EEN 1043/EE452 Wireless and Mobile Communication

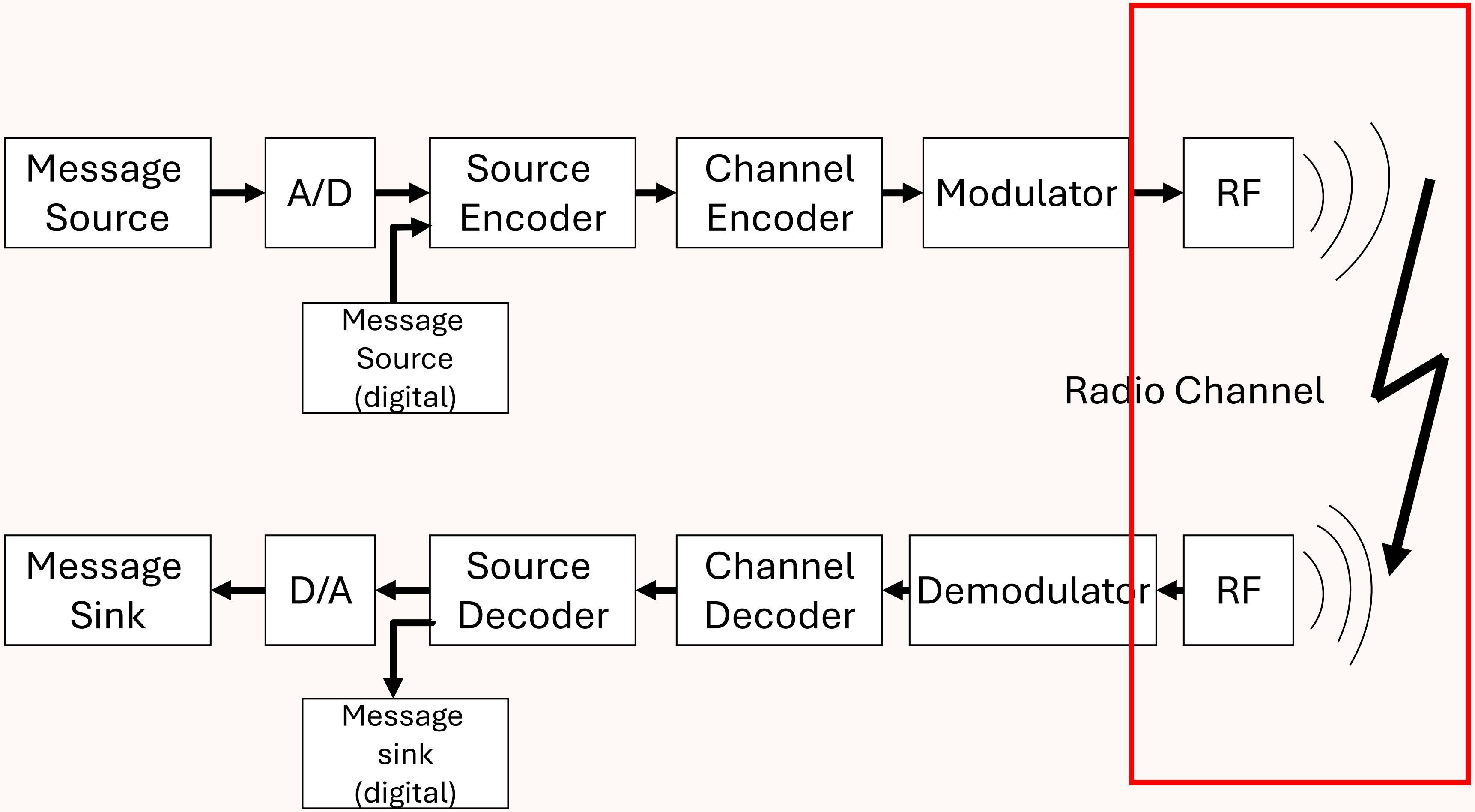
The Wireless Channel IV

Sobia Jangsher

Assistant Professor

School of Electronic Engineering

Radio Link


Message Source
A/D
Source Encoder
Channel Encoder
Modulator
RF
Message Source (digital)
Radio Channel
Message Sink
D/A
Source Decoder
Channel Decoder
Demodulator
RF
Message sink (digital)

Channel

- Channel adds noise (Additive White Gaussian Noise)

$$y(t) = x(t) + n(t)$$

- Channel delays the signal

$$y(t) = x(t - \tau) + n(t)$$

- Channel attenuates the signal

$$y(t) = hx(t - \tau) + n(t), h \boldsymbol{\alpha} \frac{\lambda}{d}$$

- Channel rotates the signal (add phase)

$$y(t) = hx(t - \tau) + n(t), h \boldsymbol{\alpha} \frac{\lambda}{d} e^{j\phi}$$

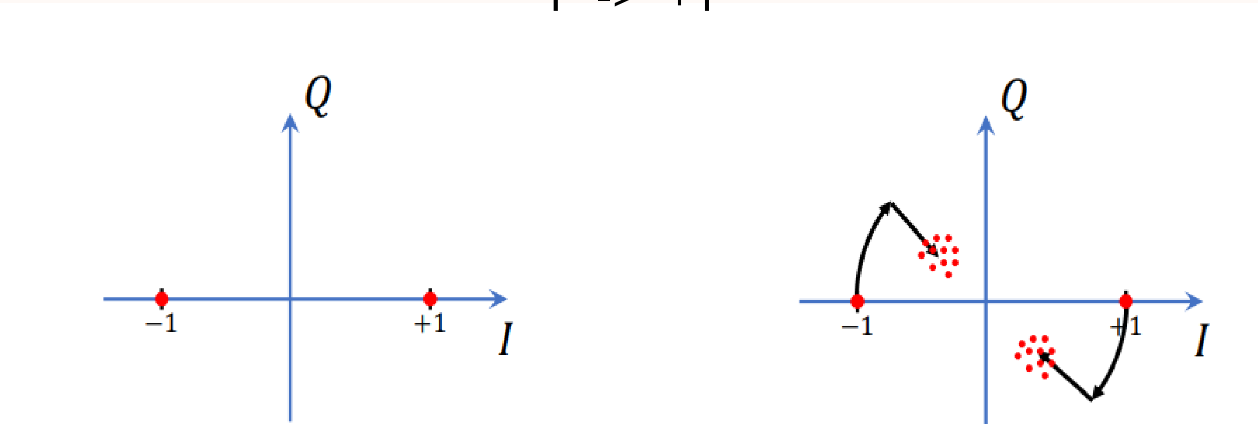
Channel Estimation and Correction

- Send training sequence (known bits)
- $x(0) = 1 \Rightarrow y(0) = 1 + \mathrm{n}(0)$
- $x(1) = 1 \Rightarrow y(1) = 1 + \mathrm{n}(1)$
- $x(2) = -1 \Rightarrow y(2) = -1 + \mathrm{n}(2)$

Consider BPSK modulation

0 -> -1

1 -> +1



$y(t) = h * x(t) + n(t)$

Estimate Channel

$$\tilde{h} = \sum_k \frac{y(k)}{x(k)}$$

Channel Correction

$$\tilde{x}(t) = \frac{y(k)}{\tilde{h}}$$

Channel Estimation

$$y = hx + n$$

- What is the known at the receiver
- $\boxed{y} = \boxed{h}\boxed{x} + \boxed{n}$

- Assume noiseless channel or high SNR
- Estimate channel h=> $\tilde{h}$

Channel Estimation Using Pilot Symbols

- $y = hx$
- Find $\tilde{h}$
- A set of pilot symbols x_p is generated and transmitted on the channel
- Pilot symbol is known at both transmitter and receiver
- Now how many unknown in $y = hx$
- Assume transmission of 1 pilot symbol
- $\frac{y_1}{x_p} = \frac{h_1 x_p}{x_p} + \boxed{\frac{n_1}{x_p}}$

Error in channel estimation

Channel Estimation Quality

- Mean Square Error (MSE)
- Error is $\frac{n_1}{x_p}$
- MSE $= ||\widetilde{h_1} - h_1||^2$
- MSE$= \frac{||n_1||^2}{\left|\left|x_p\right|\right|^2}$
- MSE$= \frac{||n_1||^2}{\left|\left|x_p\right|\right|^2} = \frac{noise\ power}{signal\ power}$
- MSE$= \frac{||n_1||^2}{\left|\left|x_p\right|\right|^2} = \frac{noise\ power}{signal\ power} = \frac{1}{SNR}$

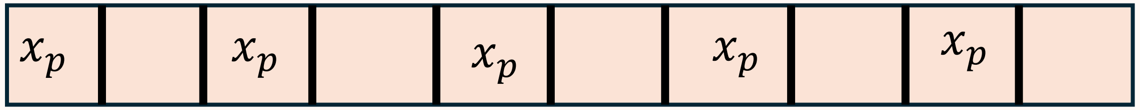
Channel Estimation Quality

- Assume transmission of 1 pilot symbol
- $\frac{y_1}{x_p} = \frac{h_1 x_p}{x_p} + \frac{n_1}{x_p}$
- $\frac{y_2}{x_p} = \frac{h_2 x_p}{x_p} + \frac{n_2}{x_p}$
- Error in two symbols = $\frac{n_1}{x_p} + \frac{n_2}{x_p}$
- Avg. error per symbol = $\frac{n_1}{2x_p} + \frac{n_2}{2x_p}$

Channel Estimation Quality

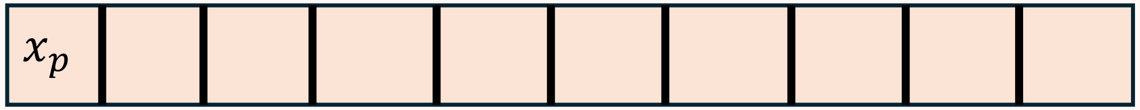
- How many times you have to estimate your channel?
- Assume you have N time slots to transmit

- A)



Wastage of resources
Better channel estimation

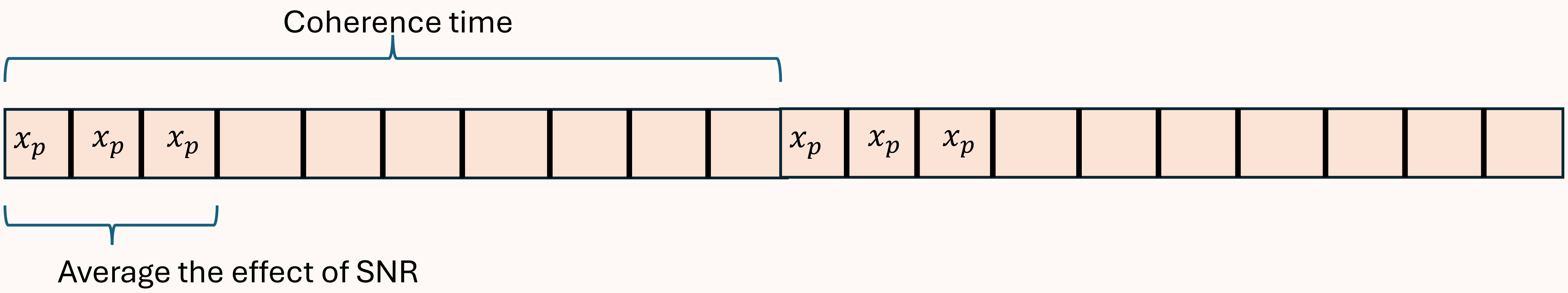
- B)



Less resources are required
Channel estimation might not be good

Channel Estimation Structure

- Depends on
  - SNR
  - Channel Coherence time



Channel Estimate

- Assume that a channel is constant for 10 symbols
- We can transmit channel with 3 symbols

x_p	x_p	x_p							

- $y_1 = hx_1 + n_1$
- $y_2 = hx_2 + n_2$
- $y_3 = hx_3 + n_3$

$$y_1 = hx_1 + n_1 \Rightarrow \frac{y_1}{x_1} = h + \frac{n_1}{x_1}$$

$$y_2 = hx_2 + n_2 \Rightarrow \frac{y_2}{x_2} = h + \frac{n_2}{x_2}$$

$$y_2 = hx_3 + n_3 \Rightarrow \frac{y_3}{x_3} = h + \frac{n_3}{x_3}$$

Least Square Channel Estimate

- $\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = h \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix} = \vec{y} = h\vec{x} + \vec{n}$

- Three equation, one unknown h,
- Solution may or may not exist
- Find h which best fits the above equations, estimate h such that the error is minimized
- $\arg\min_{h} || \vec{y} - h\vec{x}||^2$

Least Square Channel Estimate

- $\arg\min_{h} || \vec{y} - h\vec{x}||^2$
- How do you find a minima
- Derivative, put it equal to 0 and find h
- Expand $||\vec{y} - h\vec{x}||^2 = \left||\vec{y}|\right|^2 + 2\vec{x}^T h\vec{y} + h^2\ ||\vec{x}||^2$
- Differentiate w.r.t h and equate to 0 $\quad 0 + 2\vec{x}^T\vec{y} + 2h\ ||\vec{x}||^2 = 0$
- $h^* = \frac{\vec{x}^T\vec{y}}{x^T x} = (x^T x)^{-1}\ \vec{x}^T\vec{y}$

$$\boxed{h^* = (x^T x)^{-1}\ \vec{x}^T\vec{y}}$$