1. (a) $\begin{bmatrix}26 & 18\\ 18 & 74\end{bmatrix}$

   (b) $\mathrm{V} = \begin{bmatrix}1/\sqrt{10} & 3/\sqrt{10}\\ 3/\sqrt{10} & -1/\sqrt{10}\end{bmatrix}$, $\Lambda = \begin{bmatrix}80 & 0\\ 0 & 20\end{bmatrix}$

   (c) $\Sigma = \begin{bmatrix}4\sqrt{5} & 0\\ 0 & 2\sqrt{5}\end{bmatrix}$

   (d) $\mathrm{W} = \begin{bmatrix}20/\sqrt{10} & 10/\sqrt{10}\\ 20/\sqrt{10} & -10/\sqrt{10}\end{bmatrix}$ $\mathrm{U} = \begin{bmatrix}1/\sqrt{2} & 1/\sqrt{2}\\ 1/\sqrt{2} & -1/\sqrt{2}\end{bmatrix}$

   (e) pass

   (f) $\mathrm{U}^{\mathrm{T}}CV = \begin{bmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\end{bmatrix}\begin{bmatrix}5 & 5\\ -1 & 7\end{bmatrix}\begin{bmatrix}\frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}}\\ \frac{3}{\sqrt{10}} & -\frac{1}{\sqrt{10}}\end{bmatrix}$

   $= \begin{bmatrix}4/\sqrt{2} & 12/\sqrt{2}\\ 6/\sqrt{2} & -2/\sqrt{2}\end{bmatrix}\begin{bmatrix}1/\sqrt{10} & 3/\sqrt{10}\\ 3/\sqrt{10} & -1/\sqrt{10}\end{bmatrix} = \begin{bmatrix}4\sqrt{5} & 0\\ 0 & 2\sqrt{5}\end{bmatrix}$

   (g) $4\sqrt{5}\begin{bmatrix}1/\sqrt{2}\\ 1/\sqrt{2}\end{bmatrix}\begin{bmatrix}1/\sqrt{10} & 3/\sqrt{10}\end{bmatrix} + 2\sqrt{5}\begin{bmatrix}1/\sqrt{2}\\ -1/\sqrt{2}\end{bmatrix}\begin{bmatrix}3/\sqrt{10} & -1/\sqrt{10}\end{bmatrix}$

   (h) $\mathrm{C} = \mathrm{U}\Sigma\mathrm{V}^{\mathrm{T}}$

   $$\mathrm{C}_1 = \begin{bmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\end{bmatrix}\begin{bmatrix}4\sqrt{5} & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}\frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}}\\ \frac{3}{\sqrt{10}} & -\frac{1}{\sqrt{10}}\end{bmatrix} = \sqrt{4}\begin{bmatrix}1 & 3\\ 1 & 3\end{bmatrix}$$

2. (a) $\mathrm{U} = \begin{bmatrix}1/\sqrt{2} & 1/\sqrt{2}\\ 1/\sqrt{2} & -1/\sqrt{2}\end{bmatrix}$ $\Lambda = \begin{bmatrix}5 & 0\\ 0 & 3\end{bmatrix}$

   (b) same as a

3. (a) $\begin{bmatrix}3 & & \\ & 0 & \\ & & 0\end{bmatrix}$

   (b) $\begin{bmatrix}0 & 3\\ 0 & 0\end{bmatrix}$

   (c) $\mathrm{A} = \begin{bmatrix}1/\sqrt{2} & 1/\sqrt{2}\\ 1/\sqrt{2} & -1/\sqrt{2}\end{bmatrix}\begin{bmatrix}3 & \\ & 1\end{bmatrix}\begin{bmatrix}1/\sqrt{2} & 1/\sqrt{2}\\ 1/\sqrt{2} & -1/\sqrt{2}\end{bmatrix}$

   $$\mathrm{A}_1 = \begin{bmatrix}1/\sqrt{2} & 1/\sqrt{2}\\ 1/\sqrt{2} & -1/\sqrt{2}\end{bmatrix}\begin{bmatrix}3 & \\ & 0\end{bmatrix}\begin{bmatrix}1/\sqrt{2} & 1/\sqrt{2}\\ 1/\sqrt{2} & -1/\sqrt{2}\end{bmatrix} = \frac{3}{2}\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$$

(d) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

(e) $\begin{bmatrix} 4 & 4 \\ 0 & 0 \end{bmatrix}$

4. (a) M = $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 19 \\ 21 \end{bmatrix}$

(c) $\begin{bmatrix} 5/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$

(d) $\begin{bmatrix} 8 & 0 \\ 0 & 2 \end{bmatrix}$

(e) $\begin{bmatrix} 20\sqrt{2} \\ \sqrt{2} \end{bmatrix}$

(f) M$\begin{bmatrix} 19 \\ 21 \end{bmatrix}$ = $\begin{bmatrix} 20\sqrt{2} \\ \sqrt{2} \end{bmatrix}$

5. $\frac{1}{\sqrt{2}}(1,1), \frac{1}{\sqrt{2}}(1,-1)$