Weekly Lab Meeting

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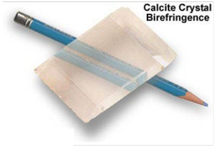
KHU NOPD LAB

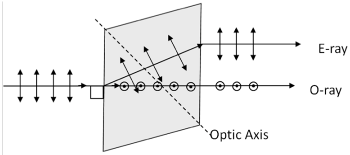
Introduction to Šolc Filter

Birefringence (Double refraction)

- **Definition**

  A birefringent, optically anisotropic material is one whose refractive index depends on polarization, giving rise to two eigenpolarizations with different refractive indices for a given propagation direction.





- **The light propagation in a birefringent crystal**

  Linear superposition of two eigenwaves. (slow and fast)

  $\rightarrow$Well-define phase velocity ($v = w/k$) and direction of polarization

- **The direction of polarization for eigenwaves**

  Mutually orthogonal for slow and fast axes.

Jones Calculus and its Application

- **Birefringent optical system**

  The passage of light through a train of **polarizers** and **retardation plates**.

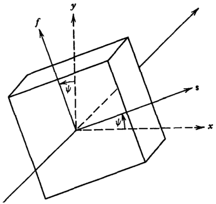
  Retardation plate (wave plate) is polarization-state converters, or transformers.

  ex) EO modulator, Šolc Filter

- **The reason why we use Jones Calculus**

  An optical system consists of many birefringence elements, each oriented at a different azimuth angle. → Compliated calculation.

  Solution : **Jones Calculus**



Jones Matrix Formulation

- **Assumption**

  There's no reflection (totally transmitted)

- **With an arbitrary single element**

  An incident light beam represented by a Jones vector in the laboratory axes (x, y), decompose "slow" and "fast" eigenwaves in crystals.

$$\begin{pmatrix} V_s \\ V_f \end{pmatrix} = \begin{pmatrix} \cos\psi & \sin\psi \\ -\sin\psi & \cos\psi \end{pmatrix} \begin{pmatrix} V_x \\ V_y \end{pmatrix} \equiv R(\psi) \begin{pmatrix} V_x \\ V_y \end{pmatrix}. \tag{1}$$

  In the "slow" and "fast" coordinate system, the passage of the light on a birefringent element,

$$\begin{pmatrix} V_s' \\ V_f' \end{pmatrix} = \begin{pmatrix} \exp\left(-in_s \frac{\omega}{c} l\right) & 0 \\ 0 & \exp\left(-in_f \frac{\omega}{c} l\right) \end{pmatrix} \begin{pmatrix} V_s \\ V_f \end{pmatrix}. \tag{2}$$

Jones Matrix Formulation

- **With an arbitrary single element**

  The phase retardation, $\Gamma = (n_s - n_f)\frac{\omega l}{c}$ and mean phase change $\phi = \frac{1}{2}(n_s + n_f)\frac{\omega l}{c}$

  $\rightarrow$ Measure of the relative change in phase, not absolute value.

$$\begin{pmatrix} V_s' \\ V_f' \end{pmatrix} = e^{-i\phi} \begin{pmatrix} e^{-i\frac{\Gamma}{2}} & 0 \\ 0 & e^{i\frac{\Gamma}{2}} \end{pmatrix} \begin{pmatrix} V_s \\ V_f \end{pmatrix}. \tag{3}$$

  Transformation axis due to retardation plate,

$$\begin{pmatrix} V_x' \\ V_y' \end{pmatrix} = R(-\psi)\, W_0\, R(\psi) \begin{pmatrix} V_x \\ V_y \end{pmatrix}$$

  where $W_0 = e^{-i\phi} \begin{pmatrix} e^{-i\Gamma/2} & 0 \\ 0 & e^{i\Gamma/2} \end{pmatrix}$. $\rightarrow e^{-i\phi}$: Not observable in interference

Jones Matrix Formulation

- **Property of retardation plate**

  An arbitrary retardation plate,

$$W = R(-\psi)\, W_0\, R(\psi) \tag{5}$$

  Jones matrix of a wave plate is unitary matrix

$$W^\dagger W = \mathrm{I} \tag{6}$$

  The passage of a polarzed light beam through a wave plate is mathematically described as a unitary transformation.

Half-Wave Retardation Plate

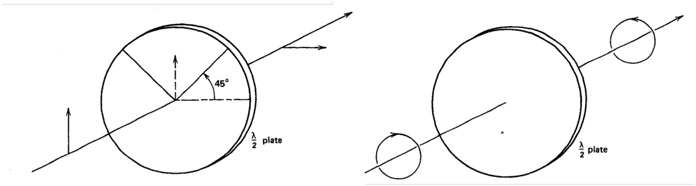
- **Half-Wave Retardation Plate (HWP)**

  HWP has a half-wave phase retardation $\Gamma = \pi$.

$$\Gamma = \frac{2\pi}{\lambda_0} t \left| n_o - n_e \right| = \pi \tag{7}$$

$$t = \frac{\lambda}{2 \left| n_e - n_o \right|} \tag{8}$$

  Denote that HWP rotate the polarization by an angle ϕ for a general azimuth angle ϕ.



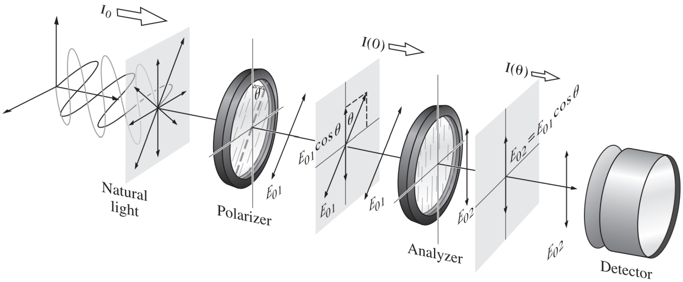
Polarizer

- **Polarizer**

  An optical device whose input is natural light and whose output is some form of polarized light is a polarizer.

  A polarizer placed in front of the system $\rightarrow$ to **prepare** a polarized light.

  A second polarizer (analyzer) $\rightarrow$ to **analyze** the polarization state of emerging beam.



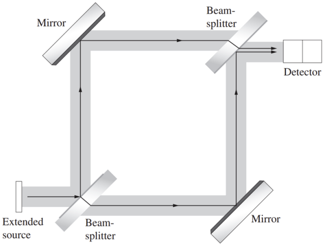
Polarizer Interference Filter

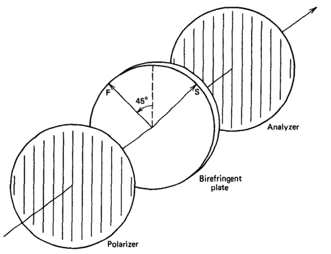
- **Polarizer Interference Filter**

  Spectral filter $\rightarrow$ based on the interference of polarized light.

  Role: extremely narrow bandwidth with wide angular field or tuning capability.

  Composition: birefringent retardation plate + polarizer



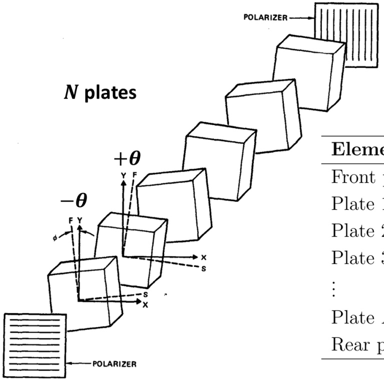


Šolc Filter

- **Šolc Filter**

  Composed of a pile of identical birefringent plates each oriented at a prescribed azimuth angle θ.

  Two types: **folded type** and fan type.



Element	Azimuth
Front polarizer	$0°$
Plate 1	θ
Plate 2	$-\theta$
Plate 3	θ
$\vdots$	$\vdots$
Plate N	$(-1)^{N-1}\theta$
Rear polarizer	$90°$

Analysis of Šolc Filter

- **Jones Matrix Analysis**

For a single layer composed $+\theta, -\theta$ birefringent retardation plate ($N = 2m$),

$$M = \left[R(\theta)\, W_0\, R(-\theta)\, R(-\theta)\, W_0\, R(\theta) \right]^m \tag{9}$$

With the crossed polarizers,

$$\begin{pmatrix} E'_x \\ E'_y \end{pmatrix} = P_y\, M\, P_x \begin{pmatrix} E_x \\ E_y \end{pmatrix}. \tag{10}$$

Transmission for x-polarized light with $\cos x = \cos 2\theta \sin \Gamma/2$,

$$T = |M_{21}|^2 = \left| \tan 2\theta \cos x \left(\frac{\sin Nx}{\sin x} \right) \right|^2 \tag{11}$$

When $\Gamma = \pi,\ 3\pi,\ 5\pi \ldots$ each plate becomes HWP.

$$T = \sin^2 2N\theta \tag{12}$$

For $T = 1$, azimuth angle $\theta = \dfrac{\pi}{4N}$

Analysis of Šolc Filter in the view of HWP

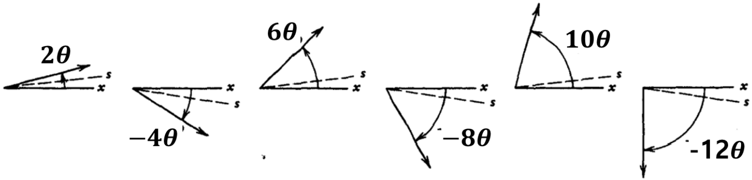
- **When each plate effectively becomes HWP**

  HWP acts like a mirror reflection for polarization.

$$\begin{pmatrix} \cos(2\theta - \phi) \\ \sin(2\theta - \phi) \end{pmatrix} = \left[R(-\theta) \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} R(\theta) \right] \begin{pmatrix} \cos\phi \\ \sin\phi \end{pmatrix} \tag{13}$$

  Input-output angle relation

$$\phi_{out} = 2\theta - \phi_{in} \tag{14}$$



Birefringent plate	1	2	3	4	5	6
Azimuth angle	7.5°	-7.5°	7.5°	-7.5°	7.5°	-7.5°
Polarization	15°	-30°	45°	-60°	75°	-90°

Analysis of Šolc Filter in the view of HWP

- **When each plate effectively becomes HWP**

  Accumulating θ, it reaches 90° final azimuth angle ($2N\theta = \frac{\pi}{2}$).

  $\rightarrow$ without any loss of intensity.

  Slightly off the HWP condition,

  $\rightarrow$ Šolc filter suffers loss.

The transmission characteristic around the peak and sidelobes

- **Slightly off Γ**

  λ_ν : the waveangle at phase retardation in $(2\nu+1)\pi$

  $\Gamma = \frac{2\pi}{\lambda}(n_e - n_o)\,d = (2\nu+1)\pi$ If λ is slightly away from λ_ν

$$\Gamma = (2\nu+1)\,d - \underbrace{\frac{(2\nu+1)\,d}{\lambda_\nu}(\lambda-\lambda_\nu)}_{\Delta\Gamma} \tag{15}$$

- **Assumption that azimuth angle $\theta = \frac{\pi}{4N}$ and $N \gg 1$**

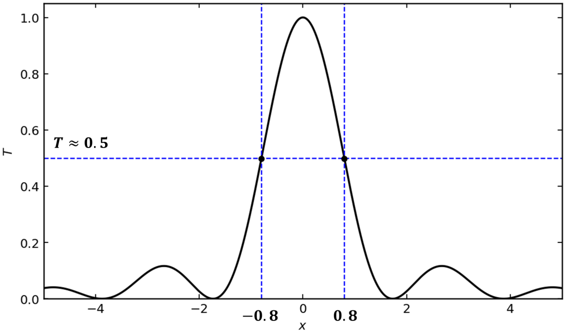
  Under these conditions, the transmission T

$$T = \left| \frac{\sin\left(\frac{1}{2}\pi\sqrt{1+(N\Delta\Gamma/\pi)^2}\right)}{1+(N\Delta\Gamma/\pi)^2} \right|^2 \tag{16}$$

The transmission characteristic around the peak and sidelobes

- **The transmission near a transmission peak where $x = N\Delta\Gamma/\pi$**

$$T = \left| \frac{\sin\left(\frac{1}{2}\pi\sqrt{1+x^2}\right)}{1+x^2} \right|^2$$



Full Width Half Maximum (FWHM)

$$\Delta\lambda = 1.60 \frac{\lambda_\nu}{(2\nu+1)\,N} \tag{17}$$