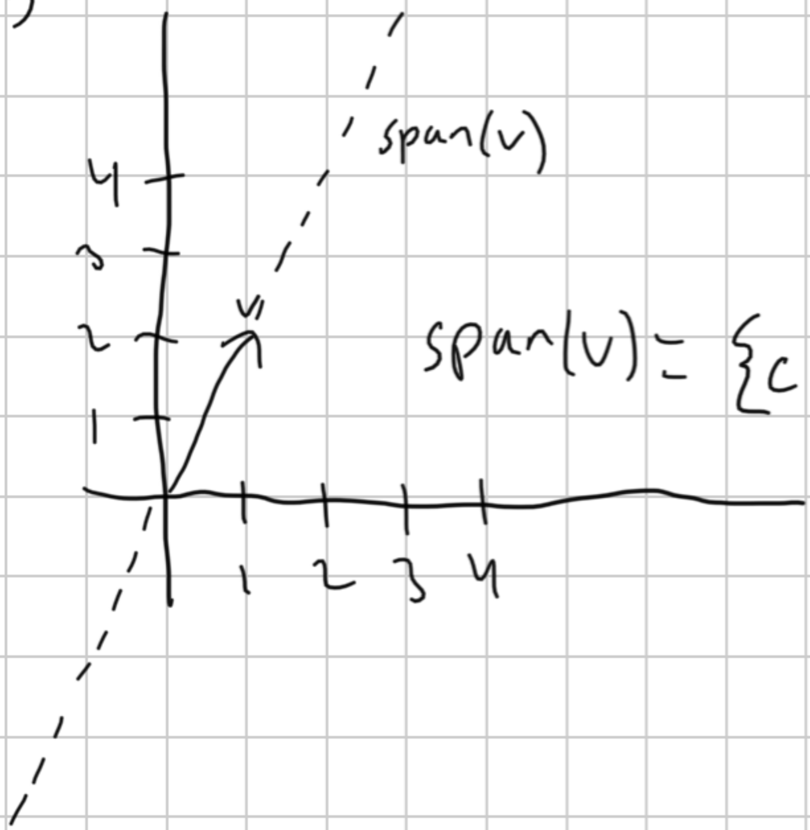
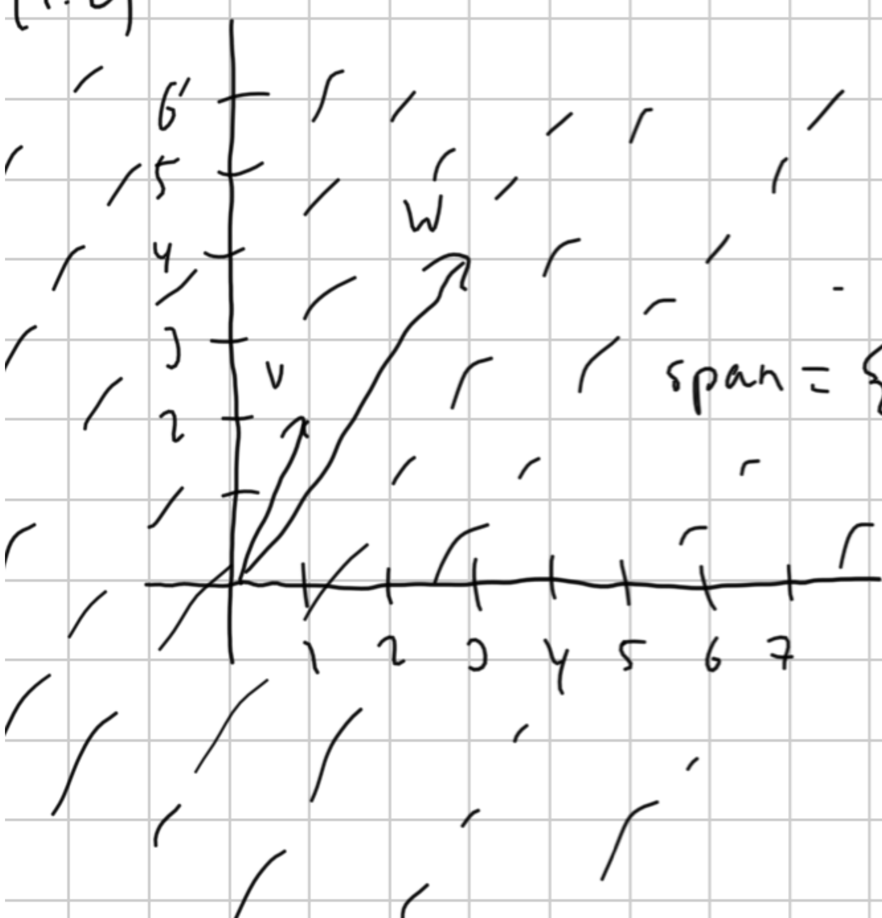
Problem 1

(1.1)



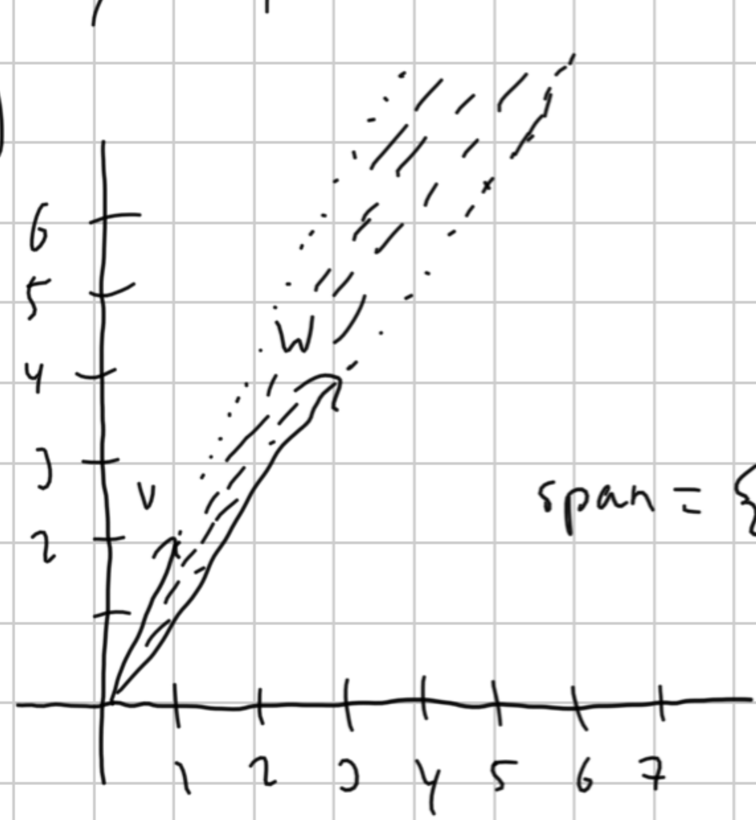
$\text{span}(v) = \{c(1,2), c \in \mathbb{R}$

(1.2)



$\text{span} = \{c_1 v + c_2 w, c_1, c_2 \in \mathbb{R}\}$

(1.3)



span $= \{c_1 v + c_2 w, c_1, c_2 \in \mathbb{R}^+\}$

Problem 2

(2.1)

The vectors (1, 1), (3, 4), and (−2, 2) are not linearly independant because we can see that the vector (-2,2) can be represented as $4 * (3, 4) - 14 * (1, 1) = (12, 16) - (14, 14) = (-2, 2)$.

(2.2)

The vectors (1, 8), (−1, −8) are not linearly independent because $(1, 8) = -1 * (-1, -8)$.

(2.3)

Two different sets of vectors that span $V = \{ax + bx^2, a, b \in \mathbb{R}\}$ are $\{x, x^2\}$ and $\{x, x + x^2\}$.

Problem 3

(3.1)

Given a set of m vectors that span $\mathbb{R}^n$, we can find n linearly independent vectors by checking each subset of n vectors of the set of m. For each subset of n vectors, we can build a matrix of the vectors and calculate the determinant of this matrix. If the determinant of the matrix is not equal to 0 then we know we have a set of n linearly independent vectors.

(3.2)

If we draw n vectors from V_n, the set of selected vectors will not always be independent. We prove this using the example of $n = 3$; we can see that if we have the vectors $(1,0,0), (1,1,0), (0,1,1)$ then the third vector can not be made from any linear combination of the other two vectors (we can see that the last part of the vector cannot be created from the last part of the two others).

(3.3)

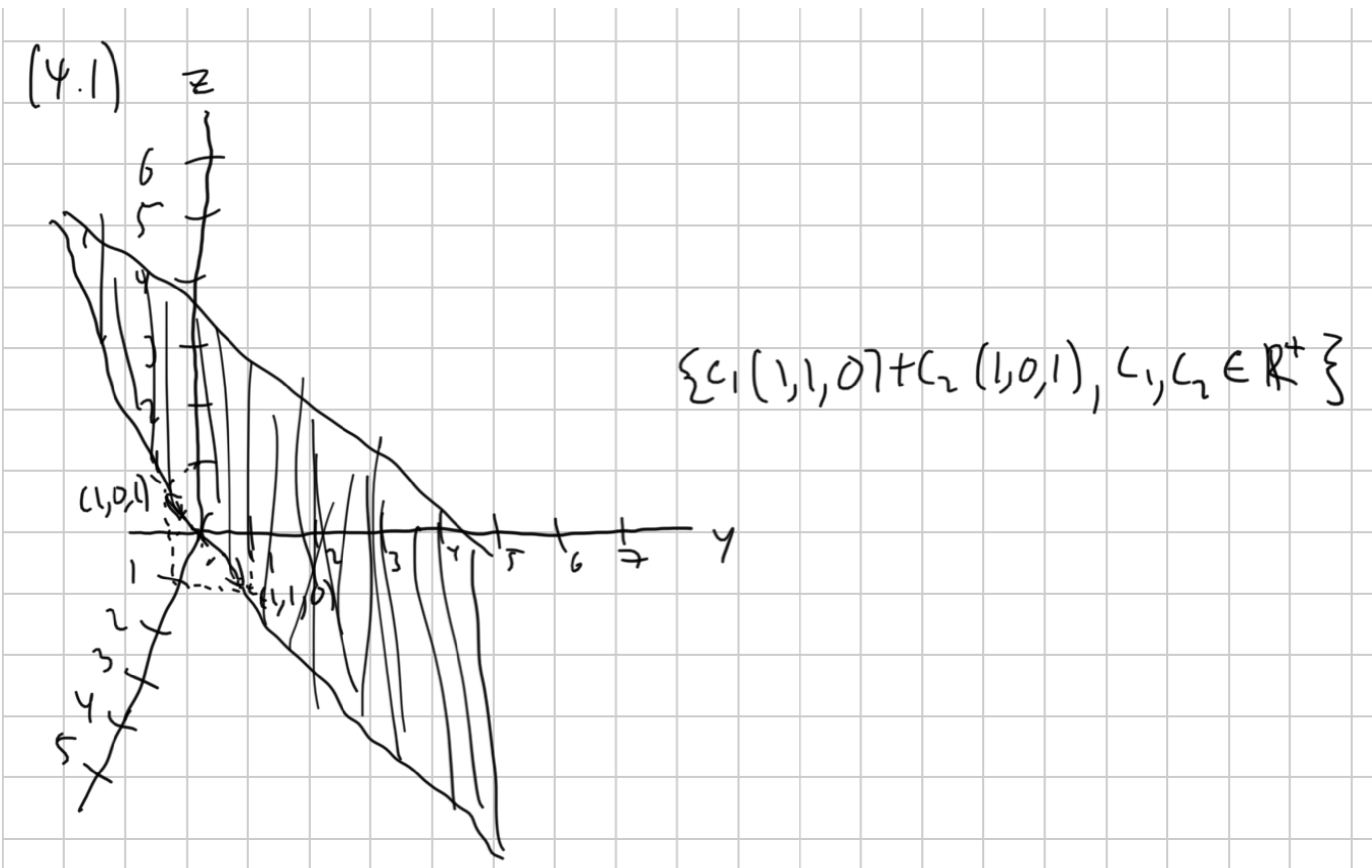
The determinant was 0 becuase the the vectors are not linearly independent; we can see that the third vector is a linear combination of the first two vectors.

(3.4)

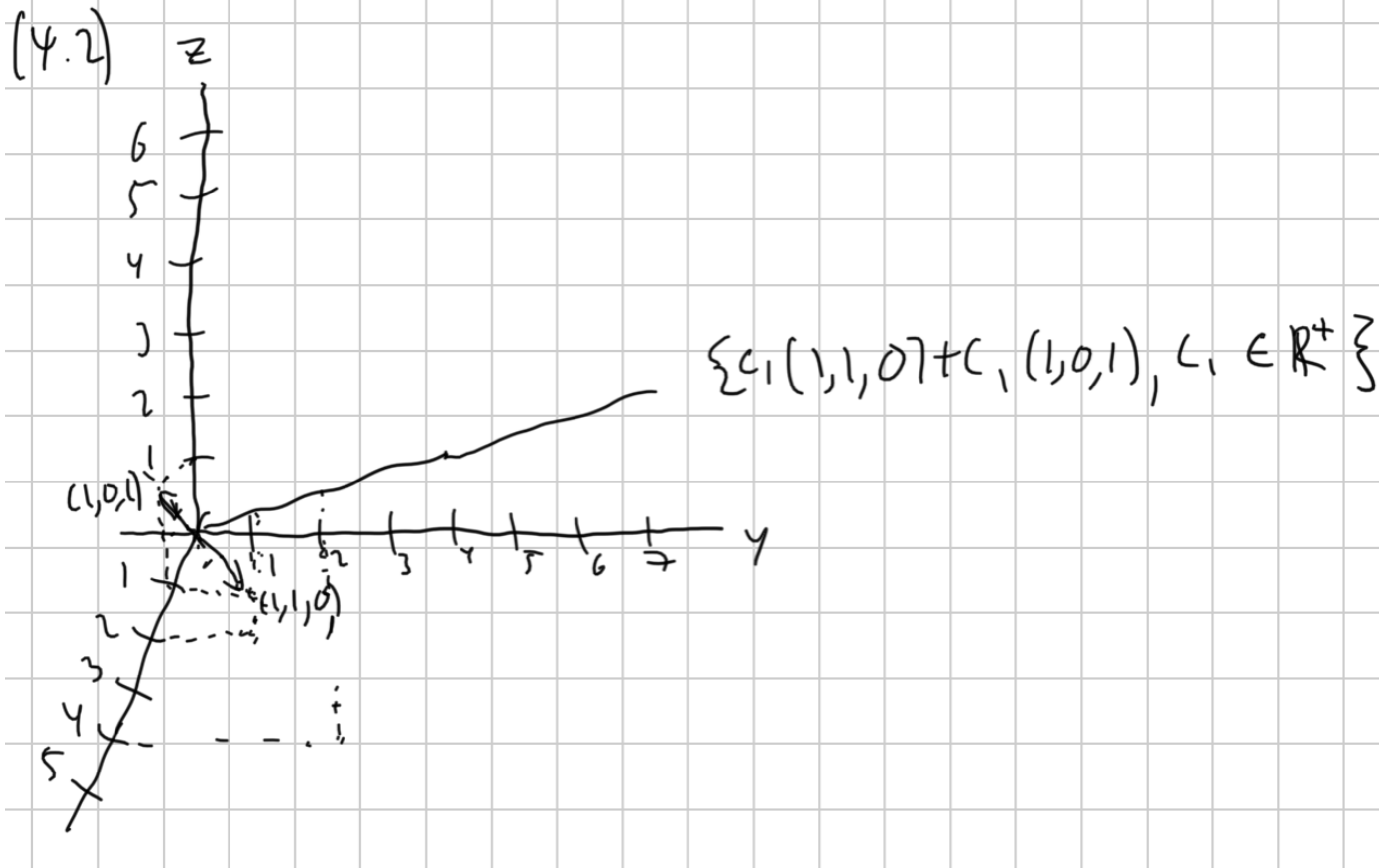
We calculated the fraction of matrices that have a 0 determinant for every n by n matrix from n=1 to n=21. This plot tells us that random binary matrices at low sizes have a fairly high fraction of matrices that have 0 determinants. As matrix size increases, the fraction of matrices that have 0 determinants decreases. Because if the matrix has a 0 determinant than the vectors are not linearly independent, this graph also shows the fraction of sets of selected vectors that are not linearly independent if we draw n vectors from V_n.

Problem 4

(4.1)



(4.2)



Problem 5

(A)

I select a vector w to be (2,7) and basis vectors v and u to be (2,1) and (-1,1).

(B)

$A = (v, u),\ c = (c_1 c_2),\ c_1 v + c_2 u = w$ so $Ac = w$.

(C)

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 0.33333333 & 0.33333333 \\ -0.33333333 & 0.66666667 \end{bmatrix}$$

(D)

$$\text{np.dot(A, Ainv)} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\text{np.dot(Ainv, A)} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

We are multiplying the matrix by its inverse so we get the identity matrix.

(E)

$c = \begin{bmatrix} 3 & 4 \end{bmatrix}$, $3 * (2,1) + 4 * (-1,1) = (2,7)$.

(F)

We have our vectors, v, u, and w, as well as our computed vector, c, that represents how much we have to scale v and u to get w. We also have the scaled v and u vectors $c_1 v$ and $c_2 u$, respectively.