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Consensus enhancement for multi-agent systems with rotating-segmentation perception

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Abstract

In this paper, we investigate the consensus problem of multi-agent systems (MASs) with a limited sensing range using two kinds of distributed neighbor selection strategies. Because each agent's convergence evolution is typically based on the "select all perceived neighbors" (SAN) framework, fragmentation into multiple clusters is likely to occur, and data storage and computational load can grow exponentially as the number of agents increases. To address this challenge, we propose a new distributed consensus framework composed of two strategies that can effectively enhance the consensus of the MAS. First, a novel representative selection with rotating-segmentation perception (RSRSP) strategy is provided for agents to intelligently select representative neighbors in each sector of the communication region for convergence evolution. Second, a distributed switching strategy is designed for each agent to synchronously switch from RSRSP to SAN when the system reaches full connectivity. We analyze the stability of the proposed consensus protocol with the common Lyapunov function and verify the superiority of the two proposed strategies through comparisons with a baseline SAN algorithm.

Keywords Multi-agent systems · Distributed consensus · Neighbor selection strategy · Lyapunov function

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1 Introduction

As an important field of distributed artificial intelligence, multi-agent systems (MASs) have attracted increasing research in control engineering [1–3], autonomous vehicles [4–6], task allocation [7–9], reinforcement learning [10–12], and other applications. Each agent in an MAS can effectively communicate and cooperate with its neighbors, thus conducting rational planning and execution by solving an optimization problem [13, 14]. The major challenge in MAS is how to coordinate a group of autonomous agents to achieve global or local consensus solely through local control [15, 16]. The primary problem here is to design the consensus protocol, which is the interaction rule that specifies the communication pattern between agents and their neighbors, such that the MAS gradually converges to a consensus [17, 18].

The interaction rule, i.e., the consensus protocol between agents, can be described by network topology, which plays a vital role in consensus theory [19, 20]. Because of the possibility of link failures or the need for energy

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efficiency, the network topologies of various systems are often modeled as switching topologies [16, 21]. Extensive studies have been conducted on the design of the consensus protocol with switching topologies. Olfati-Saber et al. [16] presented the classic convergence and performance analysis of consensus protocols with switching topologies. Hu et al. [22] addressed the bipartite consensus of MASs with switching topologies by utilizing centralized and distributed event-triggered schemes. Su et al. [23] provided distributed output feedback synthesis for both leaderless and leader-follower consensus protocols of MASs with switching topologies. Additional works on the consensus protocol of MASs with switching topologies can be found in [24–29].

In this paper, we discuss a more complex situation of switching topology, namely an MAS with dynamic topology. In this MAS, each agent has a limited sensing range and only interacts with its perceived neighbors within this range to achieve convergence evolution [30]. The state of each agent can be described as position, speed, opinion value, etc. Then, the network topology of the MAS dynamically changes according to the agents' state change during convergence evolution. MASs with dynamic topologies have been widely studied and applied in rendezvous [30, 31], flocking [32, 33], opinion management [34, 35] and cruise control [36]. Connectivity preservation is a necessary condition to ensure that an MAS with dynamic topology reaches a consensus [37]. Some studies [30, 31, 37, 38] have introduced additional connectivity preservation algorithms in the design of the consensus protocol, thus increasing the complexity of the protocol design and inevitably leading to a higher computational load and energy consumption. Furthermore, as observed by Moreau, more communication does not necessarily result in faster convergence and may even lead to a loss of convergence [19]. When the communication network of the MAS is fully connected, each agent must store the states of all members of the MAS for computing, which leads to higher computing power consumption, a wider communication bandwidth, and higher memory requirements for each agent [39]. Therefore, it is beneficial to heuristically explore and selectively utilize the information of neighbors.

In dynamic topology, each agent can select representative neighbors, or delete redundant communication links between agents without destroying the stability of the system. The research on consensus protocols with neighbor selection strategies is still in its early stages. Jadbabaie et al. [40] proposed a distributed consensus protocol using the nearest neighbor selection strategy. Cortés et al. [41] selected local neighbors for cooperation and evolution using Delaunay graphs and Gabriel graphs. Motsch et al. [42] indicated that as the heterophily dependence among agents increases, the number of clusters of the MAS decreases, and the cooperation of each agent with its far neighbors is enhanced.

This paper explores the neighbor selection strategy in the dynamic communication topology with efficient consensus guarantees. This problem can be addressed by introducing the bio-inspired heuristic "segmentation perception", which is an important mechanism to select the most representative information within the sensing range [43]. Presently, "segmentation perception" is commonly used in fields such as swarm behavior [33, 44], wireless networks [45, 46], optimization theory [47] and transportation [48]. However, few works have studied and leveraged segmentation perception in multi-agent consensus control. Therefore, we follow a simple rationale that there must be redundancy in the MAS communication, which calls for an effective consensus protocol with neighbor selection strategies to enhance the consensus of the MAS with dynamic topology. Specifically, this paper proposes a new distributed consensus framework based on two neighbor selection strategies, in which each agent evolves solely by referencing the state of some representative neighbors. The main contributions of this paper are summarized as follows:

1. We provide a novel representative selection with rotating-segmentation perception (RSRSP) strategy to select representative neighbors in each sector of a perception-based communication region. Specifically, the communication region is segmented into several sectors with an optimal rotation angle, and then the agents select the nearest neighbor in each sector for convergence evolution.
2. We design a distributed switching strategy to switch from RSRSP to the traditional multi-agent select all neighbors (SAN) framework (i.e., each agent has no optimization strategies for selecting its perceived neighbors) when the system is fully connected. Specifically, agents adopt RSRSP for the convergence evolution before the system is fully connected. Once the communication topology reaches full connectivity, all agents switch from RSRSP to SAN synchronously through the proposed distributed synchronization algorithm.
3. The stability of the proposed framework is analyzed with the conventional Lyapunov function under an undirected and dynamic communication topology.

The paper is organized as follows. Section 2 briefly outlines related matrix theory, graph theory, and the classical discrete-time consensus protocol for MAS research. Section 3 presents a discrete-time consensus protocol framework based on neighbor selection strategies for the MAS with dynamic topology. Section 4 introduces the proposed RSRSP strategy optimized for improved convergence performance. In Section 5, we present stability analyses of the framework based on different neighbor selection strategies are presented. In Section 6, we perform extensive simulations to demonstrate that the proposed framework with the two strategies can enhance the consensus of the MAS. Finally, conclusions are drawn in Section 7.

2 Preliminaries

2.1 Graphs and matrices

Graph and matrix theory are useful tools for studying MASs, and can be used to describe the communication relationships between agents. The communication network topology of the MASs in this paper is described by an undirected dynamic graph $G(V, E)$, where $V = \{1, 2, 3, \cdots, n\}$ corresponds to the n agents; $E = \{(i, j) : i, j \in V, i \neq j\}$ is the set of communication links among all agents. Because G is an undirected graph, $(i, j) \in E \Leftrightarrow (j, i) \in E$ indicates that agent i can exchange information with agent j mutually. $N_i = \{j \in V : (i, j) \in E\}$ is the neighbor set of agent i and indicates that agent i can exchange information with any agent j in N_i.

The adjacency matrix $A = \left[a_{ij}\right] (i, j \in V)$ has nonzero elements; it satisfies the symmetry property, which reflects the connectivity relationship between agents. The elements in the adjacent matrix are described as

$$a_{ij} = \begin{cases} 1, if\ (i, j) \in E \\ 0, otherwise \end{cases} \tag{1}$$

The degree matrix D of graph G is defined as $D = diag\{d_i\}$, where diagonal elements are $d_i = \sum_{j \neq i} a_{ij}$ and off-diagonal elements are zero. Accordingly, the Laplacian matrix of G is defined as $L = D - A$. The elements in $L = \left[l_{ij}\right]$ are described as

$$l_{ij} = \begin{cases} \sum_{j=1}^{n} a_{ij}, i = j \\ -a_{ij}, i \neq j \end{cases} \tag{2}$$

The graph Laplacian of the undirected graph G exhibits the following properties.

1. L is symmetric.
2. L is positive semidefinite.
3. Let $0 = \lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_3$ be the eigenvalues of L. If G is connected, then $\lambda_1 = 0, \lambda_2 \geq 0$.
4. λ_2 is called the algebraic connectivity of graph G.

2.2 Traditional discrete time distributed consensus protocol

The graph of an MAS with dynamic topology is given by $G(V(k), E(k))$, which is time-varying. The traditional discrete-time distributed consensus protocol is given by

$$\begin{aligned} x_i(k+1) &= x_i(k) + u_i(k) \\ u_i(k) &= \alpha \sum_{j \in N_i(k)} \left(x_j(k) - x_i(k)\right) \end{aligned} \tag{3}$$

The state of the ith agent at time k is denoted by $x_i(k)$, which can be considered as the position, speed, angle, opinion and so on. The set of all agents is defined as $Agent = \{1, 2, \cdots, n\}$. $X(k) = \left\{x_i(k) \in \mathbb{R}^{n \times m}, i \in Agent\right\}$ ($m = 2$ in this paper) is the vector of state of agents at time k. $u_i(k)$ is the control input, and α is a parameter affecting the MAS convergence. $r_c \in \mathbb{R}^+$ is the communication radius of all agents. The set of edges of system topology at time k is $E(k) = \left\{(i, j) : \left\|x_i(k) - x_j(k)\right\| \leq r_c, i, j \in Agent, j \neq i\right\}$. The set of neighbors of agent i at time k is defined as

$$N_i(k) = \{j : \|x_j(k) - x_i(k)\| \leq r_c, i, j \in Agent, i \neq j\} \tag{4}$$

3 The proposed framework

3.1 Problem statement

Each agent needs to select all neighbors in a communication region to update its state using a consensus protocol (3) based on the SAN algorithm. However, we note that each agent has a significant number of neighbors during each evolution period in large-scale systems, thus requiring agents to have high computational power and data storage for the system evolution, and increasing the agent costs. Moreover, more communication does not necessarily result in faster convergence and may even lead to a loss of convergence. Thus, autonomic and intelligent agents should select representative neighbors and delete redundant communication links to reduce data storage after evaluating the network topology of the system.

To overcome the drawbacks of the protocol (3), we devise a decentralized consensus framework based on two neighbor selection strategies constructed by referring to local neighbors' states for computing. The control objective is that the communication network topology is connected throughout the evolution process if the MAS evolves using a consensus protocol based on neighbor selection strategies, and the system can eventually reach a consensus if the following formula is satisfied for all agents $i \neq j$:

$$\lim_{k \to \infty} \|x_i(k) - x_j(k)\| = 0 \tag{5}$$

3.2 Consensus protocol framework based on neighbor selection strategy with dynamic topology

Because agents possess intelligence and autonomy, they have a corresponding ability to select candidate neighbor

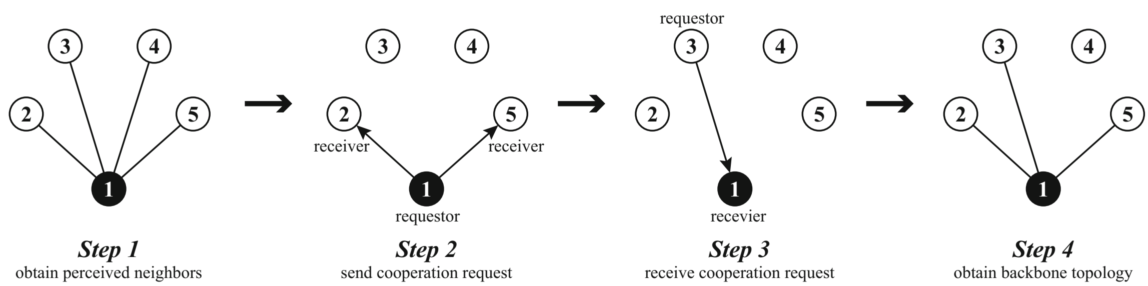


Fig. 1 Illustration of the changing process from original communication topology to backbone network. For example, agent 1 observes the states of neighbors $N_i(k) = \{2, 3, 4, 5\}$ (i.e., step 1), then it actively sends a cooperation request signal to some particular neighbors in $A_1(k) = \{2, 5\}$ (i.e., step 2) and receives the cooperation request signal from $P_1(k) = \{3\}$ (i.e., step 3). Finally, the backbone topology can be obtained according to $H_1(k) = A_1(k) \cup P_1(k) = \{2, 3, 5\}$ (i.e., step 4). $A_i(k)$ and $P_i(k)$ denote the sets of request cooperation signals sent and received, respectively, by agent i

sets for cooperation. A consensus protocol, based on the neighbor selection strategy, is given by

$$x_i(k+1) = x_i(k) + u_i(k)$$
$$u_i(k) = \alpha \sum_{j \in H_i(k)} \left(x_j(k) - x_i(k)\right) \tag{6}$$

In formula (6), it is important to determine how to obtain the candidate neighbor set $H_i(k)$, consisting of the following two parts: neighbors who receive a cooperation request signal from agent i and neighbors who actively send a cooperation request signal to agent i. For example, Fig. 1 shows the procedure that Agent 1 uses to select $H_i(k)$. The steps of the evolution cycles of the proposed framework are summarized in Table 1, where $A_i(k)$ and $P_i(k)$ denote the sets of request cooperation signals sent and received, respectively, by agent i. Here, $A_i(k) \subseteq N_i(k)$ and $P_i(k) \subseteq N_i(k)$.

The evolution cycles of the proposed MAS framework are denoted by $T = t_1 + t_2 + t_3 + t_4 + t_5$, which are designed in a distributed and synchronous fashion. $H_i(k)$, where $H_i(k) \subseteq N_i(k)$ and $H_i(k) = A_i(k) \bigcup P_i(k)$, is the neighbor set of agent i in the backbone network in the kth evolution cycles. It is also called the candidate set of agents for convergence evolution.

Accordingly, $u_i(k)$ in formula (6) is modified as

$$u_i(k) = \alpha \sum_{j \in A_i(k) \bigcup P_i(k)} \left(x_j(k) - x_i(k)\right) = \alpha \left(\sum_{j \in A_i(k)} \left(x_j(k) - x_i(k)\right) + \sum_{j \in P_i(k), j \notin A_i(k)} \left(x_j(k) - x_i(k)\right) \right) \tag{7}$$

When all agents evolve using formula (7) and satisfy

$$\lim_{k \to \infty} \|x_i(k) - x_j(k)\| = 0, \ \forall i, j \in Agent, j \neq i, k \to \infty \tag{8}$$

, the MAS reaches a consensus.

4 Neighbor selection strategy

To meet the control objectives presented in Fig. 2, we develop different strategies to obtain $H_i(k)$ under the framework of the consensus protocol based on the neighbor selection strategy. First, we introduce the communication sector division in Section 4.1. We then propose the RSRSP strategy and optimized RSRSP strategy to obtain appropriate and representative neighbors in Sections 4.2 and 4.3, respectively.

4.1 Communication sector division

The broadcast radius r_c defines a circle centered at agent i, which is the communication region S of agent i, where $S \in \mathbb{R}^2$. Agent i segments its S equally into m sectors after evaluating the distribution of all its neighbors' states in S, and $U = \{1, 2, \cdots, m\}$ corresponds to the m sectors. For mathematical convenience, the communication region of agent i is located at rectangular coordinates and centered at the origin of the coordinates, as shown in Fig. 3.(a). In Fig. 3.(b), agent i rotates all sectors β counterclockwise so that the variance of the number of neighbors in each sector is minimal. $x^+(\beta)$ represents the rotation angle of the coordinate system.

Table 1 Outline of the proposed framework

	At each time k, for each agent i with state $x_i(k)$, it	acc. to
Step 1	observes the state distribution of neighbors $N_i(k)$	formula (4)
Step 2	actively sends the cooperation request signal to all neighbors in $A_i(k)$	Section 4
Step 3	receives the cooperation request signal from $P_i(k)$	Section 4
Step 4	establishes cooperation connections with receivers and forms the backbone topology	-
Step 5	updates its state by referring to all of its neighbors in $H_i(k)$	formula (7)

In the kth evolution cycle, the neighbor set of agent i in the uth sector is defined as

$$N_i^u(k) = \left\{ j : \frac{2\pi(u-1)}{m} < \theta\left(\overrightarrow{ij}, x^+(\beta)\right) \leqslant \frac{2\pi u}{m}, \ and \ \left\| x_j(k) - x_i(k) \right\| \leqslant r_c \right\} \tag{9}$$

where $i, j \in Agent$ and $i \neq j$; $\overrightarrow{ij}$ is the vector from agent i to agent j; $\theta\left(\overrightarrow{ij}, x^+(\beta)\right)$ is the radian between vector $\overrightarrow{ij}$ and $x^+(\beta)$; $\frac{2\pi(u-1)}{m}$ and $\frac{2\pi u}{m}$ are radian boundaries of sector u. Therefore, the neighbor set of agent i in the kth evolution cycle is also defined as

$$N_i(k) = N_i^1(k) \bigcup N_i^2(k) \bigcup \cdots \bigcup N_i^m(k) \tag{10}$$

Agent i actively selects the most representative neighbor in each sector to construct the backbone network. This reduces the amount of information on neighbors' states that agent i must store for evolution.

In summary, formula (7) can be modified as

$$u_i(k) = \alpha \left(\begin{array}{l} \sum\limits_{u=1}^{m} \left(x_{i^u}(k) - x_i(k) \right) \\ + \sum\limits_{c \in P_i(k), c \notin A_i(k)} \left(x_c(k) - x_i(k) \right) \end{array} \right) \tag{11}$$

The above consensus protocol is illustrated as follows:

1. i^u, where $i^u \in N_i^u(k)$ and $i^u \in A_i(k)$, is a neighbor of agent i. Agent i actively sends a cooperation request signal to neighbor i^u in the uth sector. Then, agent i puts i^u in $H_i(k)$. Here, $x_{i^u}(k)$ is the state of agent i^u in the kth evolution cycle.
2. Agent i is a neighbor of agent c, where $i \in N_c^v(k)$, $i \in A_c(k)$ and $c \neq i^u$. Agent c actively sends a cooperation request signal to neighbor i in the vth sector. Subsequently, agent i agrees to cooperate with agent c and puts $c \in P_i(k)$ in $H_i(k)$. Here, $x_c(k)$ is the state of agent c in the kth evolution cycle.
3. α is the gaining parameter, where $\alpha > 0$.

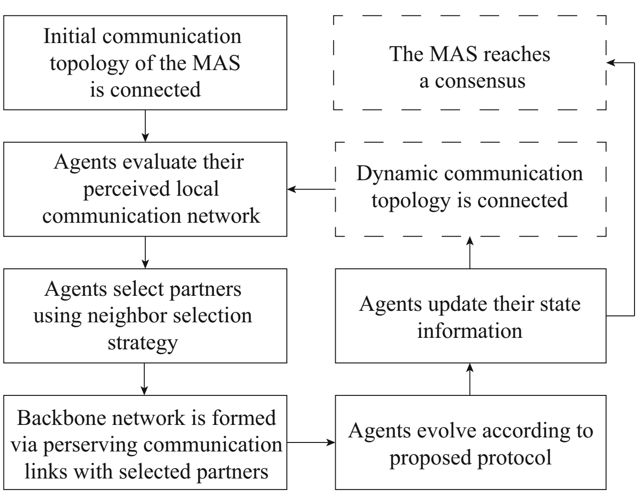


Fig. 2 Distributed closed-loop feedback control system

4.2 RSRSP strategy

In this section, we present the RSRSP strategy. First, agent i segments its communication region S equally into $m = 4$ sectors. Agent i rotates all sectors γ degrees after observing the distribution of the neighbors' states to make the number of neighbors in each sector as uniform as possible. Then, agent i actively selects the agent in each sector from which it has the minimum difference in states in each sector to join $H_i(k)$. For example, if agent c has the minimum difference in states from agent i in the vth sector, then agent i sends a cooperation request signal to agent c and puts c in $H_i(k)$.

Figure 4 shows the procedures that agent i uses to select neighbors to join $H_i(k)$ for evolution by Strategy 1. It is possible that there are no neighbors in the uth sector of agent i, i.e., $N_i^u(k) = \emptyset$. Therefore, the neighbors actively selected by agent i can number no more than 4, i.e., $|A_i(k)| \leq 4$. The number of c is unknown.

Strategy 1 Representative selection with rotating-segmentation perception.

In the kth evolution cycle,

1: Agent i segments its communication region S equally into four sectors after evaluating the distribution of states of $N_i(k)$.

2: After evaluating the distribution of states of $N_i(k)$, agent i rotates all sectors $\gamma = \arg\min D_i(NumF(\beta))$, where β denotes the angle of rotation, $NumF(\beta)$ is a set representing the number of agents in the four sectors after rotating β degrees, and D_i represents the variance, so that the number of neighbors in each sector is as uniform as possible.

3: Agent i actively selects agent $\min(i^u) = \arg\min \|x_{i^u}(k) - x_i(k)\|$ in each sector to join $H_i(k)$ for updating its states.

4: $\exists c$, where $c \neq \min(i^u)$and $c \in P_i(k)$. Agent c actively selects agent $i = \min(c^v) = \arg\min \|x_{c^v}(k) - x_c(k)\|$ to cooperate; Then, agent i selects c to join $H_i(k)$ for updating its states.

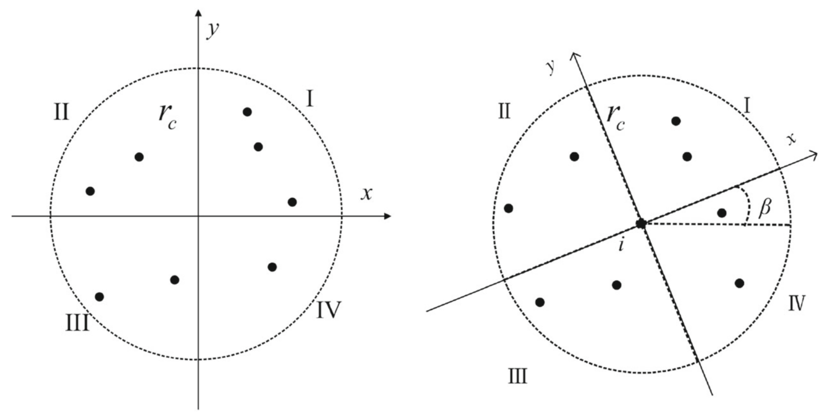


Fig. 3 Illustration of communication sector division

(a) Communication sector $x^+(0)$ (b) Optimization communication sector $x^+(\beta)$

From the above analysis, formula (11) can be modified as

$$u_i\left(k\right)=\alpha\left(\begin{array}{l}\sum\limits_{u=1}^{m}\left(x_{i^u_{min}}\left(k\right)-x_i\left(k\right)\right)\\+\sum\limits_{c\in P_i(k)c\notin A_i(k)}\left(x_c\left(k\right)-x_i\left(k\right)\right)\end{array}\right)\tag{12}$$

where i^u_{min} represents the agent with the minimum difference in states from agent i in the uth sector.

4.3 Optimized RSRSP strategy

Algorithm 1 Synchronous switching of strategies.

Each agent i has two parameters: $tag_i(k)$ and $Token_i(k)$.
$tag_i(k)$: if the number of agent i's neighbors is $n-1$, then $tag_i(k)$=1, else $tag_i(k)$=0.
$Token_i(k)$=[$tag_1(k), \cdots, tag_i(k), \cdots, tag_n(k)$] records the surrounding connectivity.

```
Initialization: For each agent i, Token_i(k)=[0, 0, ···, 0] and update its own tag_i(k) according to the number of its neighbors.
for i = 1, 2, ···, n
  if tag_i(k)=1, then
    Agent i collects tag_j(k) from N_i(k) and
    Agent i updates Token_i(k);
    if all(Token_i(k))==1, then
      Agent i changes the RSRSP into the SAN
    else
      Agent i still adopts the RSRSP
    end if
  else
    Agent i still adopts the RSRSP
  end if
end for
```

Strategy 1 is an effective way to enhance consensus when the system is not yet fully connected. However, when the system is fully connected, the SAN algorithm has a better convergence performance. Therefore, we expect that through some simple mechanisms, the system can switch strategies after reaching full connectivity to optimize its entire convergence process. Because of the lack of a central controller in the distributed MAS, the MAS cannot broadcast a command for all agents to switch the neighbor selection strategy concurrently. Agents can only exchange information with neighbors within a limited communication range. Consequently, we design a distributed algorithm, i.e., Algorithm 1, which is used in Strategy 2 to ensure that all agents in the system can switch the neighbor selection strategy at the same time k.

Strategy 2: the MAS evolves using Strategy 1 before the communication network topology of the MAS is fully connected. Once the communication network topology of the MAS reaches full connectivity, all agents switch from RSRSP to SAN for evolution (realized by Algorithm 1).

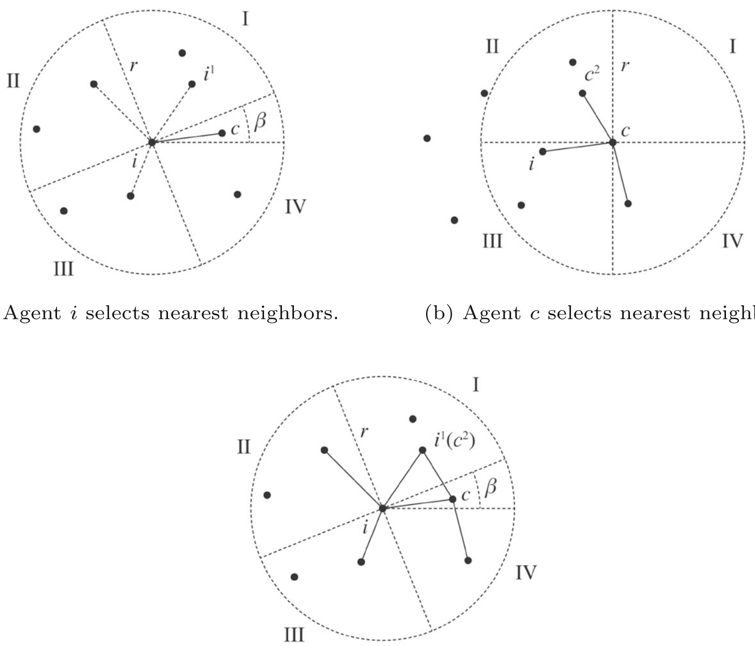
From the above analysis, formula (11) can be rewritten as

$$\begin{array}{l}\exists i,\, N_i\left(k\right)<n-1,\\ \quad u_i\left(k\right)=\alpha\left(\begin{array}{l}\sum\limits_{u=1}^{m}\left(x_{\min i^u}\left(k\right)-x_i\left(k\right)\right)+\\ \sum\limits_{c\in P_i(k),c\notin A_i(k)}\left(x_c\left(k\right)-x_i\left(k\right)\right)\end{array}\right)\\ \forall i,\, N_i\left(k\right)=n-1,\\ \quad u_i\left(k\right)=\alpha\sum\limits_{j\in N_i(k)}\left(x_j\left(k\right)-x_i\left(k\right)\right)\end{array}\tag{13}$$

5 Stability analysis

For the convenience of analysis, we assume that the initial communication network is connected in this section.

Fig. 4 The procedures that agent i uses to select neighbors to join $H_i(k)$ by the RSRSP



(a) Agent i selects nearest neighbors.

(b) Agent c selects nearest neighbors.

(c) Agent i obtains $H_i(k)$

5.1 Parameter range of strategy 1

Lemma 1. *A necessary and sufficient condition for a negative semi-definite matrix is that all eigenvalues of a real symmetric matrix are no more than 0* [49].

Using Lemma 1, we can establish primary results as follows.

Theorem 1. *Assume that the communication network of the MAS is always connected throughout the convergence evolution and the MAS evolves by Strategy 1.*

If $n \le 14$ *and the parameter satisfies* $0 < \alpha \le 2/n$ *, then the MAS is asymptotically stable and reaches a consensus.*

If $n > 14$ *and the parameter satisfies* $0 < \alpha \le 2/15$ *, then the MAS is asymptotically stable and reaches a consensus.*

Proof of Theorem 1 Formula (6) can be modified as

$$X(k+1) = X(k) - \alpha\overline{L}(k)X(k) = (I - \alpha\overline{L}(k))X(k) \tag{14}$$

Lyapunov function is given by

$$V(X(k)) = \frac{1}{2}X^T(k)X(k) \tag{15}$$

which is positive definite. Then, we derive the following

$$\begin{aligned}
\Delta V(X(k)) &= V(X(k+1)) - V(X(k)) \\
&= \tfrac{1}{2}X^T(k+1)X(k+1) - \tfrac{1}{2}X^T(k)X(k) \\
&= \tfrac{1}{2}X^T(k)(I-\alpha\overline{L}(k))^T(I-\alpha\overline{L}(k))X(k) - \tfrac{1}{2}X^T(k)X(k) \\
&= \tfrac{1}{2}X^T(k)(I-2\alpha\overline{L}(k)+\alpha^2\overline{L}^2(k))X(k) - \tfrac{1}{2}X^T(k)X(k) \\
&= \tfrac{1}{2}X^T(k)\left(-2\alpha\overline{L}(k)+\alpha^2\overline{L}^2(k)\right)X(k) \\
&= \alpha X^T(k)\left(-\overline{L}(k)+\tfrac{1}{2}\alpha\overline{L}^2(k)\right)X(k)
\end{aligned} \tag{16}$$

Let $W(k) = -\overline{L}(k) + \frac{1}{2}\alpha\overline{L}^2(k)$. The MAS is asymptotically stable in the sense of Lyapunov when $W(k)$ is a negative semidefinite matrix based on the Lyapunov stability theory. $\overline{L}(k)$can be expressed as

$$\overline{L}(k) = [l_1(k), l_2(k), l_3(k), \cdots, l_n(k)]^T, \tag{17}$$

so $\overline{L}^2(k)$ can be expressed as

$$\overline{L}^2(k) = [l_1(k), l_2(k), l_3(k), \cdots, l_n(k)]^T \cdot [l_1(k), l_2(k), l_3(k), \cdots, l_n(k)]. \tag{18}$$

Namely,

$$\overline{L}^2(k) = \begin{bmatrix} \langle l_1(k), l_1(k)\rangle & \langle l_1(k), l_2(k)\rangle & \cdots & \langle l_1(k), l_n(k)\rangle \\ \langle l_2(k), l_1(k)\rangle & \langle l_2(k), l_2(k)\rangle & \cdots & \langle l_2(k), l_n(k)\rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle l_n(k), l_1(k)\rangle & \langle l_n(k), l_2(k)\rangle & \cdots & \langle l_n(k), l_n(k)\rangle \end{bmatrix} \tag{19}$$

where $\langle l_i(k), l_j(k)\rangle$ represents the element at row i and column j of $\overline{L}^2(k)$ at time k.

Because the graph is undirected, the corresponding Laplacian matrix $\overline{L}^2(k)$ is symmetric, and $\overline{L}^2(k)$ is also symmetric.

The diagonal elements of $\overline{L}^2(k)$ are

$$\langle l_i(k), l_i(k)\rangle = n_i^2(k) + n_i(k) \tag{20}$$

where $n_i(k)$ represents the number of neighbors of agent i at time k. The sum of off-diagonal elements in the same row of $\overline{L}^2(k)$ is

$$\sum_{j\neq i}\left\langle l_i(k), l_j(k)\right\rangle = \left\langle l_i(k), \sum_{j\neq i} l_j(k)\right\rangle = \tag{21}$$
$$\langle l_i(k), -l_i(k)\rangle = -\langle l_i(k), l_i(k)\rangle = -(n_i{}^2(k) + n_i(k)).$$

Thus, in $W(k)$, the diagonal elements are

$$\begin{aligned} w_{ii}(k) &= -l_{ii}(k) + \frac{\alpha}{2}\langle l_i(k), l_i(k)\rangle \\ &= -n_i(k) + \frac{\alpha}{2}\left(n_i{}^2(k) + n_i(k)\right). \end{aligned} \tag{22}$$

And the off-diagonal elements in the same row satisfies

$$\sum_{j\neq i} w_{ij}(k) = n_i(k) - \frac{\alpha}{2}\left(n_i{}^2(k) + n_i(k)\right). \tag{23}$$

According to the Gerschgorin theorem, all eigenvalues of matrix $W(k)$ are in the union of n discs:

$$|\lambda - w_{ii}(k)| \leq \sum_{j\neq i} |w_{ij}| \tag{24}$$

Under Lemma 1, if matrix $W(k)$ is a negative semidefinite matrix, then all eigenvalues of $W(k)$ are no more than 0.

First, ensuring that the centers of all discs are to the left of the complex plane, namely, $w_{ii}(k) \leq 0$, we obtain

$$\begin{aligned} &w_{ii}(k) = -n_i(k) + \frac{\alpha}{2}\left(n_i{}^2(k) + n_i(k)\right) \leq 0 \\ &\Rightarrow \alpha \leq 2/(n_i(k) + 1) \end{aligned} \tag{25}$$

If agents use the SAN algorithm, then $\max(n_i(k)) = n - 1$, and we can derive $\alpha \leq 2/n$.

Second, the radius of the discs is $\sum_{j\neq i} |w_{ij}|$, which can be rewritten as

$$\sum_{j\neq i} |w_{ij}| = \sum_{j\neq i} w_{ij} = n_i(k) - \frac{\alpha}{2}\left(n_i{}^2(k) + n_i(k)\right) = -w_{ii}(k) \tag{26}$$

In summary, we derive the Euclidean distance from the center of the discs to the origin of the coordinates as the radius of the discs. $W(k)$ is a negative semidefinite matrix if $W(k)$ satisfy inequality (25), that is, the centers of all discs are to the left of the complex plane. Then, if the above condition (25) is met, the MAS is asymptotically stable in the sense of Lyapunov.

Additionally, by observing a significant number of simulation results in Section 6.4, we can determine that the max degree of agent i is $max(n_i(k)) \leqslant 14$. Therefore, the theorem is proved. □

5.2 Parameter range of strategy 2

Theorem 2. *Assume that the communication network of the MAS is always connected throughout the convergence evolution and the MAS evolves by Strategy 2. If the parameter* α *satisfies Theorem 1 before the communication network is fully connected and the parameter satisfies* $0 < \alpha \leq 2/n$ *when the communication network is fully connected, then the system is asymptotically stable and reaches a consensus.*

Proof of Theorem 2 Before the communication network is fully connected, agent i selects neighbors to join $H_i(k)$ by Strategy 1. Therefore, the parameter should satisfy Theorem 1. When the communication network is fully connected, the parameter should satisfy $0 < \alpha \leq 2/n$ because the MAS will select $n - 1$ neighbors to form $H_i(k)$. Then, the MAS is asymptotically stable and reaches a consensus. □

6 Simulations

In this section, we perform a series of simulations to accomplish the following targets. We refer readers to https://github.com/kyoran/RSRSP for more details of the simulations.

1. Explore the relationship between different initial topologies and convergence performance.
2. Validate the consensus protocol, based on the neighbor selection strategy, by observing the evolution of MASs and convergence of the communication topology.
3. Compare convergence rate and resource consumption, including the data storage and computational load between two neighbor selection strategies.

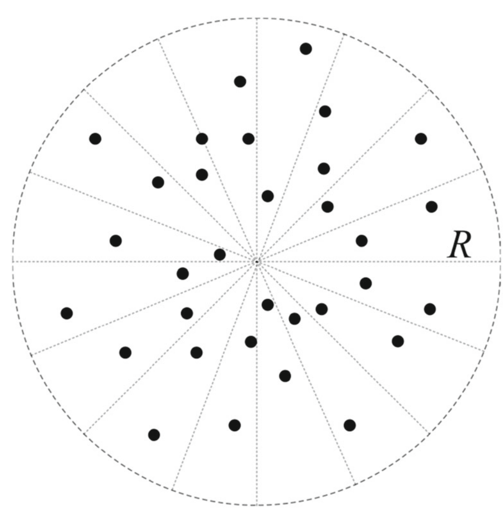


Fig. 5 The initial distribution of the agents

Table 2 Comparisons between the SAN algorithm and the two proposed strategies

initial topology	$DAS(n, R)$				
	(n, R)	$(\geq 20, 1.5)$	$(\geq 35, 2)$	$(\geq 50, 2.5)$	$(\geq 100, 3)$
SAN	$\overline{cn}$	1.18	2.94	5	7.2
Strategy 1	$\overline{cn}$	1	1	1.14	1.08
Strategy 2	$\overline{cn}$	1	1	1.14	1.08

4. Obtain the maximum number of neighbors that each agent can select by Strategy 1.

6.1 Relationship between different initial topologies and convergence performance

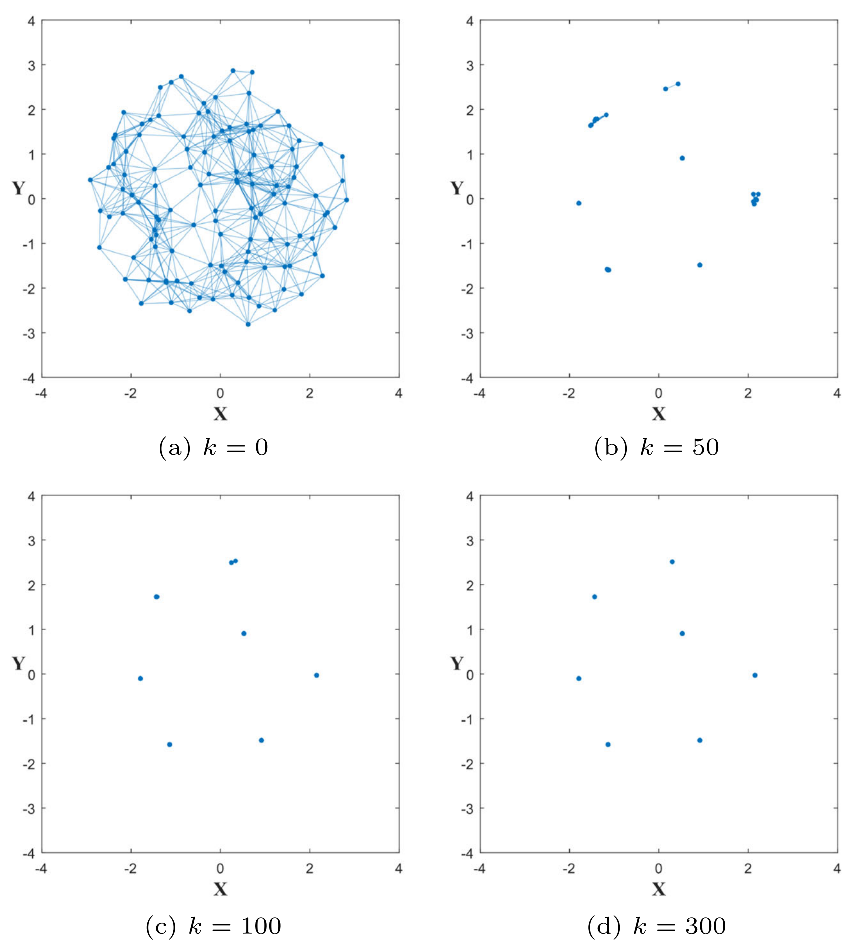
The initial distribution of the MASs shown in Fig. 5 is called distribution by average sector, $DAS(n, R)$. All agents are randomly distributed in a circular region of radius R, and the number of agents distributed randomly in each sector is the same. We evaluate the convergence performance by the number of clusters when the MAS is stable [24]. A smaller cluster number indicates better convergence performance.

Supposing that the MASs' communication topology is initially connected, for each n-R, we repeat the simulation 50 times to obtain stable results in which the number of agents in each simulation is evenly distributed according to the range in the second row of Table 1. The results are presented in Table 2, where $\overline{cn}$ is the average number of clusters. We observe that $\overline{cn}$ of the SAN algorithm gradually increases with increasing number of agents while Strategies 1 and 2 can finally stabilize to fewer clusters without the connectivity preservation algorithm. Therefore, our strategies can efficiently enhance the consensus without additional complicated connectivity preservation algorithms.

6.2 Validation of consensus protocols based on three neighbor selection strategies

We consider that the MASs consist of $n = 120$ agents with communication radius $r_c = 1$. These agents are represented by dots on a 2-D coordinate plane and are randomly distributed on the circular region of radius $R = 3$, and centered at the origin of the coordinates. The positions of all agents in the 2-D coordinates are set artificially to guarantee

Fig. 6 Variation in the communication network during evolution using the SAN algorithm



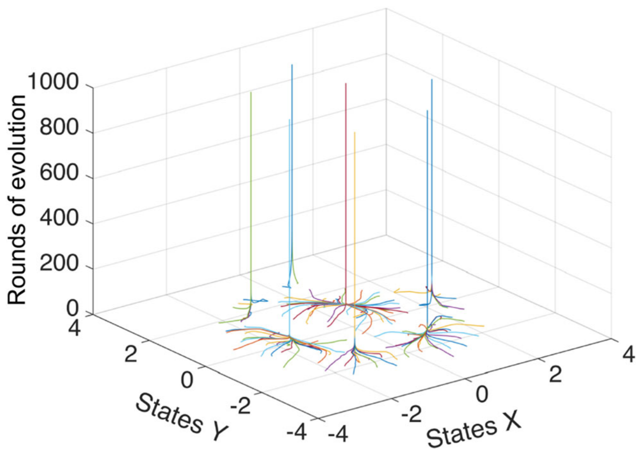


Fig. 7 The evolution of the agents' states using the SAN algorithm

the initial connectivity of communication topology. We verify the convergence effect of the two proposed strategies, and visually compare them with the convergence effect of the SAN algorithm on the same initial distribution of agents.

6.2.1 SAN

The MASs evolve according to formula (3) based on the SAN algorithm, and the parameter satisfies $\alpha = 0.01$. Figures 6 and 7 show that the MASs with an initially connected communication topology, cannot preserve connectivity throughout the evolution and converge quickly to seven clusters.

6.2.2 Strategy 1

The MASs evolve according to formula (12) based on Strategy 1. The parameter satisfies $\alpha = 0.1$, and the communication region of each agent is segmented equally into four sectors. Figures 8 and 9 show that the MASs' communication topology is connected throughout the evolution. The MASs converge to a single cluster asymptotically.

6.2.3 Strategy 2

The MASs evolve according to formula (13) based on Strategy 2. The parameters satisfy $\alpha_1 = 0.1$ and $\alpha_2 = 0.01$, and the communication region of each agent is segmented equally into four sectors. Figure 10 shows that the communication topology of the MASs with initial connectivity is connected throughout the evolution. The convergence of Strategy 1 is similar to that of Strategy 2 before communication topology is fully connected. Once the communication topology of the MASs becomes fully connected, the MASs reach a consensus quickly.

From the above simulations, we observe that the MASs converge to a single cluster under the proposed strategies whereas they converge to seven clusters under the SAN algorithm. Thus, Strategy 1 and Strategy 2 enhance the consensus of the MASs and reduce the number of clusters after the MASs converge. Additionally, the convergence speed of Strategy 2 is faster than that of Strategy 1.

6.3 Comparison between SAN and proposed strategies

The second smallest eigenvalue of the network topology's Laplacian matrix, λ_2, is the algebraic connectivity, and it can act as an indicator of the connectivity of the MAS [50]. The greater the algebraic connectivity is, the faster the convergence rate.

As shown in Fig. 11, under Strategies 1 and 2, the algebraic connectivity of the communication network gradually becomes bigger; the connectivity of the communication network improves before the network reaches full connectivity. When MASs evolve using the SAN algorithm, the algebraic connectivity of the communication network quickly decreases to 0 because of the destruction of the connectivity of the communication network.

As shown in Fig. 12, the algebraic connectivity of Strategy 2 is the same as that of Strategy 1 before the communication network reaches full connectivity. Once the communication topology of the MASs is fully connected, the algebraic connectivity of Strategy 2 is higher than that of Strategy 1. We also observe that the convergence rate of Strategy 2 is the same as that of Strategy 1 before the communication topology reaches full connectivity. Subsequently, the convergence of Strategy 2 is faster than that of Strategy 1, once the communication network reaches full connectivity.

Additionally, we use the sum of the edges of the backbone network to reflect the data storage and computational loads of the MASs. As shown in Fig. 13, the data storage and computational loads of the MASs based on the SAN algorithm and the two proposed strategies are approximately equal to 1600 and 200, respectively. However, data storage and computational load in the two new strategies are around 200. Therefore, the data storage and computational loads of Strategy 1 are significantly reduced compared with those of the SAN algorithm.

From the simulations, we conclude the following:

1. Strategies 1 and 2 enhance connectivity and convergence. If the parameter α satisfies Theorem 1 and $n - R$ satisfies Table 2, the MASs can asymptotically converge to smaller cluster in finite time based on Strategy 1 or Strategy 2.

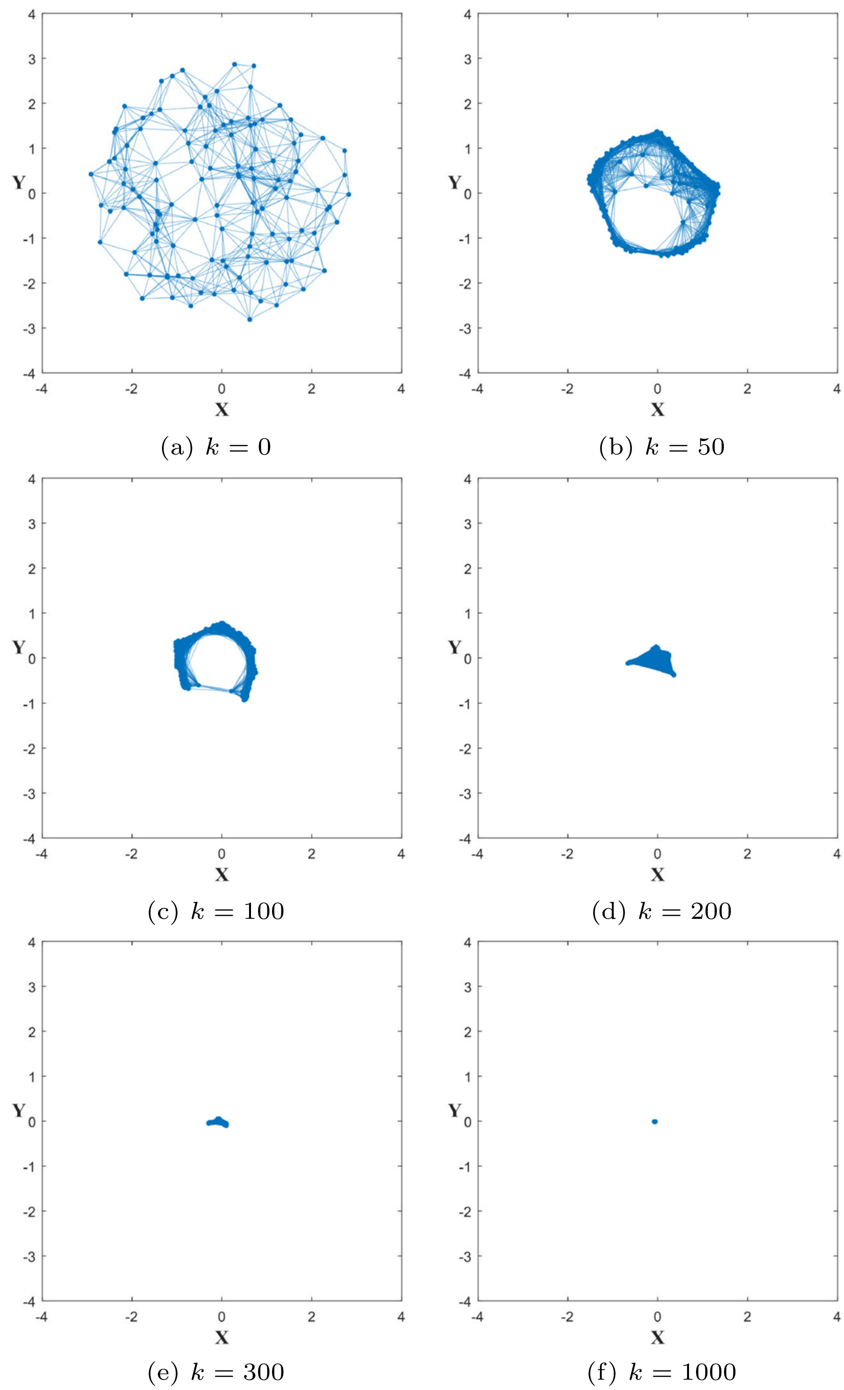


Fig. 8 Variation in the communication network during evolution using Strategy 1

2. The data storage and computational load of RSRSP are significantly reduced compared with those of the SAN algorithm.

6.4 Obtaining the maximum number of neighbors each agent can select

In this section, n agents are placed in the circle of radius R, and the possible distributions of agents are shown in Fig. 14.

We perform a series of simulations to obtain the maximum number of neighbors each agent can select by Strategy 1. For each $n - R$ in Fig. 14, 50 simulations are conducted. The convergence condition of the MASs is $\left\| x_j(k) - x_i(k) \right\| \leq 0.02$ and $i, j \in Agent$. The maximum numbers of neighbors that an agent can select by Strategy 1 are observed, and the results are shown in Table 3. Interestingly, we observe that $|H_i(k)| \leq 14$ if the MASs select neighbors to form the candidate neighbor set $H_i(k)$

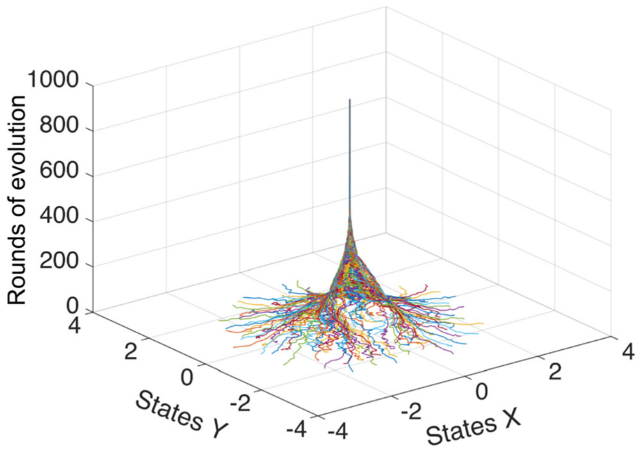


Fig. 9 The evolution of the agents' states using Strategy 1

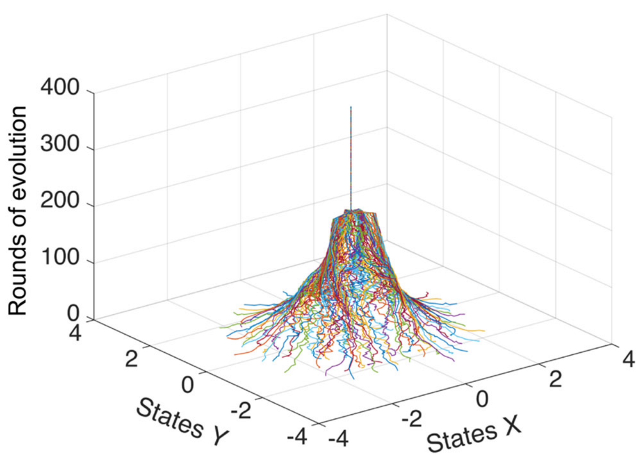


Fig. 10 The evolution of the agents' states using Strategy 2

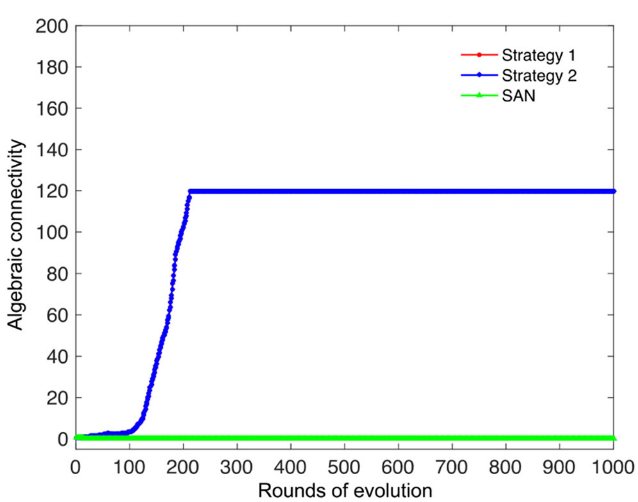


Fig. 11 Variation in the algebraic connectivity of the communication network during evolution using the SAN algorithm and the two proposed strategies

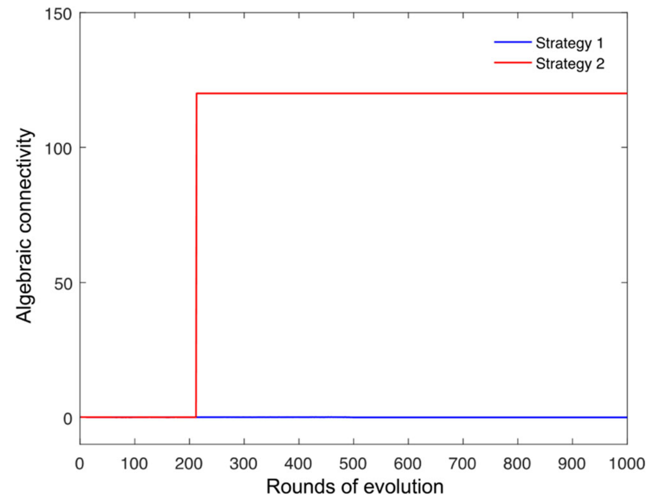


Fig. 12 Variation in the algebraic connectivity of the backbone topology during evolution using Strategy 1 and Strategy 2

based on Strategy 1 or Strategy 2. In other words, the maximum degree of each agent is no more than 14, i.e., $\max(n_i(k)) \leq 14$.

6.5 Further exploration of resource consumption of the three strategies

In this section, we compare the numbers of agents corresponding to the different degrees of backbone topology after the 120-agent MASs obtain different backbone topologies based on the SAN algorithm, Strategy 1, and Strategy 2. As mentioned in Section 4.3, once MASs become fully connected, agents will switch from RSRSP to SAN. Therefore, the resource consumption of Strategy 2 is the same as that of Strategy 1.

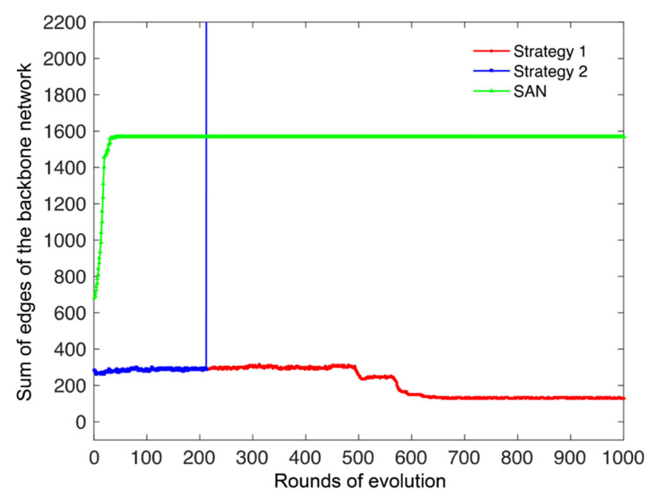


Fig. 13 Variation in the sum of the edges of the backbone network during evolution using the SAN algorithm and the two proposed strategies

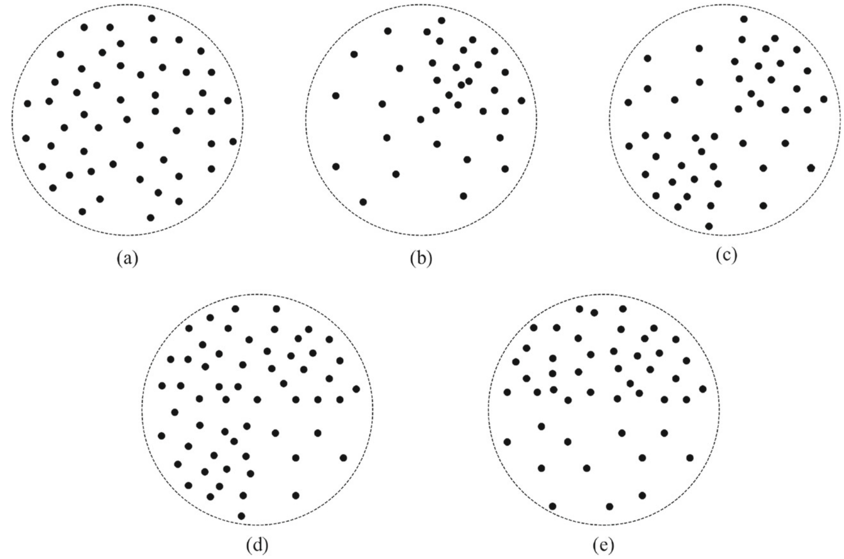


Fig. 14 The distributions of agents: (a) all agents are distributed relatively equally in the circular region; (b) most agents are distributed in the first quadrant;(c) most agents are distributed in the first quadrant and the third quadrant; (d) most agents are distributed in the first, second, and third quadrants; (e) most agents are distributed in the first and second quadrants

Table 3 The maximum degree $\max(n_i(k))$ of agents

initial topology	R: the radius of the distribution of MASs					
	0.01	0.1	0.5	1	2	3
$n = 100$	12	12	13	13	12	11
$n = 200$	13	12	13	13	13	12
$n = 300$	12	12	13	12	14*	12
$n = 400$	13	13	12	13	12	11
$n = 500$	13	12	12	13	13	13
$n = 1000$	13	12	13	12	13	13

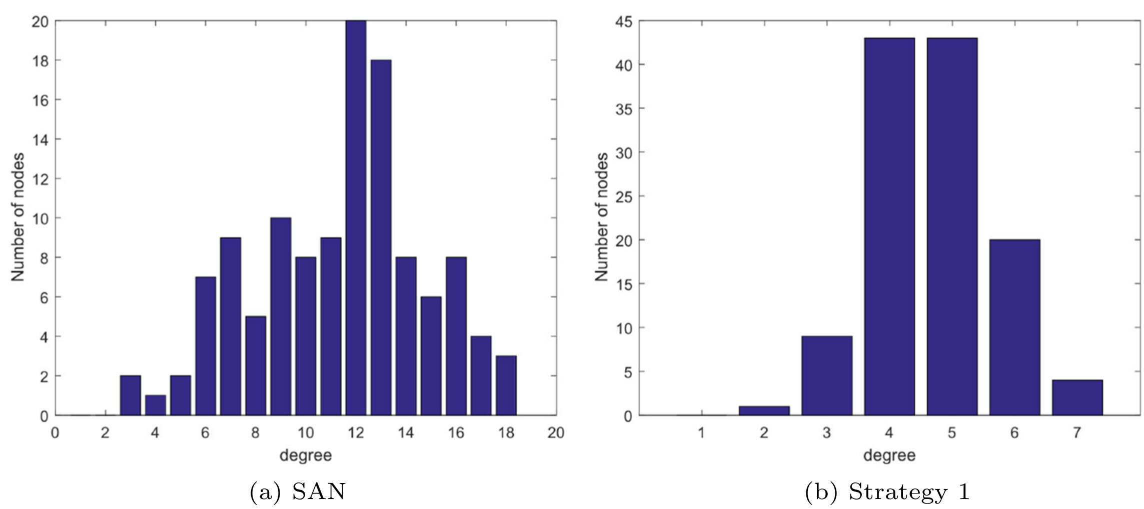


Fig. 15 The degrees of two backbone topologies

Table 4 The data storage and computational loads using the SAN algorithm and the two proposed strategies

strategies	data storage	computational load
SAN	2 to n	1 to $n-1$
Strategy 1	2 to 15	1 to $n-1$
Strategy 2	2 to n	1 to $n-1$

Figure 15 demonstrates that the maximum degree of backbone topology based on the two novel strategies is significantly lower than that based on the SAN algorithm. According to Table 3 and Fig. 15b, we obtain that the maximum degree of Strategy 1 is no more than 14, so that the data storage capacity of each agent ranges from 2 to 15 and the computational loads range from 1 to 14. The data storage and computational loads of the three strategies are shown in Table 4; each agent's data storage ranges from 2 to n, and the computational load ranges from 1 to $n-1$.

7 Conclusion

This paper focuses on a distributed discrete-time consensus protocol of the MAS with an undirected and dynamic topology. A framework composed of two types of neighbor selection strategies is developed, which harnesses the intelligence and ability of agents to assess the situation. First, the RSRSP strategy is proposed, in which each agent segments its communication region equally into four sectors and actively selects the neighbor in each sector with which it has the minimum difference of states after evaluating the distribution of neighbors. Second, the RSRSP strategy is optimized to a distributed switching strategy to improve the average convergence speed of the MAS.

One limitation of the proposed framework is that necessary and sufficient conditions for preserving connectivity cannot be derived using mathematical methods. From simulation results, we observe that if the agents of the MASs are distributed across a large area, connectivity cannot be preserved and convergence is prevented. In addition, an extensive number of agents in the MAS can lead to a slower convergence speed. Therefore, our future work will focus on developing a consensus protocol that preserves connectivity preservation at all times during convergence evolution and accelerates convergence.

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Declarations

Conflict of Interests The authors have no competing interests to declare that are relevant to the content of this article.

References

1. Li X, Yu Z, Li Z, Wu N (2021) Group consensus via pinning control for a class of heterogeneous multi-agent systems with input constraints. Inf Sci 542:247–262. https://doi.org/10.1016/j.ins.2020.05.085
2. Hu X, Zhang Z, Li C (2021) Consensus of a new multi-agent system with impulsive control which can heuristically construct the communication network topology. Applied intelligence. pp 1–16. https://doi.org/10.1007/s10489-021-02644-4
3. Zhang J, Zhang H, Sun S, Gao Z (2021) Leader-follower consensus control for linear multi-agent systems by fully distributed edge-event-triggered adaptive strategies. Inf Sci 555:314–338. https://doi.org/10.1016/j.ins.2020.10.056
4. Li K, Li SE, Gao F, Lin Z, Li J, Sun Q (2020) Robust distributed consensus control of uncertain multiagents interacted by eigenvalue-bounded topologies. IEEE Internet Things J 7(5):3790–3798. https://doi.org/10.1109/JIOT.2020.2973927
5. Li Y, Tang C, Li K, He X, Peeta S, Wang Y (2019) Consensus-based cooperative control for multi-platoon under the connected vehicles environment. IEEE Trans Intell Transp Syst 20(6):2220–2229. https://doi.org/10.1109/TITS.2018.2865575
6. Guo J, Cheng S, Liu Y (2021) Merging and diverging impact on mixed traffic of regular and autonomous vehicles. IEEE Trans Intell Transp Syst 22(3):1639–1649. https://doi.org/10.1109/TITS.2020.2974291
7. Nelke SA, Okamoto S, Zivan R (2020) Market clearing-based dynamic multi-agent task allocation. ACM Trans Intell Syst Technol 11(1):4–1425. https://doi.org/10.1145/3356467
8. Niu M, Cheng B, Feng Y, Chen J (2020) GMTA: A geo-aware multi-agent task allocation approach for scientific workflows in container-based cloud. IEEE Trans Netw Serv Manag 17(3):1568–1581. https://doi.org/10.1109/TNSM.2020.2996304
9. Wu H, Shang H (2020) Potential game for dynamic task allocation in multi-agent system. ISA Transactions 102:208–220. https://doi.org/10.1016/j.isatra.2020.03.004
10. Jiang H, Shi D, Xue C, Wang Y, Wang G, Zhang Y (2021) Multi-agent deep reinforcement learning with type-based hierarchical group communication. Appl Intell 51(8):5793–5808. https://doi.org/10.1007/s10489-020-02065-9
11. Vinyals O, Babuschkin I, Czarnecki WM, Mathieu M, Dudzik A, Chung J, Choi DH, Powell R, Ewalds T, Georgiev P, Oh J, Horgan D, Kroiss M, Danihelka I, Huang A, Sifre L, Cai T, Agapiou JP, Jaderberg M, Vezhnevets AS, Leblond R, Pohlen T, Dalibard V, Budden D, Sulsky Y, Molloy J, Paine TL, Gülçehre, Ç, Wang Z, Pfaff T, Wu Y, Ring R, Yogatama D, Wünsch, D, McKinney K, Smith O, Schaul T, Lillicrap TP, Kavukcuoglu K, Hassabis D, Apps C, Silver D (2019) Grandmaster level in starcraft II using multi-agent reinforcement learning. Nat 575(7782):350–354. https://doi.org/10.1038/s41586-019-1724-z
12. Liang L, Ye H, Li GY (2019) Spectrum sharing in vehicular networks based on multi-agent reinforcement learning. IEEE J Sel Areas Commun 37(10):2282–2292. https://doi.org/10.1109/JSAC.2019.2933962
13. Wang A, Dong T, Liao X (2019) Distributed optimal consensus algorithms in multi-agent systems. Neurocomputing 339:26–35. https://doi.org/10.1016/j.neucom.2019.01.044

14. Li X, Xie L, Hong Y (2021) Distributed aggregative optimization over multi-agent networks. In: IEEE Transactions on Automatic Control. https://doi.org/10.1109/TAC.2021.3095456
15. Bond AH (1988) An analysis of problems and research in dai. Readings in distributed artificial intelligence. pp 3–35
16. Olfati-Saber R, Fax JA, Murray RM (2007) Consensus and cooperation in networked multi-agent systems. Proc IEEE 95(1):215–233. https://doi.org/10.1109/JPROC.2006.887293
17. Wang Y, Lei Y, Bian T, Guan Z (2019) Distributed control of nonlinear multiagent systems with unknown and nonidentical control directions via event-triggered communication. IEEE Trans Cybern. pp 1–13. https://doi.org/10.1109/TCYB.2019.2908874
18. Li J, Ye Y, Papadaskalopoulos D, Strbac G (2021) Distributed consensus-based coordination of flexible demand and energy storage resources. IEEE Trans on Power Syst 36(4):3053–3069. https://doi.org/10.1109/TPWRS.2020.3041193
19. Moreau L (2005) Stability of multiagent systems with time-dependent communication links. IEEE Trans Autom Control 50(2):169–182. https://doi.org/10.1109/TAC.2004.841888
20. Nedic A, Olshevsky A, Rabbat MG (2018) Network topology and communication-computation tradeoffs in decentralized optimization. Proc IEEE 106(5):953–976. https://doi.org/10.1109/JPROC.2018.2817461
21. Lin P, Wang Y, Qi H, Hong Y (2018) Distributed consensus-based k-means algorithm in switching multi-agent networks. Journal of Systems Science and Complexity 31(5):1128–1145. https://doi.org/10.1007/s11424-018-7102-3
22. Hu A, Wang Y, Cao J, Alsaedi A (2020) Event-triggered bipartite consensus of multi-agent systems with switching partial couplings and topologies. Inf Sci 521:1–13. https://doi.org/10.1016/j.ins.2020.02.038
23. Su Y, Lee T-C (2022) Output feedback synthesis of multiagent systems with jointly connected switching networks: A separation principle approach. IEEE Transactions on Automatic Control 67(2):941–948. https://doi.org/10.1109/TAC.2021.3077352
24. Xie G, Chen J, Li Y (2021) Hybrid-order network consensus for distributed multi-agent systems. J Artif Intell Res 70:389–407. https://doi.org/10.1613/jair.1.12061
25. Chen L, Gao Y, Bai L, Cheng Y (2020) Scaled consensus control of heterogeneous multi-agent systems with switching topologies. Neurocomputing 408:13–20. https://doi.org/10.1016/j.neucom.2019.09.017
26. Zou W, Shi P, Xiang Z, Shi Y (2020) Finite-time consensus of second-order switched nonlinear multi-agent systems. IEEE Trans Neural Networks Learn Syst 31(5):1757–1762. https://doi.org/10.1109/TNNLS.2019.2920880
27. Jiang J, Jiang Y (2020) Leader-following consensus of linear time-varying multi-agent systems under fixed and switching topologies. Autom 113:108804. https://doi.org/10.1016/j.automatica.2020.108804
28. Dai J, Guo G (2018) Event-triggered leader-following consensus for multi-agent systems with semi-markov switching topologies. Inf Sci 459:290–301. https://doi.org/10.1016/j.ins.2018.04.054
29. Song W, Feng J, Sun S (2021) Data-based output tracking formation control for heterogeneous MIMO multiagent systems under switching topologies. Neurocomputing 422:322–331. https://doi.org/10.1016/j.neucom.2020.10.017
30. Dong Y, Xu S (2020) A novel connectivity-preserving control design for rendezvous problem of networked uncertain nonlinear systems. IEEE Trans Neural Networks Learn Syst 31(12):5127–5137. https://doi.org/10.1109/TNNLS.2020.2964017
31. Dong Y, Su Y, Liu Y, Xu S (2018) An internal model approach for multi-agent rendezvous and connectivity preservation with nonlinear dynamics. Autom 89:300–307. https://doi.org/10.1016/j.automatica.2017.12.018
32. Zou Y, An Q, Miao S, Chen S, Wang X, Su H (2021) Flocking of uncertain nonlinear multi-agent systems via distributed adaptive event-triggered control. Neurocomputing 465:503–513. https://doi.org/10.1016/j.neucom.2021.09.005
33. Liang H, Fu Y, Gao J (2021) Bio-inspired self-organized cooperative control consensus for crowded UUV swarm based on adaptive dynamic interaction topology. Appl Intell 51(7):4664–4681. https://doi.org/10.1007/s10489-020-02104-5
34. Dong Y, Zha Q, Zhang H, Kou G, Fujita H, Chiclana F, Herrera-Viedma E (2018) Consensus reaching in social network group decision making: Research paradigms and challenges. Knowl Based Syst 162:3–13. https://doi.org/10.1016/j.knosys.2018.06.036
35. Li Y, Liu M, Cao J, Wang X, Zhang N (2021) Multi-attribute group decision-making considering opinion dynamics. Expert Syst Appl 184:115479. https://doi.org/10.1016/j.eswa.2021.115479
36. Bai W, Lin Z, Dong H, Ning B (2019) Distributed cooperative cruise control of multiple high-speed trains under a state-dependent information transmission topology. IEEE Trans Intell Transp Syst 20(7):2750–2763. https://doi.org/10.1109/TITS.2019.2893583
37. Feng Z, Sun C, Hu G (2017) Robust connectivity preserving rendezvous of multirobot systems under unknown dynamics and disturbances. IEEE Trans Control Netw Syst 4(4):725–735. https://doi.org/10.1109/TCNS.2016.2545869
38. Loizou S, Lui DG, Petrillo A, Santini S (2021) Connectivity preserving formation stabilization in an obstacle-cluttered environment in the presence of time-varying communication delays. IEEE Transactions on Automatic Control. pp 1–1. https://doi.org/10.1109/TAC.2021.3119003
39. Kumar K, Liu J, Lu Y-H, Bhargava B (2013) A survey of computation offloading for mobile systems. Mob Networks Appl 18(1):129–140. https://doi.org/10.1007/s11036-012-0368-0
40. Jadbabaie A, Lin J, Morse AS (2003) Coordination of groups of mobile autonomous agents using nearest neighbor rules. IEEE Trans Autom Control 48(6):988–1001. https://doi.org/10.1109/TAC.2003.812781
41. Cortés J, Martínez S, Bullo F (2006) Robust rendezvous for mobile autonomous agents via proximity graphs in arbitrary dimensions. IEEE Trans Autom Control 51(8):1289–1298. https://doi.org/10.1109/TAC.2006.878713
42. Motsch S, Tadmor E (2014) Heterophilious dynamics enhances consensus. SIAM Rev 56(4):577–621. https://doi.org/10.1137/120901866
43. Anderson JR, Crawford J (1980) Cognitive Psychology and Its Implications
44. Liu Y, Passino KM, Polycarpou MM (2003) Stability analysis of m-dimensional asynchronous swarms with a fixed communication topology. IEEE Trans Autom Control 48(1):76–95. https://doi.org/10.1109/TAC.2002.806657
45. Ghorai C, Banerjee I (2018) A robust forwarding node selection mechanism for efficient communication in urban vanets. Vehicular Communications 14:109–121. https://doi.org/10.1016/j.vehcom.2018.10.003
46. Călinescu G, Măndoiu II, Wan P-J, Zelikovsky AZ (2004) Selecting forwarding neighbors in wireless ad hoc networks. Mob Netw Appl 9(2):101–111. https://doi.org/10.1023/B:MONE.0000013622.63511.57
47. Ding S, He X, Wang J (2017) Multiobjective optimization model for service node selection based on a tradeoff between quality of service and resource consumption in mobile crowd sensing. IEEE Internet of Things Journal 4(1):258–268. https://doi.org/10.1109/JIOT.2017.2647740
48. Spanos D, Murray RM (2004) Robust connectivity of networked vehicles. In: 43rd IEEE Conference on Decision and Control, CDC 2004, Nassau, Bahamas, December 14-17, pp 2893–2898. https://doi.org/10.1109/CDC.2004.1428904

49. Horn RA, Johnson CR (2012) Matrix Analysis, 2nd Ed. https://doi.org/10.1017/CBO9781139020411
50. Fiedler M (1973) Algebraic connectivity of graphs. Czechoslovak Mathematical Journal 23(2):298–305

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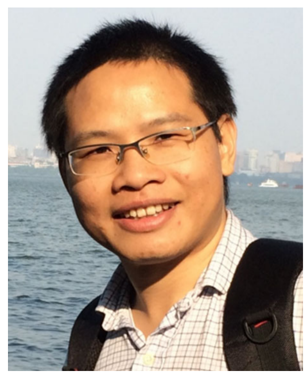

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