1. $LU = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

2. $P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, $PA = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 5 & 9 \\ 1 & 2 & 0 \end{bmatrix} = LU = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 7 \\ 0 & 0 & -1 \end{bmatrix}$

3. $$[A\ I] = \begin{bmatrix} 1 & -2 & 3 & 1 & 0 & 0 \\ 2 & -3 & 6 & 0 & 1 & 0 \\ 0 & 2 & 1 & 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 1 & 0 \\ 0 & 2 & 1 & 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 1 & 0 \\ 0 & 0 & 1 & 0 & -2 & 1 \end{bmatrix}$$
$$\Rightarrow \begin{bmatrix} 1 & -2 & 0 & 1 & 6 & -3 \\ 0 & 1 & 0 & -2 & 1 & 0 \\ 0 & 0 & 1 & 0 & -2 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & -3 & 8 & -3 \\ 0 & 1 & 0 & -2 & 1 & 0 \\ 0 & 0 & 1 & 0 & -2 & 1 \end{bmatrix} = [I\ A^{-1}]$$
$$\Rightarrow A^{-1} = \begin{bmatrix} -15 & 8 & -3 \\ -2 & 1 & 0 \\ 4 & -2 & 1 \end{bmatrix}$$

4. $$[A\ I] = \begin{bmatrix} -1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} -1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 2 & 0 & 1 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} -1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -.5 & 1 \\ 0 & 0 & 1 & 0 & .5 & 0 \end{bmatrix}$$
$$\Rightarrow \begin{bmatrix} -1 & 0 & 0 & 1 & .5 & -2 \\ 0 & 1 & 0 & 0 & -.5 & 1 \\ 0 & 0 & 1 & 0 & .5 & 0 \end{bmatrix}$$
$$A^{-1} = \begin{bmatrix} -1 & -.5 & 2 \\ 0 & -.5 & 1 \\ 0 & .5 & 0 \end{bmatrix}$$

5. $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} x_1 + \begin{bmatrix} 3 \\ 2 \\ 1 \\ 3 \end{bmatrix} x_2 + \begin{bmatrix} 3 \\ 2 \\ 2 \\ 3 \end{bmatrix} x_3 = 0$

$$i)\ x_1 + 3x_2 + 3x_3 = 0$$
$$ii)\ x_1 + 2x_2 + 2x_3 = 0$$
$$iii)\ x_1 + x_2 + 2x_3 = 0$$

$$ii) - iii) = x_2 = 0$$
$$i) - ii) = x_2 + x_3 = 0 \Rightarrow x_3 = 0$$

따라서, $x_1 = x_2 = x_3 = 0$ 이므로 주어진 벡터는 1 차독립

6. c

7. A = $\begin{bmatrix}1 & 2 & 2 & 2\\2 & 4 & 6 & 8\\3 & 6 & 8 & 10\end{bmatrix} \Rightarrow \begin{bmatrix}1 & 2 & 2 & 2\\0 & 0 & 2 & 4\\0 & 0 & 2 & 4\end{bmatrix} \Rightarrow \begin{bmatrix}1 & 2 & 2 & 2\\0 & 0 & 1 & 2\\0 & 0 & 0 & 0\end{bmatrix} \Rightarrow \begin{bmatrix}1 & 2 & 0 & -2\\0 & 0 & 1 & 2\\0 & 0 & 0 & 0\end{bmatrix}$

rref 가 다음과 같은 형태이므로

기저는 {(1, 2, 2, 2,), (2, 4, 6, 8)}

8. {(1, 2, 3), (2, 6, 8)}

9. {(-2, 1, 0, 0), (2, 0, -2, 1)}

10. c(A) 와 N(A^{T})의 관계에 의해, {(1,1,-1)}

11. $\begin{bmatrix}1 & 3 & 0 & 2 & 1\\0 & 0 & 1 & 4\,| & 6\\1 & 3 & 1 & 6 & 7\end{bmatrix} \Rightarrow \begin{bmatrix}1 & 3 & 0 & 2 & 1\\0 & 0 & 1 & 4\,| & 6\\0 & 0 & 1 & 4 & 6\end{bmatrix} \Rightarrow \begin{bmatrix}1 & 3 & 0 & 2 & 1\\0 & 0 & 1 & 4\,| & 6\\0 & 0 & 0 & 0 & 0\end{bmatrix}$

Particular solution: (1, 0, 6, 0)

Homogeneous solution: (-3,1,0,0)$\mathrm{x}_1 + (-2,0,-4,1)\mathrm{x}_2$

Complete solution: $(-3\mathrm{x}_1 - 2x_2 + 1, x_1, -4x_2 + 6, x_2)$

12. $\mathrm{A}^{\mathrm{T}}A = \begin{bmatrix}3 & 3\\3 & 5\end{bmatrix} \Rightarrow \left(\mathrm{A}^{\mathrm{T}}A\right)^{-1} = \frac{1}{6}\begin{bmatrix}5 & -3\\-3 & 3\end{bmatrix}$

$$\mathrm{A}^{\mathrm{T}}b = (6,0)$$

$$\left(\mathrm{A}^{\mathrm{T}}A\right)^{-1}(6,0) = (5,-3)$$

$$\mathrm{A}(5,-3) = (5,2,-1)$$

13. B = b – A = (1, 1, -2)
C = c- 3A + B = (1, 1, 1)

$$\mathrm{q}_1 = \frac{A}{\sqrt{2}} = (\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0)$$
$$\mathrm{q}_2 = \frac{B}{\sqrt{6}} = (\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, -\frac{2}{\sqrt{6}})$$
$$\mathrm{q}_3 = \frac{C}{\sqrt{3}} = (\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$$

14.

$$\mathrm{q}_1^T a = \sqrt{2},$$
$$q_1^T b = \sqrt{2}, q_2^T b = \sqrt{6}$$
$$\mathrm{q}_1^T c = 3\sqrt{2}, q_2^T c = -\sqrt{6}, q_3^T c = \sqrt{3}$$
$$\mathrm{QR} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ 0 & -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} \end{bmatrix} \begin{bmatrix} \sqrt{2} & \sqrt{2} & 3\sqrt{2} \\ 0 & \sqrt{6} & -\sqrt{6} \\ 0 & 0 & \sqrt{3} \end{bmatrix}$$

15. $\begin{vmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ 2 & 3 & 0 \end{vmatrix} = 0\begin{vmatrix} 0 & -3 \\ 3 & 0 \end{vmatrix} - 1\begin{vmatrix} -1 & -3 \\ 2 & 0 \end{vmatrix} + 2\begin{vmatrix} -1 & 0 \\ 2 & 3 \end{vmatrix} = -12$

16. b

17. $\begin{vmatrix} -\lambda & 2 \\ 2 & 3-\lambda \end{vmatrix} = \lambda^2 - 3\lambda - 4 = 0 \Rightarrow \lambda = 4\,, -1$

$$\lambda = -1, \qquad \begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix} \mathrm{x} = 0$$
$$\mathrm{x} = (-2, 1)$$
$$\lambda = 4, \qquad \begin{vmatrix} -4 & 2 \\ 2 & -1 \end{vmatrix} x = 0$$
$$\mathrm{x} = (1,2)$$

18. $\begin{bmatrix} \frac{1}{\sqrt{5}} & -\frac{2}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ -\frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{bmatrix}$

19. $A = \frac{1}{5}\begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 4 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$

$$A^5 = \frac{1}{5}\begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 4^5 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$$

$$= \frac{1}{5}\begin{bmatrix} 2^{10} & 2 \\ 2^{11} & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 2^{10} - 4 & 2^{11} + 2 \\ 2^{11} + 2 & 2^{12} - 1 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 1020 & 2050 \\ 2050 & 4095 \end{bmatrix} = \begin{bmatrix} 204 & 410 \\ 410 & 819 \end{bmatrix}$$

20. $\begin{vmatrix} 25-\lambda & 0 & 0 \\ 0 & 7-\lambda & -24 \\ 0 & -24 & -7-\lambda \end{vmatrix} = (25-\lambda)(\lambda^2 - 625) = (25-\lambda)(\lambda - 25)(\lambda + 25)$

$$\lambda = 25, \quad x = (1,0,0), (0,4,-3)$$
$$\lambda = -25, \quad x = (0,3,4)$$

21. $\lambda^2 - 6\lambda + 8 = 0 \Rightarrow \lambda = 4,2$

$$\lambda = 4, x = (1,1)$$
$$\lambda = 2, x = (1,-1)$$

22. $Bx = M^{-1}AMx = \lambda x \Rightarrow A(Mx) = \lambda(Mx)$

따라서 Mx 는 A 의 고유벡터

$$M^{-1} = \begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix}$$

$$M^{-1}(1,1) = (-3,2)$$

$$M^{-1}(1,-1) = (-7,4)$$

23. b, d