CSE 250A HW 2
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1. (a) First compute $P(A=1|E=1) = P(A=1, B=0|E=1) + P(A=1, B=1|E=1)$

$= P(A=1|B=0, E=1)P(B=0|E=1) + P(A=1|B=1, E=1)P(B=1|E=1)$ 「By HW1 1.(a)」

$= 0.29\,P(B=0) + 0.95\,P(B=1)$ 「$B \perp\!\!\!\perp E$」

$= 0.29 \times 0.999 + 0.95 \times 0.001$

$= 0.29066$

Then $P(A=1) = P(A=1|E=1)P(E=1) + P(A=1|E=0)P(E=0)$

$= 0.29066 \times 0.002 + 0.998\big(P(A=1|B=0, E=0)P(B=0|E=0) + P(A=1|B=1, E=0)P(B=1|E=0)\big)$ 「HW1, 1(a)」

$= 0.00058 + 0.998\,(0.001 \times 0.999 + 0.94 \times 0.001)$

$= 0.0025$

Then $P(E=1|A=1) = \dfrac{P(A=1|E=1)P(E=1)}{P(A=1)} = \dfrac{0.29066 \times 0.002}{0.0025} = \underline{0.2325}$

(b) $P(E=1|A=1, B=0) = \dfrac{P(A=1|E=1, B=0)P(E=1|B=0)}{P(A=1|B=0)}$ 「HW1, 1(b)」,

where $P(A=1|B=0) = P(A=1, E=1|B=0) + P(A=1, E=0|B=0)$

$= P(A=1|E=1, B=0)P(E=1|B=0) + P(A=1|E=0, B=0)P(E=0|B=0)$

$= 0.29 \times 0.002 + 0.001 \times 0.998$

$= 0.001578$

So $P(E=1|A=1, B=0) = \dfrac{0.29 \times 0.002}{0.001578} = \underline{0.3676}$.

(c) $P(A=1|M=1) = \dfrac{P(M=1|A=1)P(A=1)}{P(M=1|A=1)P(A=1) + P(M=1|A=0)P(A=0)}$

$= \dfrac{0.7 \times 0.0025}{0.7 \times 0.0025 + 0.01 \times (1-0.0025)} = \underline{0.1493}$

(d) $P(A=1|M=1, J=0) = P(A=1|J=0, M=1) = \dfrac{P(J=0|A=1, M=1)P(A=1|M=1)}{P(J=0|M=1)}$

$P(J=0|M=1) = P(J=0|A=1, M=1)P(A=1|M=1) + P(J=0|A=0, M=1)P(A=0|M=1)$

$= P(J=0|A=1)P(A=1|M=1) + P(J=0|A=0)P(A=0|M=1)$

$= 0.1 \times 0.1493 + 0.95 \times (1-0.1493) = 0.8231$

So $P(A=1|M=1, J=0) = \dfrac{0.1 \times 0.1493}{0.8231} = \underline{0.0181}$

(e) $P(A=1|M=0) = \dfrac{P(M=0|A=1)P(A=1)}{P(M=0|A=1)P(A=1)+P(M=0|A=0)P(A=0)}$

$= \dfrac{0.3\times 0.0025}{0.3\times 0.0025+0.99\times(1-0.0025)}$

$= \underline{0.00076}$

(f) $P(A=1|M=0,B=1) = \dfrac{P(M=0|A=1,B=1)P(A=1|B=1)}{P(M=0|A=1,B=1)P(A=1|B=1)+P(M=0|A=0,B=1)P(A=0|B=1)}$

$B\to A\to M$ d-sep (1) $\Rightarrow P(M=0|A=1,B=1) = P(M=0|A=1) = 0.3$

$P(M=0|A=0,B=1) = P(M=0|A=0) = 0.99$

$P(A=1|B=1) = P(A=1,E=0|B=1)+P(A=1,E=1|B=1)$

$= P(A=1|E=0,B=1)P(E=0|B=1)+P(A=1|E=1,B=1)P(E=1|B=1)$

$= 0.94\times 0.998+0.95\times 0.002$

$= 0.94002$

So $P(A=1|M=0,B=1) = \dfrac{0.3\times 0.94002}{0.3\times 0.94002+0.99\times(1-0.94002)} = \underline{0.8261}$

(b) > (a) is reasonable since $B=0$ rules out the prob. that burglar triggers the alarm.

(d) < (c) is reasonable since $J=0$ tends to reduce $P(A=1)$.

(f) > (e) is reasonable since $B=1$ is the extra cause of alarm.

2. (a) $P(D=0|S_1=1,\cdots,S_k=1)$

$= \dfrac{P(S_1=1,\cdots,S_k=1|D=0)P(D=0)}{P(S_1=1,\cdots,S_k=1|D=0)P(D=0)+P(S_1=1,\cdots,S_k=1|D=1)P(D=1)}$

where $P(S_1=1,\cdots,S_k=1|D=0) = \prod_{i=1}^{k} P(S_i=1|D=0)$

$= \prod_{i=2}^{k} P(S_i=1|D=0) = \prod_{i=2}^{k}\dfrac{f(i-1)}{f(i)} = \dfrac{f(1)}{f(k)} = \dfrac{1}{2^k+(-1)^k}$

$P(S_1=1,\cdots,S_k=1|D=1) = \prod_{i=1}^{k} P(S_i=k|D=1) = \dfrac{1}{2^k}$,

then $P(D=0|S_1=1,\cdots,S_k=1) = \left(\dfrac{1}{2^k+(-1)^k}\cdot\dfrac{1}{2}\right)/\left(\dfrac{1}{2^k+(-1)^k}\cdot\dfrac{1}{2}+\dfrac{1}{2^k}\cdot\dfrac{1}{2}\right)$

$P(D=1|S_1=1,\cdots,S_k=1) = \left(\dfrac{1}{2^k}\cdot\dfrac{1}{2}\right)/\left(\dfrac{1}{2^k}\cdot\dfrac{1}{2}+\dfrac{1}{2^k+(-1)^k}\cdot\dfrac{1}{2}\right)$

So $r_k = \left(\dfrac{1}{2^k+(-1)^k}\cdot\dfrac{1}{2}\right)/\left(\dfrac{1}{2^k}\cdot\dfrac{1}{2}\right) = \dfrac{2^k}{2^k+(-1)^k}$

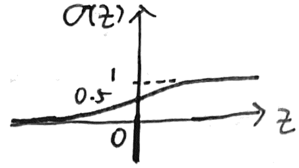
$2^k > 2^k+(-1)^k$ when k is odd;

$2^k < 2^k+(-1)^k$ when k is even.

So doctor diagnoses $D=0$ on odd days, $D=1$ on even days.

(b) Less certain, since the numerator and denominator always differ by 1, as k get larger, r_k will converge to 1 (i.e. $\lim_{k\to\infty}\dfrac{2^k}{2^k+(-1)^k}=1$), then the two probabilities will become more closer, so it is harder to diagnose.

3. (a) $\sigma'(z) = -(1+e^{-z})^{-2}(-e^{-z})$



$$= \frac{e^{-z}}{(1+e^{-z})^2}$$

$$\sigma(z)\sigma(-z) = \frac{1}{(1+e^{-z})(1+e^{z})}$$

$$= \frac{\frac{1}{1+e^{z}}}{1+e^{-z}} = \frac{\frac{1+e^{-z}}{1+e^{z}}}{1+e^{-z}} = \frac{e^{-z}}{1+e^{-z}} \quad (\text{since } e^{-z}(1+e^{z}) = e^{-z}+1).$$

$$= \sigma'(z)$$

(b) $$\sigma(-z)+\sigma(z) = \frac{1}{1+e^{z}} + \frac{1}{1+e^{-z}}$$

$$= \frac{1+e^{-z}+1+e^{z}}{1+e^{-z}+e^{z}+e^{0}} = 1$$

(c) $$L(\sigma(z)) = \log\left(\frac{\sigma(z)}{1-\sigma(z)}\right)$$

$$= \underbrace{\log\left(\frac{\sigma(z)}{\sigma(-z)}\right)}_{\text{by (b)}} = \log \frac{1+e^{z}}{1+e^{-z}} = \log e^{z} = z.$$

(d) $p_i = P(Y=1 \mid X_i = 1 \text{ only}) = \sigma(w_i)$,

So $L(p_i) = L(\sigma(w_i)) = w_i$ by (c).

4. (1) X = month, Y = pud. E = {rain, sp.}
(2) Same XY, E = {rain, sp., fall}.
(3) X = rain, Y = sp. E = {month}
(4) X = month, Y = fall, E = {rain, sp.}.
(5) Same XY, E = {rain, sp, pud.}
(6) Same XY, E = {pud.}
(7) Same XY, E = {rain, pud.}
(8) Same XY, E = {sp., pud.}
(9) X = rain, Y = fall, E = {pud.}
(10) Same XY, E = {pud., sp.}
(11) Same XY, E = {pud., sp., month}
(12) Same XY, E = {pud., month}
(13) X = sp., Y = fall, E = {pud.}
(14) Same XY, E = {pud., rain}
(15) Same XY, E = {pud., month}
(16) Same XY, E = {pud., month, rain}.

No case where $E = \emptyset$ since the graph is connected and no d.sep case 3 exists for all pairs of nodes.

5. For all cases, we prove the last step is blocked when going from the point BACK to X.
This is sufficient since all X's adjacent nodes are in B_X.
(1) Via parent's parent: all such paths will be blocked by some parents of X w/ d.sep (1).
(2) Via child's child: all such paths will be blocked by some children of X w/ d.sep (1)
(3) Via parent's child: all such paths will go through parent's child ← parent → X, which is d.sep (2)
(4) Via spouse's child: all such paths go through spouse → child ← X, where child cannot have any child in B_X, which is d.sep (3)
(5) Via spouse's parent: same as (4), d.sep case (3).

6. F T T F T T F T T F

7. (1) $S=\{D\}$ (2) $S=\{B,D,F,E,C\}$ (3) $S=\{D,E,F\}$
(4) $S=\{A\}$ (5) $S=\{F\}$ (6) $S=\{D,F\}$
(7) $S=\{A,B,C,D\}$ (8) $S=\emptyset$ (9) $S=\{A,D\}$
(10) $S=\{D,E,A,B,C\}$

8. (a) < (b) < (c) > (d) <
(e) = (f) > (g) <

9. (a)
$$P(C|A,B,D) = \frac{P(D|A,B,C)P(C|A,B)}{P(D|A,B)}$$

$$= \frac{P(D|B,C)P(C|A)}{\sum_c P(C=c,D|A,B)} \quad \ulcorner A \perp\!\!\!\perp D \mid B,C,\ d.sep\ (1)$$
$$B \perp\!\!\!\perp C \mid A,\ d.sep\ (3)\lrcorner$$

$$= \frac{P(D|B,C)P(C|A)}{\sum_c P(C=c|A,B)P(D|C=c,A,B)}$$

$$= \frac{P(D|B,C)P(C|A)}{\sum_c P(C=c|A)P(D|B,C=c)}$$

(b)
$$P(E|A,B,D) = \sum_c P(E,C=c|A,B,D)$$
$$= \sum_c P(C=c|A,B,D)P(E|C=c,A,B,D)$$
$$= \sum_c P(C=c|A,B,D)P(E|C=c) \quad \ulcorner E \perp\!\!\!\perp A,B,D \mid C$$
d.sep (1) and (2)⌟

(c)
$$P(G|A,B,D) = \sum_e P(G,E=e|A,B,D)$$
$$= \sum_e P(E=e|A,B,D)P(G|A,B,D,E=e)$$
$$= \sum_e P(E=e|A,B,D)P(G|E) \quad \ulcorner G \perp\!\!\!\perp A,B,D \mid E$$
d. sep (1)⌟

(d)
$$P(F|A,B,D,G) = \frac{P(G|A,B,D,F)P(F|A,B,D)}{P(G|A,B,D)}$$

$$= \frac{\sum_e P(G,E=e|A,B,D,F)P(F,E=e|A,B,D)}{P(G|A,B,D)}$$

$$= \frac{\sum_e P(E=e|A,B,D,F)P(G|A,B,D,F,E=e)P(E=e|A,B,D)P(F|A,B,D,E=e)}{P(G|A,B,D)}$$

$$= \frac{\sum_e P(E=e|A,B,D)^2 P(G|E=e,F)P(F)}{P(G|A,B,D)}$$
$\ulcorner E \perp\!\!\!\perp F \mid A,B,D$ d.sep (3)
$G \perp\!\!\!\perp A,B,D \mid E,F$, d.sep (1)
$F \perp\!\!\!\perp A,B,D,E$, d.sep(3)⌟