

선대 숙제 4 답

13.

$$1. \quad \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{pmatrix} \cdot \left(\begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{pmatrix}^T \cdot \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{pmatrix} \right)^{-1} \cdot \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{pmatrix}^T$$

$$= \frac{1}{6} \begin{pmatrix} 5 & 2 & -1 \\ 2 & 2 & 2 \\ -1 & 2 & 5 \end{pmatrix}$$

2.

$$Pb = \begin{pmatrix} \frac{5}{6} & \frac{1}{3} & -\frac{1}{6} \\ \frac{1}{6} & \frac{1}{3} & \frac{1}{6} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ -\frac{1}{6} & \frac{1}{3} & \frac{5}{6} \end{pmatrix} \cdot \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \\ -1 \end{pmatrix}$$

$$PPb = \begin{pmatrix} \frac{5}{6} & \frac{1}{3} & -\frac{1}{6} \\ \frac{1}{6} & \frac{1}{3} & \frac{1}{6} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ -\frac{1}{6} & \frac{1}{3} & \frac{5}{6} \end{pmatrix} \cdot \begin{pmatrix} 5 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \\ -1 \end{pmatrix}$$

$$3. \quad Q Q^T = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$Q^T Q = \frac{1}{9} \begin{pmatrix} 5 & 4 & -2 \\ 4 & 5 & 2 \\ -2 & 2 & 8 \end{pmatrix}$$

$$4. \quad v_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, w_1 = \begin{bmatrix} \frac{1}{3} \\ -\frac{1}{3} \\ \frac{5}{3} \end{bmatrix}, \|v_1\| = \sqrt{3}, \|w_1\| = \sqrt{3},$$

$$5. \quad v_2 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, w_2 = \begin{bmatrix} \frac{1}{3} \\ -\frac{4}{3} \\ \frac{8}{3} \end{bmatrix}, \|v_2\| = 3, \|w_2\| = 3,$$

$$6. \quad \frac{v_1^T}{\|v_1\|} \cdot \frac{v_2}{\|v_2\|} = \frac{5}{9} \sqrt{3}$$

$$7. \quad \frac{w_1^T}{\|w_1\|} \cdot \frac{w_2}{\|w_2\|} = \frac{5}{9} \sqrt{3}, \text{ 문제 6과 같다.}$$

$$8. \quad \sqrt{v_1^2 + v_2^2}$$

$$9. \quad \cos \theta$$

$$10. \quad A = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}, C = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$11. \quad q_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, q_2 = \frac{1}{\sqrt{6}} \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}, q_3 = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$12. \quad q_1 \cdot q_2 = 0, q_1 \cdot q_3 = 0, q_2 \cdot q_3 = 0. \text{ 서로 직교한다.}$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & -3 \\ 0 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ -1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ 0 & -2/\sqrt{6} & 1/\sqrt{3} \end{bmatrix} \begin{bmatrix} \sqrt{2} & \sqrt{2} & \sqrt{18} \\ 0 & \sqrt{6} & -\sqrt{6} \\ 0 & 0 & \sqrt{3} \end{bmatrix} = QR.$$