$\dot{\underline{y}} = \underline{\underline{A}} \cdot \underline{y}$

Solve ODE system

1. Find eigenvalue λ_i and eigenvectors v_i

   → solve $\det(A - \lambda \mathbb{I})$

   → solve for $\underline{v}_i$: $\left[\underline{\underline{A}} - \lambda_i \underline{\underline{\mathbb{I}}}\right] \underline{v}_i = \underline{0}$

2. If λ_i is real:

   $\underline{y}(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t} + \dots$

   If λ_i is complex ($\lambda_{1,2} = a \pm ib$):

   $$\underline{y}(t) = C_1 e^{at} \begin{pmatrix} v_{1,1} \cos(bt) \\ -v_{1,2} \sin(bt) \end{pmatrix} + C_2 e^{at} \begin{pmatrix} v_{1,1} \sin(bt) \\ v_{1,2} \cos(bt) \end{pmatrix}$$

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Tips:

3a) Express ODE with $y = \begin{pmatrix} x \\ \dot{x} \end{pmatrix}$

3c) → Explicit Euler = Forward Euler

→ $F(y_n) = A \cdot y_n$

→ $N = \frac{T}{dt}$ → output arrays include t=0

↳ size of array: $N+1$

→ set size with: array.resize(N+1);

3d) → Implicit Euler = Backward Euler

4d) Documentation: bit.ly/31mOA7v