THE UNIVERSITY OF AUCKLAND
NEW ZEALAND

COMPSCI.220.C.S2 – Algorithms and Data Structures

Assignment 1 – Analysing Time Complexity of Algorithms

Out: Friday, 25**th** of July, 2014
Due: Friday, 8**st** of August, 2014

Assignment 1 (Lectures 1 – 6) is worth **60 marks** representing **6%** of your total course grade.

Objectives. Learning basic techniques for analysing time complexity, in particular, evaluating complexity of a given pseudocode; exploring algorithm performance with "Big-Oh", "Big-Omega", and "Big-Theta" tools, and solving basic recurrences describing the performance.

Requirements. You should give clear and detailed answers to the following questions:

1. (*10 marks*) Work out time complexity $T(n)$ of the following piece of pseudocode in terms of a total number of specified elementary operations for a given integer n:

   for $i = 1$ **step** $i \leftarrow n * i$ **while** $i < n^4$ **do**
       for $j = 0$ **step** $j \leftarrow j + 2$ **while** $j < 4 * n$ **do**
           for $k = 1$ **step** $k \leftarrow 3 * k$ **while** $k < n$ **do**
               Constant number C of specified operations
           end for
       end for
   end for

2. (*10 marks*) You have found empirically that methods A of complexity $\Theta(n^2)$ and B of complexity $\Theta(n\sqrt{n})$ process an array of $n = 100$ objects for $T_A(n) = 2$ and $T_B(n) = 10$ microseconds, respectively. Find out how long will each method process an array of $n = 1,000,000$ objects?

3. (*10 marks*) Assuming $n = 5^m$ with the integer $m = \log_5 n$, derive a closed-form formula for $T(n)$ by solving the recurrence $T(n) = 5T(n/5) + 5$ with the base condition $T(1) = 0$.

4. (*15 marks*) **Proposition** 4 [Exercise 1.3.4 of the textbook]: Given two nonnegative-valued functions, $f(n)$ and $g(n)$, defined on nonnegative integers, n, prove that $f(n)$ is $\Theta(g(n))$ if and only if both $f(n)$ is $O(g(n))$ and $f(n)$ is $\Omega(g(n))$.

5. (*15 marks*) The processing time $T(n)$ of a certain algorithm is both $\Omega(n)$ and $\mathrm{O}(n^3)$. Decide whether the conclusion that $T(n)$ is $\Theta(n^2)$ is correct or not and prove your decision.

Your report should be clearly structured and provide detailed answers to Questions 1–3 and proofs of Proposition 4 and your decision regarding Question 5, including relevant math expressions. The proofs should exploit basic properties of "Big-Oh", "Big-Theta", and "Big-Omega" tools like, e.g., in the proof for Exercise 1.3.6 of the textbook.

The report has to be prepared as an electronic document, e.g., by using a LaTeX document preparation system or Microsoft Word text editor, and submitted in a single PDF file with a fixed specification: `CS220assign1.pdf` (only this PDF file will be evaluated by markers, so that you must check that it can be read by PCs in the departmental Computer Labs). Note that **scanned handwritten documents are strictly forbidden** (even as images in an electronic text) and will not be accepted for marking.

Submission

Submit your report electronically to `http://www.cs.auckland.ac.nz/automated-marker` (this automated marking system, abbreviated AMS below, will be open for submission from July 25 to August 10, 2014; 08:30 pm).

The due date is Friday, 8st of August 2014, 08:30 p.m. (AMS time). If submitted after due date but before 9th of August, 08:30 p.m., the penalty is 10%. If submitted after this date, but before 10th of August, 08:30 p.m., the penalty is 50%; and no submission afterwards.

Marking scheme

For Questions 1 – 3	% of marks
Clear structure of your report and detailed explanations	up to 20
Correctness of your final answers	up to 20
Correctness of your intermediate steps in deriving these answers	up to 20
Detailed explanations of all steps with references, if necessary, to the textbook	up to 40
	Total: up to 100

For Proposition 4 and Question 5	
Clear structure of your report and detailed explanations	up to 20
Correct usage of “Big-{Oh, Omega, Theta} tools with due references to the textbook	up to 30
Correctness of your proof	up to 50
	Total: up to 100