CSE 250A HW 4

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1.(a) $\mathcal{L} = \log P(data) = \log \prod_{d=1}^{2D} P_d^{C_d} = \sum_{d=1}^{2D} C_d \log P_d$.

(b) max. $\mathcal{L}$, s.t. $\sum P_d = 1$, $P_d \geq 0$.

$$Lag(P_1, \cdots, P_{2D}, \lambda) = \sum_{d=1}^{2D} C_d \log P_d + \lambda\left(\sum_{d=1}^{2D} P_d - 1\right)$$

$$\frac{\partial Lag}{\partial P_i} = \frac{C_i}{P_i} + \lambda = 0, \quad \frac{\partial Lag}{\partial \lambda} = \sum_{d=1}^{2D} P_d - 1 = 0.$$

↳ implies $\frac{C_1}{P_1} = \cdots = \frac{C_{2D}}{P_{2D}}$,

$\sum P_d = 1 \Rightarrow \hat{P}_i = \frac{C_i}{\sum_{i=1}^{2D} C_i} \geq 0$

(c) $P(X_{even}) = P_2 + P_4 + \cdots + P_{2D}$, $P(X_{odd}) = P_1 + P_3 + \cdots + P_{2D-1}$

$P(X_{even}) = P(X_{odd}) \Leftrightarrow -P_1 + P_2 - P_3 + \cdots - P_{2D+1} + P_{2D} = 0$

$\Leftrightarrow \sum_{d=1}^{2D} (-1)^d P_d = 0$

(d) We have one more constraint, so

$$Lag(P_1, \cdots, P_{2D}, \lambda, \mu) = \sum_{d=1}^{2D} C_d \log P_d + \lambda\left(\sum_{d=1}^{2D} P_d - 1\right) + \mu\left(\sum_{d=1}^{2D} (-1)^d P_d\right)$$

$$\frac{\partial Lag}{\partial P_i} = \frac{C_i}{P_i} + \lambda + (-1)^i \mu = 0, \quad \frac{\partial Lag}{\partial \lambda} = \sum P_d - 1 = 0, \quad \frac{\partial Lag}{\partial \mu} = \sum (-1)^d P_d = 0.$$

For odd i, $\frac{C_i}{P_i} + \lambda - \mu = 0$; for even i, $\frac{C_i}{P_i} + \lambda + \mu = 0$.

then $\frac{C_1}{P_1} = \frac{C_3}{P_3} = \cdots = \frac{C_{2D-1}}{P_{2D-1}}$, $\frac{C_2}{P_2} = \frac{C_4}{P_4} = \cdots = \frac{C_{2D}}{P_{2D}}$.

Since $\sum_{i\,odd} P_i = \sum_{i\,even} P_i$, we have $P_i = \begin{cases} \frac{C_i}{2C_{odd}}, & \text{if } i \text{ odd} \\ \frac{C_i}{2C_{even}}, & \text{if } i \text{ even} \end{cases}$

then $\sum_{i\,odd} P_i = \sum_{i\,even} P_i = \frac{1}{2}$.

2.(a) $P_{ML}(X_1 = x) = \frac{Count_1(X)}{T}$, $i \geq 1$: $P_{ML}(X_{i+1} = x' \mid X_i = x) = \frac{Count_i(X, X')}{count_i(X)}$

(b) $P_{ML}(X_n = x) = \frac{Count_n(X)}{T}$, $i < n$: $P_{ML}(X_i = x \mid X_{i+1} = x') = \frac{Count_i(X, X')}{count_{i+1}(X')}$

(c) $P_{G_1}(X_1 = x_1, \cdots, X_n = x_n) = P(X_1 = x_1) P(X_2 = x_2 \mid X_1 = x_1) \cdots P(X_n = x_n \mid X_{n-1} = x_{n-1})$

$$= \frac{Count_1(X_1)}{T} \prod_{i=1}^{n-1} \frac{Count_i(X_i, X_{i+1})}{Count_i(X_i)} = \frac{\cancel{Count_1(X_1)}}{T} \frac{Count_1(X_1, X_2)}{\cancel{Count_1(X_1)}} \cdots \frac{Count_{n-1}(X_{n-1}, X_n)}{Count_{n-1}(X_{n-1})}$$

$P_{G_2}(X_1 = x_1, \cdots, X_n = x_n) = P(X_n = x_n) P(X_{n-1} = x_{n-1} \mid X_n = x_n) \cdots P(X_1 = x_1 \mid X_2 = x_2)$

$$= \frac{Count_n(X_n)}{T} \prod_{i=1}^{n-1} \frac{Count_i(X_i, X_{i+1})}{Count_{i+1}(X_{i+1})} = \frac{\cancel{Count_n(X_n)}}{T} \frac{Count_1(X_1, X_2)}{Count_2(X_2)} \cdots \frac{Count_{n-1}(X_{n-1}, X_n)}{\cancel{Count_n(X_n)}}$$

So P_{G_1}, P_{G_2} have the same joint distribution

(d) Since $P(X_{n-1} \mid X_{n-2}) \neq P(X_{n-1} \mid X_{n-2}, X_{n-3})$ 「d.sep(3) fails」, then we can not expand the joint probability as conditional only on one previous node.

CSE 250A HW4 Code

October 28, 2021

[1]:
```python
import numpy as np
import matplotlib.pyplot as plt


def load_data():
    word = []
    uni = []
    bi = {}
    word_path = "hw4_vocab.txt"
    uni_path = "hw4_unigram.txt"
    bi_path = "hw4_bigram.txt"
    with open(word_path, "r") as w, \
            open(uni_path, "r") as u, \
            open(bi_path, "r") as b:
        for line in w.readlines():
            line = line.split("\n")
            word.append(line[0])
        for line in u.readlines():
            line = line.split("\n")
            uni.append(int(line[0]))
        for line in b.readlines():
            line = line.strip().split("\t")
            tup = (int(line[0]) - 1, int(line[1]) - 1)
            bi[tup] = int(line[2])
        return word, uni, bi


words, unigram, bigram = load_data()
total_words = 0
for count in unigram:
    total_words += count


# Part A MLE
def unigram_mle(word):
    return unigram[word] / total_words
```

```python
# Part B MLE
def bigram_mle(prev, curr):
    if (prev, curr) not in bigram.keys():
        return 0
    return bigram[(prev, curr)] / unigram[prev]


def unigram_prob(sentence):
    sentence = sentence.split(" ")
    for i in range(len(sentence)):
        if sentence[i] not in words:
            sentence[i] = "<UNK>"
    uni = 1
    for i in range(1, len(sentence) - 1):
        uni *= unigram_mle(words.index(sentence[i]))
    return np.log(uni)


def bigram_prob(sentence):
    sentence = sentence.split(" ")
    for i in range(len(sentence)):
        if sentence[i] not in words:
            sentence[i] = "<UNK>"
    bi = 1
    for i in range(1, len(sentence) - 1):
        prob = bigram_mle(words.index(sentence[i - 1]), words.index(sentence[i]))
        if prob == 0:
            print("Not observed: " + sentence[i - 1] + " " + sentence[i])
        bi *= prob
    return np.log(bi)


def mix_mle(lam, prev, curr):
    return lam * unigram_mle(curr) + (1 - lam) * bigram_mle(prev, curr)


def mix_prob(lam, sentence):
    sentence = sentence.split(" ")
    for i in range(len(sentence)):
        if sentence[i] not in words:
            sentence[i] = "<UNK>"
    mix = 1
    for i in range(1, len(sentence) - 1):
        mix *= mix_mle(lam, words.index(sentence[i - 1]), words.
↪index(sentence[i]))
    return np.log(mix)
```

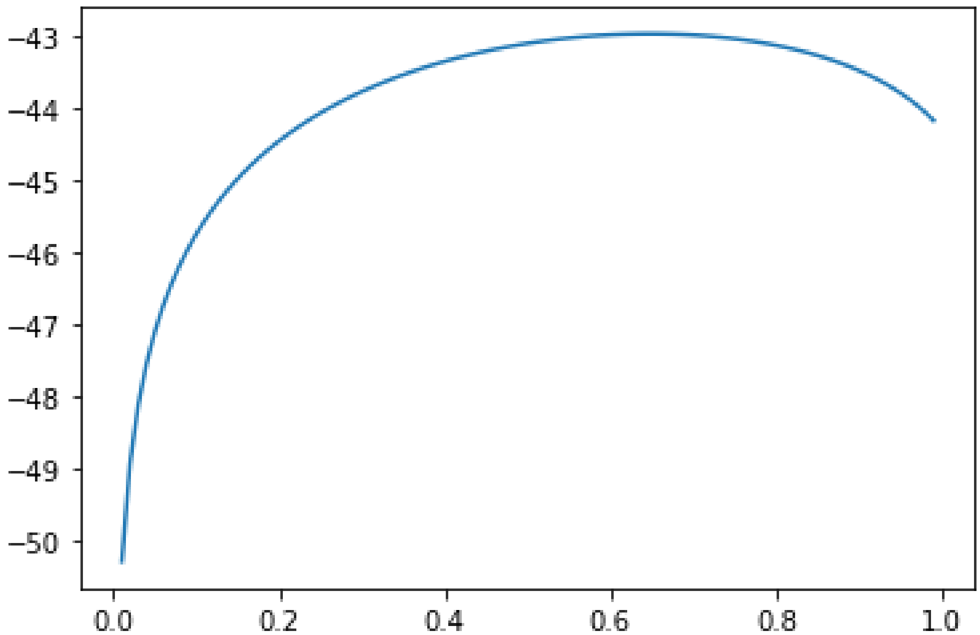
```python
def plot(sentence):
    lambda_range = np.linspace(0.01, 0.99, 100)
    mixture = []
    dic = {}
    for i in range(len(lambda_range)):
        mixture.append(mix_prob(lambda_range[i], sentence))
        dic[lambda_range[i]] = mixture[i]
    dic = sorted(dic.items(), key=lambda x: x[1], reverse=True)
    plt.plot(lambda_range, mixture)
    plt.show()
    return dic


def main():
    # 4.3.a
    print("4.3.a")
    part_a = {}
    for i in range(len(words)):
        if words[i][0].upper() == 'M':
            part_a[words[i]] = unigram[i] / total_words
    print(part_a)

    # 4.3.b
    print("4.3.b")
    part_b = {}
    k = words.index("THE")
    for key in bigram.keys():
        if key[0] == k:
            part_b[words[key[1]]] = bigram_mle(key[0], key[1])
    part_b_sorted = sorted(part_b.items(), key=lambda x: x[1], reverse=True)[:10]
    print(part_b_sorted)

    # 4.3.c
    print("4.3.c")
    sentence_c = "<s> THE STOCK MARKET FELL BY ONE HUNDRED POINTS LAST WEEK </s>"
    print("Unigram: ", unigram_prob(sentence_c))
    print("Bigram: ", bigram_prob(sentence_c))

    # 4.3.d
    print("4.3.d")
    sentence_d = "<s> THE SIXTEEN OFFICIALS SOLD FIRE INSURANCE </s>"
    print("Unigram: ", unigram_prob(sentence_d))
    print("Bigram: ", bigram_prob(sentence_d))
    # This makes the log likelihood becomes negative infinity

    # 4.3.e
```

```python
print("4.3.e")
best = plot(sentence_d)[0][0]
best = round(best, 2)
print(f"Optimal Lambda is {best}")

# 4.4.a
nas0 = []
nas1 = []
with open("nasdaq00.txt", "r") as n0, \
        open("nasdaq01.txt", "r") as n1:
    for line in n0.readlines():
        nas0.append(float(line[:-1]))
    for line in n1.readlines():
        nas1.append(float(line[:-1]))
A = np.zeros((3, 3))
b = np.zeros((3, 1))
for i in range(len(nas0) - 3):
    xt = np.array([nas0[i], nas0[i + 1], nas0[i + 2]]).reshape((3, 1))
    A = A + np.dot(xt, xt.T)
    yt = nas0[i + 3]
    b += yt * xt
coef = np.flip(np.dot(np.linalg.inv(A), b)).reshape((1, 3))
print("4.4.a")
print("[a1, a2, a3] = " + str(coef))

# 4.4.b
valid0 = np.array(nas0[3:])
valid1 = np.array(nas1[3:])
test0 = []
test1 = []
for i in range(len(nas0) - 3):
    prev = np.array([nas0[i + 2], nas0[i + 1], nas0[i]]).reshape((3, 1))
    test0.append((np.dot(coef, prev))[0][0])
for i in range(len(nas1) - 3):
    prev = np.array([nas1[i + 2], nas1[i + 1], nas1[i]]).reshape((3, 1))
    test1.append((np.dot(coef, prev))[0][0])
test0 = np.array(test0)
test1 = np.array(test1)

mse0 = np.sqrt(np.mean(((test0 - valid0) ** 2)))
mse1 = np.sqrt(np.mean(((test1 - valid1) ** 2)))

# lower MSE does not justify that the model works better in 2001,
# it only demonstrates that the linear regression preserves through 2001
print("4.4.b")
print(f"mse 2000: {mse0}")
print(f"mse 2001: {mse1}")
```

```python
if __name__ == '__main__':
    main()
```

```
4.3.a
{'MILLION': 0.002072759168154815, 'MORE': 0.0017088989966186725, 'MR.':
0.0014416083492816956, 'MOST': 0.0007879173033190295, 'MARKET':
0.0007803712804681068, 'MAY': 0.0007298973156289532, 'M.':
0.0007034067394618568, 'MANY': 0.0006967290595970209, 'MADE':
0.0005598610827336895, 'MUCH': 0.0005145971758110562, 'MAKE':
0.0005144626437991272, 'MONTH': 0.00044490959363187093, 'MONEY':
0.00043710673693999306, 'MONTHS': 0.0004057607781605526, 'MY':
0.0004003183467688823, 'MONDAY': 0.00038198530259784006, 'MAJOR':
0.00037089252670515475, 'MILITARY': 0.00035204581485220204, 'MEMBERS':
0.00033606096579846475, 'MIGHT': 0.00027358919153183117, 'MEETING':
0.0002657374141083427, 'MUST': 0.0002665079156312084, 'ME':
0.00026357267173457725, 'MARCH': 0.0002597935452176646, 'MAN':
0.0002528834918776787, 'MS.': 0.0002389900041002911, 'MINISTER':
0.00023977273580605944, 'MAKING': 0.00021170446604452378, 'MOVE':
0.0002099555498894477, 'MILES': 0.00020596851026319035}
4.3.b
[('<UNK>', 0.6150198100055118), ('U.', 0.013372499432610317), ('FIRST',
0.011720260675031612), ('COMPANY', 0.011658788055636611), ('NEW',
0.009451480076516552), ('UNITED', 0.008672308141231398), ('GOVERNMENT',
0.006803488635995202), ('NINETEEN', 0.006650714911000876), ('SAME',
0.006287066757449016), ('TWO', 0.006160749602827221)]
4.3.c
Unigram:  -64.50944034364878
Bigram:  -40.91813213378977
4.3.d
Unigram:  -44.291934473132606
Not observed: SIXTEEN OFFICIALS
Not observed: SOLD FIRE
Bigram:  -inf
4.3.e
```

```
<ipython-input-1-34c698bd51c1>:68: RuntimeWarning: divide by zero encountered in
log
  return np.log(bi)
```




```
Optimal Lambda is 0.64
4.4.a
[a1, a2, a3] = [[0.95067228 0.01560333 0.03189472]]
4.4.b
mse 2000: 117.9083331254247
mse 2001: 54.6360532458946
```

[ ]: