• Risk, Return, and the Capital Asset Pricing Model (CAPM)

We assume that rational people invest in relatively risky assets when they expect to receive relatively high returns.

*** How to measure/analyze an asset's risk?

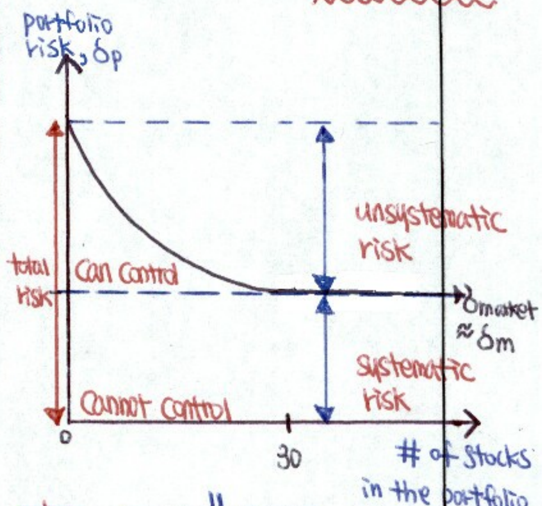
① A Stand-Alone basis;
asset is considered in isolation; one asset

Case1) "Damieon" buys $10,000 of "Short-term Treasury bill" with an expected return of 5%.

Case2) "Aisha" buys $10,000 of "Bitcoin" with an expected return of 100%.

- risk-free asset? "Short-term Treasury bill"
- risky asset? "Bitcoin"
- Damieon? "risk-averse lady"
- Aisha? "risk-lover (risk-premium)"

② A portfolio basis;
asset is held as one of # of assets in a portfolio; multiple asset



*** ① Stand-Alone Risk

ⓐ Expected Returns → What do you expect the return to be?

ⓑ Standard deviations → Risk

ⓒ Coefficients of variation (CV) → CV shows the risk per unit of return, and it provides a more meaningful measure when the expected returns on two alternatives are not same!

*** ② Portfolio Risk

ⓐ Expected Portfolio Returns → What do you expect the portfolio return to be?

ⓑ Portfolio Risk → Diversification?

① Stand-Alone Risk

ⓐ Expected Returns

ex) An investment has a 50% chance of producing a 20% return, a 25% chance of producing a 3% return and a 25% chance of producing a −7% return. What is its expected return?

$(50\% \times 20\%) + (25\% \times 3\%) + (25\% \times -7\%) =$ Expected Return

$10\% + 0.75\% + (-1.75\%) = 9\%$

ⓑ Standard deviation $= \sqrt{\text{Variance}} =$ Risk

ⓒ Coefficient of Variation (CV)

$$CV = \frac{\text{Standard deviation}}{\text{Expected Return}} = \frac{\delta}{\hat{r}} = \text{the standardized measure of the risk/unit}$$

ex) Company A vs. Company B

$\delta = 10, \hat{r} = 1$ $\quad$ $\delta = 20, \hat{r} = 4$

$CV_A = \frac{10}{1} = 10$ $\quad$ $CV_B = \frac{20}{4} = 5$

Company A is about 2 times riskier than Company B on the basis of this criterion.

※ Before we discuss ② Portfolio Risk, let's talk about * CAPM.

* Required Return on $Stock_i = R_f + \beta_i (R_m - R_f) = ER_i$

ex) A stock has a beta of 1.8. Assume that the risk-free rate is 5% and the market premium is 7%. Find its expected return.

$ER_i = 5\% + 1.8 \cdot (7\%) = 17.6\%$

R_f = risk-free rate, ER_i = expected return of investment, β_i = Beta of the investment
R_m = Expected return on market, $R_m - R_f$ = market risk premium.

Stand-Alone Risk

Case 1.

Step 1. Probability Distributions and Expected Returns

Shin's BBQ

Economy, Which Affects Demand	Probability of This Demand Occurring	Rate of Return if This Demand Occurs	Product
Strong	0.30	80%	24%
Normal	0.40	10%	4%
Weak	0.30	-60%	-18%
	1.00	Expected return =	10%

Barry's BBQ

Probability of This Demand Occurring	Rate of Return if This Demand Occurs	Product
0.30	15%	4.5%
0.40	10%	4.0%
0.30	5%	1.5%
1.00	Expected return =	10.0%

Step 2. **1) Calculating Shin's BBQ' Standard Deviation**

Economy, Which Affects Demand	Probability of This Demand Occurring	Rate of Return if This Demand Occurs	Deviation Actual Minus Expected Return	Deviation Squared	Squared Deviation × Probabiliaty
Strong	0.30	80%	70%	0.4900	0.1470
Normal	0.40	10%	0%	0.0000	0.0000
Weak	0.30	-60%	-70%	0.4900	0.1470
	1.00			Variance	0.2940
				Standard deviation	0.5422
				Standard deviation (%)	54.22%

1) Calculating Barry's BBQ' Standard Deviation

Economy, Which Affects Demand	Probability of This Demand Occurring	Rate of Return if This Demand Occurs	Deviation Actual Minus Expected Return	Deviation Squared	Squared Deviation × Probabiliaty
Strong	0.30	15%	5%	0.0025	0.0007
Normal	0.40	10%	0%	0.0000	0.0000
Weak	0.30	5%	-5%	0.0025	0.0008
	1.00			Variance	0.0015
				Standard deviation	0.0387
				Standard deviation (%)	3.87%

Step 3. Summarize and Calculate Coefficients of Variations

Std Dev for Shin's BBQ	54.22%
Std Dev for Barry's BBQ	3.87%

CV for Shin's BBQ	5.42
CV for Barry's BBQ	0.39

Case 2.

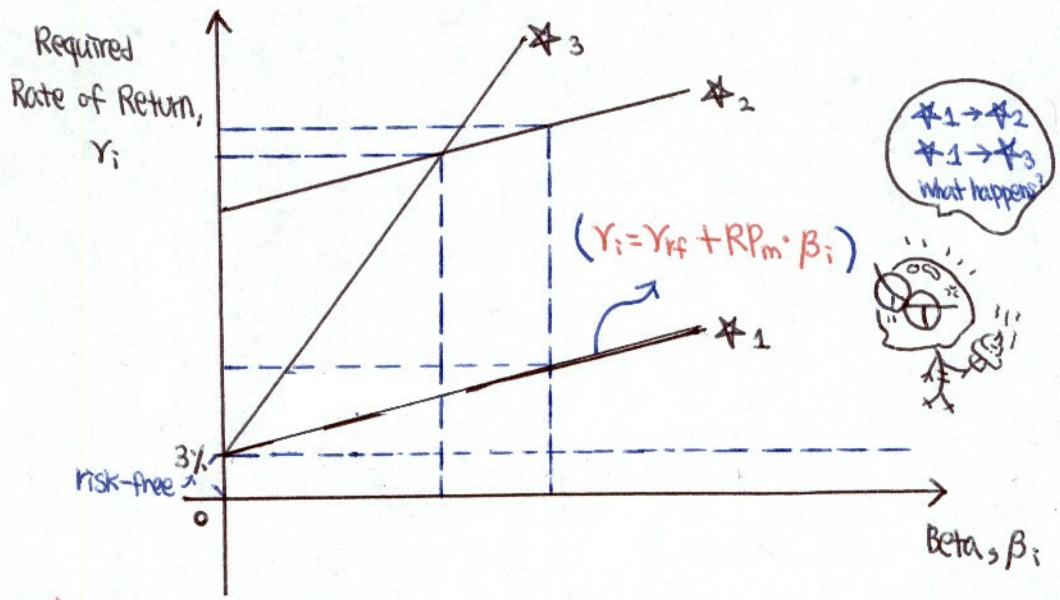
Standard Deviations Based on Historical Data

Year	Return	Deviation from Average		Squared Deviation
2016	20.0%	15.3%		0.0233
2017	-9.0%	-13.8%		0.0189
2018	18.0%	13.3%		0.0176
2019	-10.0%	-14.8%		0.0218
Average	4.75%		Sum of Squared Deviations	**0.0815**
			Variance	**0.0272**
			Standard Deviation	**16.48%**

The relationship between Risk and Rates of Return.

The CAPM posits that only market risk matters and an asset's required return should consist of a risk-free component plus a risk-premium that compensates for the asset's market risk. The asset's risk-premium is the product of the market risk-premium and the particular asset's exposure to the market risk component.

- Security Market Line (SML) shows the relationship between the stock's beta and its required return, as predicted by the CAPM.



*** How to calculate required rate of returns for three different stocks?

we need more information! $r_m = 6\%$, Stock 1's $\beta = 0.5$, Stock 2's $\beta = 1.5$, Stock 3's $\beta = 1.0$, then,,, use $r_i = r_{rf} + RP_m \cdot \beta_i$

$(\uparrow \beta, \uparrow \text{return}?)$

Stock 1's required rate of return $= r_1 = 3\% + 0.5 \times (6\% - 3\%) = 4.5\%$

Stock 2's required rate of return $= r_2 = 3\% + 1.5 \times (6\% - 3\%) = 7.5\%$

Stock 3's required rate of return $= r_3 = 3\% + 1.0 \times (6\% - 3\%) = 6\%$

② A portfolio basis (multiple assets);
The portfolio's risk, δ_p, is usually smaller than the average of stock's' δs because *diversification lowers the portfolio risk.

Let's have some cases (all three cases are very important!)

(Case1) Returns with perfect correlation, $\rho = 1$

Year	Stock Ⓕ	Stock Ⓣ	Portfolio ⒻⓉ
2017	30%	30%	30%
2018	-22%	-22%	-22%
2019	15%	15%	15%
Average Return	7.67%	7.67%	(7.67%)
Standard Deviation	26.76%	26.76%	(26.76%)
Correlation Coefficient, ρ			(1)

Please practice these!

$$\frac{30\% + (-22\%) + 15\%}{3}$$

$$\sqrt{\frac{(30\% - 7.67\%)^2 + (-22\% - 7.67\%)^2 + (15\% - 7.67\%)^2}{3-1}}$$

Stock Ⓕ − Average of Ⓕ

30% − 7.67% = 22.33%
−22% − 7.67% = −29.67%
15% − 7.67% = 7.33%

Stock Ⓣ − Average of Ⓣ

30% − 7.67% = 22.33%
−22% − 7.67% = −29.67%
15% − 7.67% = 7.33%

$$\begin{aligned} 22.33 &\times 22.33 &= 498.78 \\ -29.67 &\times -29.67 &= 880.11 \\ 7.33 &\times 7.33 &= 53.78 \end{aligned}$$

$\sum (F - \bar{F}) \cdot (T - \bar{T})$ 1432.67

$$\text{Cov}(Ⓕ, Ⓣ) = \frac{1432.67}{3-1} = 716.33$$

$$\rho = \frac{\text{Cov}(Ⓕ, Ⓣ)}{\delta_{Ⓕ} \cdot \delta_{Ⓣ}} = \frac{716.33}{26.76 \cdot 26.76} = 1$$

Perfect positive correlation
Diversification has NO effect :p

(Case 2) Returns with Perfect Negative Correlation, $\rho = -1$

Year	Stock F	Stock T	Portfolio F, T
2016	30%	-15%	7.5%
2017	-15%	30%	7.5%
2018	30%	-15%	7.5%
2019	-15%	30%	7.5%
Average Return	7.5%	7.5%	7.5%
Standard Deviation	25.98%	25.98%	Ø
Correlation Coefficient, ρ			-1

$\rho = -1$; when two stocks are negatively correlated, diversification is its strongest. In this case, the portfolio return is a certain (no risk) 7.5%.

(Case 3) Returns with somewhat partial correlation, $\rho = -0.5$

Year	Stock F	Stock T	Portfolio F, T
2017	30%	-15%	7.5%
2018	-15%	20%	2.5%
2019	20%	30%	25%
Average Return	11.67%	11.67%	11.67%
Standard Deviation	23.63%	23.63%	11.81%
Correlation Coefficient, ρ			-0.5

$\rho = -0.5$; diversification is effective in lowering portfolio risk. In this case, the portfolio return is an average of the stock returns and risk is reduced from 23.63% per stock to 11.81% for the portfolio.

※ You must understand how to calculate average return(s), standard deviation(s), and correlation coefficient(s), and meanings as well!