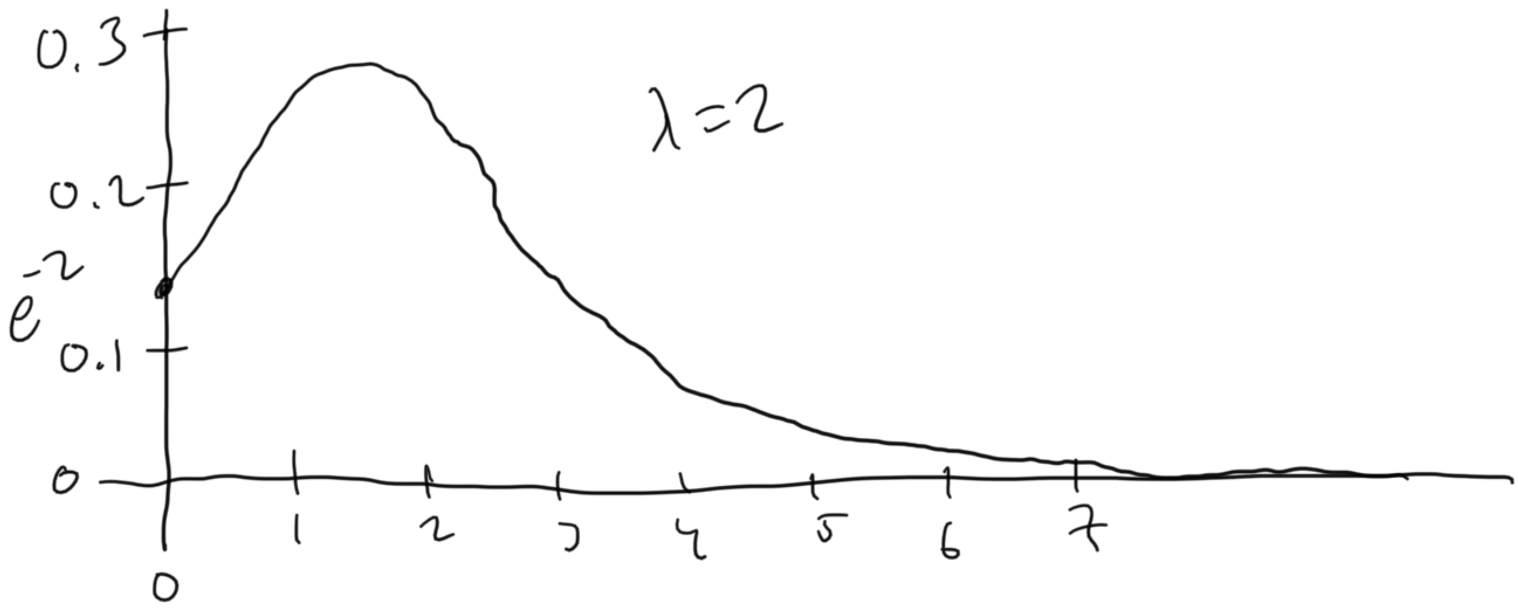
$$\frac{\sigma}{\mu} = \frac{\sqrt{np(1-p)}}{np} = \sqrt{\frac{1-p}{np}} \qquad \frac{\sigma^2}{\mu} = \frac{np(1-p)}{np} = 1-p$$

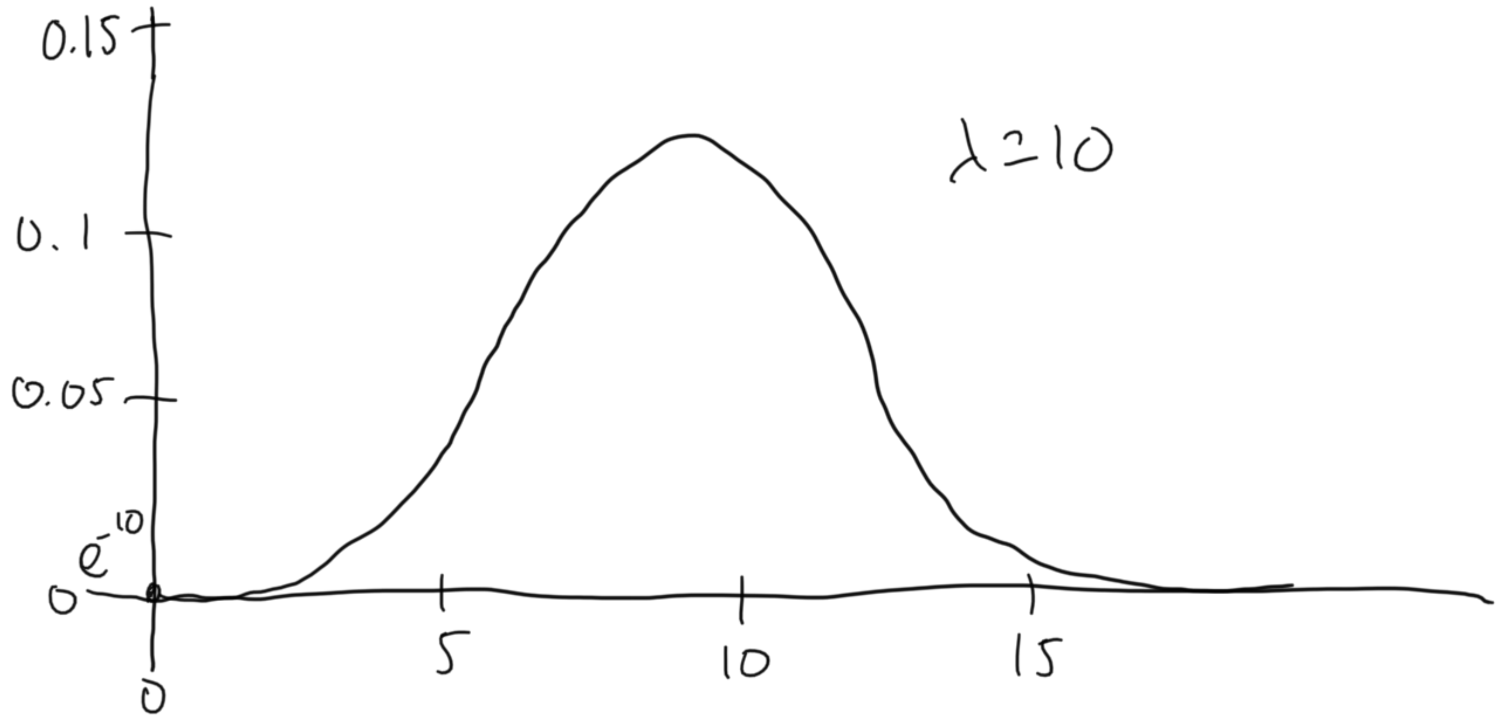
5. $P(x) = \frac{\lambda^x e^{-\lambda}}{x!}$

a) $P(0) = \frac{\lambda^0 e^{-\lambda}}{0!} = e^{-\lambda}$

b) $P(x) = \frac{0.1^x e^{-0.1}}{x!}, \frac{2^x e^{-2}}{x!}, \frac{10^x e^{-10}}{10!}$







c) $\langle x \rangle = E[x] \quad \langle x^2 \rangle = E[x^2]$

$E[x^n] = g^{(n)}(0)$ $\quad g(s)$ is moment-generating function

$$g(s) = \sum_x P(x) e^{sx} = \sum_x \frac{\lambda^x e^{-\lambda}}{x!} e^{sx} = e^{-\lambda} \sum_x \frac{(\lambda e^s)^x}{x!}$$

$$= e^{-\lambda} e^{\lambda e^s} = e^{\lambda(e^s-1)}$$

$$g'(s) = \frac{d}{ds} e^{\lambda(e^s-1)} = e^{\lambda(e^s-1)} \frac{d}{ds} \lambda(e^s-1) = e^{\lambda(e^s-1)} \lambda e^s$$

$$g''(s) = \frac{d}{ds} e^{\lambda(e^s-1)} \lambda e^s = e^{\lambda(e^s-1)} \lambda^2 e^{2s} + e^{\lambda(e^s-1)} \lambda e^s = e^{\lambda(e^s-1)} \lambda e^s (\lambda e^s + 1)$$

$$E(x) = g'(0) = e^{\lambda(1-1)} \lambda e^0 = \lambda$$

$$E[x^2] = g''(0) = e^{\lambda(1-1)} \lambda e^0 (\lambda e^0 + 1) = \lambda(\lambda+1) = \lambda^2 + \lambda$$

$$\mu = E[x] = \lambda$$

$$\sigma^2 = E[(x - E[x])^2] = E[x^2] - E[x]^2 = \lambda^2 + \lambda - \lambda^2 = \lambda$$

6. $\mathcal{X} = \{1, \dots, n\} \quad P(x) = 1/n$

$$H[P(x)] = E[-\log(P(x))] = \sum_{x \in \mathcal{X}} -\log(P(x)) \cdot P(x) = \sum_{x=1}^{n} -\log(1/n) \cdot 1/n$$

$$= n \cdot (-\log(1/n)\, 1/n) = -\log(1/n) = \log(n)$$