Time Value of Money (TVM)

Time = Money!

The primary objective of financial management is to maximaze the value of the firm's stock ; Stock values depend on the timing of the cash flows investors expect from an investment. in other words, a dollar expected soon is worth more than a dollar expected in the distant future. Reason why financial managers must understand the time value of money and its impact on stock prices.

1. Present values and Future values

PV $\xrightarrow{*}$ Compounding → FV

*

$$PV = \frac{FV}{(1+\text{interest})^N}$$

$$FV = PV \cdot (1+\text{Interest})^N$$

Periods: 0, 1, 2, 3

PV=100 — 105.00 — 110.25 — 115.76

Compounding process

$PV_0 = 100$
Interest(I) = 5%
Number of periods(N) = 3

$FV_1 = 105$
$FV_2 = 110.25$
$FV_3 = 115.76$

(a dollar in hand today is worth more than a dollar to be received in the future! why?)

→ A stream of equal payments at fixed intervals expected to continue forever!

2. "Annuities" if the payments are equal and are made at fixed intervals.

(auto loans, Student loans, and mortgages)

① Perpetuity (≈ annuity with an extended life) $\left(PVP = \frac{PMT}{I}\right)$

② Ordinary (deferred) annuity

Periods: 0, 1, 2, 3

I=5%. -100 (at 1), -100 (at 2), -100 (at 3)

③ Annuity due

Periods: 0, 1, 2, 3

I=5%. -100 (at 0), -100 (at 1), -100 (at 2)

(FV / PV of Ordinary annuity / Annuity due exist!!)

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ⓐ Future Value of an Ordinary Annuity

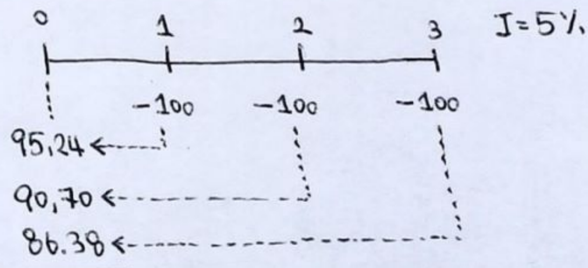
$$FVA_N = PMT(1+Interest)^{N-1} + PMT(1+Interest)^{N-2} + \cdots + PMT(1+Interest)^{0}$$

$$= PMT \cdot \left[\frac{(1+Interest)^N - 1}{Interest} \right]$$

The future value of an annuity over N periods!

ⓑ Future value of an Annuity Due ; * $FVA_{due} = FVA_{ordinary}(1+Interest)$

ⓒ Present value of an Ordinary Annuity



$PVA_N = 272.32$

$$PVA_N = \frac{PMT}{(1+Interest)^1} + \frac{PMT}{(1+Interest)^2} + \cdots + \frac{PMT}{(1+Interest)^N}$$

$$= PMT \cdot \left[\frac{1 - \frac{1}{(1+Interest)^N}}{Interest} \right]$$

3. Uneven Cash Flows (Non-Constant Cash flows)

$\updownarrow$

Annuity (Constant Cash flows)

Uneven cash flows

Annuity + additional final payment;

Periods	0	1	2	3
$	0	100	100	100
				1000
				1100

I = 10%

Irregular cash flows;

Periods	0	1	2	3
$	0	100	300	200

I = 10%

ⓐ PV of an Uneven Cash flow Stream; I = 12%

Periods	0	1	2	3	4	5
	$0	$100	$300	$300	$300	$500

PV		FV
	$0	0
$89,29	$100	157,35
$239,16	$300	421,48
$213.53	$300	376.32
$190,66	$300	336,00
$281,71	$500	500
$1016.35		1791.15

*$1016.35 ≈ PV of cash flow stream ≈ Value of the asset

ⓑ FV of an Uneven Cash flow Stream; I = 12%

* IRR? Internal Rate of Return; IRR means the rate of return the investment provides :)

	① Annually	② Semi-annually	③ Quarterly	④ Monthly
PV	$ 100	$100	$100	$100
N	5	10	20	60
I/Y	12	6	3	1
PMT	0	0	0	0
FV	$ 176.23	$179.08	$180.61	181.67

Compounding more frequently than annually!

*** How to construct its loan amortization schedule, $100,000 at 6% for 5yrs.

Step ① find PMT

$$\$100{,}000 = \frac{PMT}{(1.06)^1} + \frac{PMT}{(1.06)^2} + \frac{PMT}{(1.06)^3} + \frac{PMT}{(1.06)^4} + \frac{PMT}{(1.06)^5}$$

PMT = -23,739.64

Step ② Construct its loan schedule; Amount borrowed : $100,000
Years : 5
Rate : 6%
PMT : -$23,739.64

Year	Beginning Amount	Payment	Interest	Repayment of Principal	Ending Balance
1	$ 100,000	$ 23,739.64	$6,000	$ 17,739.64	$82,260.36
2	82,260.36	23,739.64	4,935.62	18,804.02	63,456.34
3	63,456.34	23,739.64	3,807.38	19,932.26	43,524.08
4	43,524.08	23,739.64	2,611.44	21,128.20	22,395.89
5	22,395.89	23,739.64	1,343.75	22,395.89	0

4. How to Compare interest rate?

→ EAR represents the annual rate of return actually being earned after adjustments have been made for different compounding periods.

① annual percentage rate (APR) ($\approx$ quoted or stated rate) ; the periodic rate times the # of periods per year. also called Nominal Interest Rate (I_{NOM})

② effective annual rate (EFF% or EAR) ; the annual rate of interest actually being earned, as opposed to the quoted rate.

* $EAR = \left[1 + \frac{I_{NOM}}{M}\right]^{M} - 1$; ex) nominal rate is 9% with semi-annual compounding ; to solve this, employ EAR formula.

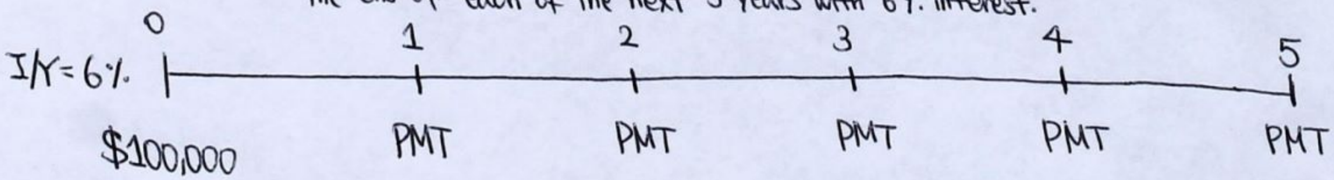
$$EAR = \left[1 + \frac{0.09}{2}\right]^{2} - 1 = ?$$

*** (If a loan or an investment uses annual compounding, its nominal rate is also its effective rate. However, if compounding occurs more than once a year, the EAR is higher than I_{NOM}.)

5. Amortized Loans? ;

A loan that is to be repaid in *equal amounts on a monthly, quartely, or annual basis!

* example! ; You borrow $100,000 on a car loan, and it is to be repaid in five equal payments at the end of each of the next 5 years with 6% interest.



Loan Amortized Loans

Suppose you borrowed $30,000 on a student loan at a rate of 8% and must repay it in 3 equal installments at the end of each of the next 3 years. How large would your payments be, how much of the first payment would represent interest, how much would be principal, and what would your ending balance be after the first year?

N 3
I 8%
PV $30,000
FV 0

Loan Amortization Schedule, $30,000 at 8% for 3 Years

Amount borrowed: $30,000

Years: 3

Rate: 8%

PMT: – $11,641.01

Year	Beginning Amount	Payment	Interest	Repayment of Principal	Ending Balance
1	$30,000	$ 11641.01	$ 2400	$ 9241.01	$20,758.99
2	$ 20,758.99	$ 11641.01	$ 1,660.72	$ 9980.29	$ 10,778.71
3	$ 10,778.71	$ 11641.01	$ 862.32	$ 10,778.71	$ 0.00

The Time Value of Money

1. 0 — I/Y = 9% — N = 15

PV = −1000 FV = ?

N = 15
I/Y = 9
PV = −1000
PMT = 0
* FV = 3,642.48

2. 0 — I/Y = 20% — N = 50

PV = −10 FV = ?

N = 50
I/Y = 20
PV = −10
PMT = 0
* FV = 91,004.38

3. Since no interest is earned, * $100 is needed today to have $100 in three years

4. 0 — I/Y = 8% — N = 7

PV = ? FV = 10,000

N = 7
I/Y = 8
PMT = 0
FV = 10,000
* PV = 5,834.90

5. 0 — I/Y = ? — N = 8

PV = −3.00 FV = 4.50

N = 8
* I/Y = 5.20%
PMT = 0
PV = −3.00
FV = 4.50

6. 0 — I/Y = 12% — N = ?

PV = −5,000 FV = 10,000

* N = 6.12
I/Y = 12
PMT = 0
PV = −5,000
FV = 10,000

*

7. Two ways to get solution :)

① $CF_0 = 0$;
$CF_1 = 500$;
$CF_2 = 200$;
$CF_3 = 800$;
I/Y = 12;
* NPV = 1175.29

② 0 — 1 — 2 — 3

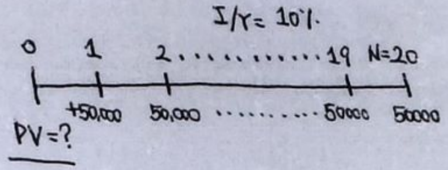
I/Y = 12% 0 500 200 800

PV_1 = 446.43
PV_2 = 159.44
PV_3 = 569.42

$PV_1 + PV_2 + PV_3$ = * PV_{total}

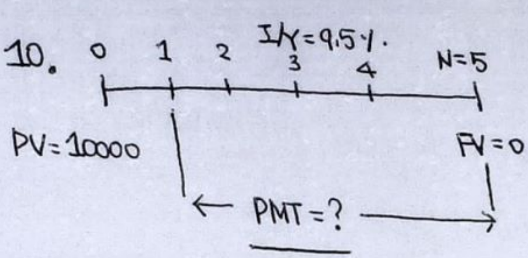
446.43 + 159.44 + 569.42 = 1175.29

8. $CF_0 = 0$; $CF_1 = 10$; $CF_2 = -20$; $CF_3 = 10$; $CF_4 = 150$; $I/Y = 8.5$; * NPV=108.?

9. 

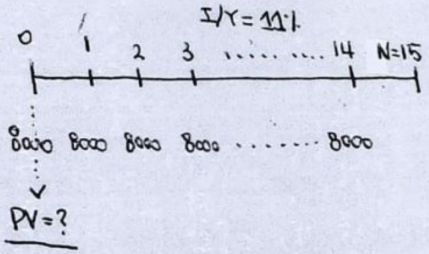
N=20
I/Y = 10
PMT = 50,000

FV=0
*PV = 425,678.19

10. 

PV = −10,000
FV = 0
I/Y = 9.5
N = 5

*PMT = 2604.36

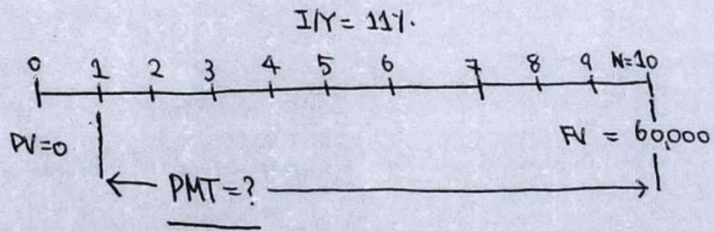
11. 

Two steps; ① N=15; I/Y= 11 ; PMT=-8000 ; FV=0

PV = 57,526.96

② Annuity Due = Aunity$_{ord}$ x (1+interest);

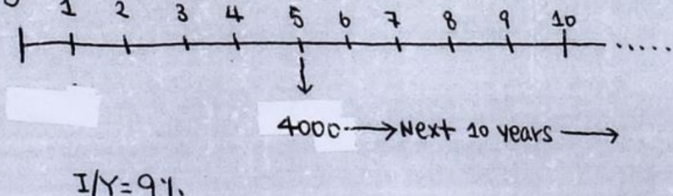
57526.96 x (1+11%) = 63,854.92

12. 

I/Y= 11
*PMT = 3588.09
N = 10

PV=0
FV = 60,000

*

13. 

Two steps ;

① Find PV of the 10-year annuity

N= 10 ; I/Y=9 ; PMT = 4000 ; FV=0 ;

PV= 25670.63 (This value as the end of year 4)

② Discount PV of Annuity back!

N=4
FV=-25670.63
I/Y=9

PMT = 0
*PV = 18,185.72

14. Annuity Due 2 Solutions ;

Solution 1 ; N=4 ; PMT=-1000 ; PV=0 ; I/Y = 12

PV = 4779.33

Annuity Due = Annuity $_{ori}$ X (1+ interest)

= 4779.33 X (1 + 12%) = * 5352.85

Solution 2 ; Sum of PVs

$PV_{today} = 1000 \times (1+12\%)^1 = 1120$

$PV_1 = 1000 \times (1+12\%)^2 = 1254.40$

$PV_2 = 1000 \times (1+12\%)^3 = 1404.93$

$PV_3 = 1000 \times (1+12\%)^4 = 1573.52$

* 5352.85

*15. $PV_{Annuity} = PMT \left[\frac{1 - \frac{1}{(1+Interest)^N}}{Interest} \right]$

N = 30 X 12 = 360

I/Y = 9/12 = 0.75

FV = 0

PV = -150.000 (1-20%) = -120,000

* PMT = 965.55

16. PV = -5,000

I/Y = 12/365 = 0,03

N = 1 X 365 = 365

PMT = 0

* FV = 5,637.37

17. $\frac{9}{0.11} = 81.82$; $\frac{PMT}{I/Y} = PV_{perpetuity}$.

18. $\frac{4.5}{65} = 0.07 = 7\%$; Rate of Return for a perpetuity $= I/Y = \frac{PMT}{PV_{perpetuity}}$

* 19. PV = 10,000 ; FV = 0 ; I/Y = 10 ; N = 10 ; * PMT = 1627.45

amortization question!

First year interest = ? = 10000 × 10% = 1000

Principle Balance going into second year is 10000 − (1627.45 − 1000)

= 9372.55

Second year interest = ? = 9372.55 × 10% = 937.255

20. $EAR = [(1+(0.18/12)]^{12} - 1 = 19.56\%$

$EAR = (1 + \text{Periodic Rate})^m - 1$

$\text{periodic rate} = \frac{\text{Stated annual rate}}{\text{\# of compounding periods / year}}$