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Forecast Combination With Entry and Exit of Experts

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Combination of forecasts from survey data is complicated by the frequent entry and exit of individual forecasters which renders conventional least squares regression approaches infeasible. We explore the consequences of this issue for existing combination methods and propose new methods for bias-adjusting the equal-weighted forecast or applying combinations on an extended panel constructed by back-filling missing observations using an EM algorithm. Through simulations and an application to a range of macro-economic variables we show that the entry and exit of forecasters can have a large effect on the real-time performance of conventional combination methods. The bias-adjusted combination method is found to work well in practice.

KEY WORDS: Bias-adjustment; EM algorithm; Real-time Data; Survey of Professional Forecasters.

1. INTRODUCTION

Survey forecasts provide an ideal data source for investigating real-time forecasting performance. By construction, such forecasts were computed in real time and so do not suffer from the potential look-ahead biases associated with forecasts constructed ex post from an econometric model due to the effects of parameter estimation, model selection (Pesaran and Timmermann 2005), or data revisions (Amato and Swanson 2001; Croushore and Stark 2001).

Surveys include multiple participants and so a natural question becomes how best to select or combine individual forecasts. Moreover, a largely ignored issue is that most surveys take the form of unbalanced panels due to the frequent entry, exit, and reentry of individual forecasters. This is a general problem and affects, inter alia, the Livingston survey, the Survey of Professional Forecasters, Blue Chip forecasts, the survey of the Confederation of British Industry, Consensus Forecasts, and surveys of financial analysts' forecasts.

As an illustration of this problem, Figure 1 shows how participation in the Survey of Professional Forecasters evolved over the 5-year period from 1995 to 1999. Each quarter, participants are asked to predict the implicit price deflator for the Gross Domestic Product. Forecasters constantly enter, exit, and reenter following a period of absence, creating problems for standard combination approaches that rely on estimating the covariance matrix for the individual forecasts. Such approaches are not feasible with this type of data since many forecasters may not have overlapping data and so the covariance matrix cannot be estimated.

This paper considers ways to recursively select or combine survey forecasts in the presence of such missing observations. We consider both conventional methods and some new approaches. The first category includes the previous best forecast, the equal-weighted average, odds ratio methods in addition to least squares, and shrinkage methods modified by trimming

forecasts from participants who do not report a minimum number of data points. The second category includes a method that projects the realized value on a constant and the equal-weighted forecast. This projection performs a bias correction in response to the strong evidence of biases in macroeconomic survey forecasts (Zarnowitz 1985; Davies and Lahiri 1995) and among financial analysts (Hong and Kubik 2003). Although this method is parsimonious and only requires estimating an intercept and a slope parameter, we also consider using the SIC to choose between the simple and bias-adjusted average forecast. Finally, we use the EM algorithm to fill out past missing observations when forecasters leave and rejoin the survey and combine the forecasts from the extended panel.

We compare the ("pseudo") real-time forecasting performance of these methods through Monte Carlo simulations in the context of a common factor model that allows for bias in the individual forecasts, dynamics in the common factors, and heterogeneity in individual forecasters' ability. In situations with a balanced panel of forecasts, the least squares combination methods perform quite well unless the cross-section of forecasts (N) is large relative to the length of the time series (T). If the parameters in the Monte Carlo simulations are chosen so that equal weights are sufficiently suboptimal in population, least-squares combination methods dominate the equal-weighted forecast. Interestingly, the simple bias-adjusted mean outperforms regression-based and shrinkage combination forecasts in most experiments.

In the simulations that use an unbalanced panel of forecasts calibrated to match actual survey data, the simulated

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ID	95.1	95.2	95.3	95.4	96.1	96.2	96.3	96.4	97.1	97.2	97.3	97.4	98.1	98.2	98.3	98.4	99.1	99.2	99.3	99.4
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40	x	x	x	x	x					x						x		x	x	x
65	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
84	x	x	x	x	x		x		x	x		x	x		x		x	x	x	x
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99	x	x		x		x		x		x		x		x			x			x
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Figure 1. Participants in the survey of professional forecasters (PGDP). Notes: The ID corresponds to the identification number assigned to each forecaster in the survey. The columns represent the quarter when the survey was taken. The xs show when a particular forecaster responded to the PGDP part of the survey and provided a one-step-ahead forecast for inflation.

real-time forecasting performance of the least squares combination methods deteriorates relative to that of the equal-weighted combination. This happens because the panel of forecasters must be trimmed to get a balanced subset of forecasts from which the combination weights can be estimated by least squares methods. This step entails a loss of information relative to using the equal-weighted forecast which is based on the complete set of individual forecasts. The bias-adjusted

mean forecast continues to perform well in the unbalanced panel. We finally evaluate the selection and combination methods using survey data on 14 time series covering a range of macroeconomic variables. Consistent with other studies (e.g., Stock and Watson 2001, 2004), we find that most methods are dominated by the simple equal-weighted average. However, there is evidence for around half of the variables that the bias-adjusted

combination method—particularly when refined by the selection step based on the SIC—improves upon the equal-weighted average. We show that this is related to evidence of biases in the equal-weighted average.

The plan of the paper is as follows. Section 2 describes methods for estimating combination weights. Section 3 conducts the Monte Carlo simulation experiment while Section 4 provides the empirical application. Section 5 concludes.

2. SELECTION AND COMBINATION METHODS

This section introduces the methods for selecting individual forecasts or combining multiple forecasts that will be used in the simulations and empirical application. We let $\hat{Y}_{t+h|t}^i$ be the i th survey participant’s period- t forecast of the outcome variable Y_{t+h} , where h is the forecast horizon, $i = 1, \dots, N_t$ and N_t is the number of forecasts reported at time t . Individual forecast errors are then given by $e_{t+h|t}^i = Y_{t+h} - \hat{Y}_{t+h|t}^i$, while the vector of forecast errors is $\mathbf{e}_{t+h|t} = \mathbf{Y}_{t+h} - \hat{\mathbf{Y}}_{t+h|t}$, where $\hat{\mathbf{Y}}_{t+h|t} = (\hat{Y}_{t+h|t}^1, \dots, \hat{Y}_{t+h|t}^{N_t})'$ and $\mathbf{1}$ is an $N_t \times 1$ vector of ones. Suppose that $Y_{t+h} \sim (\mu_y, \sigma_y^2)$, $\hat{\mathbf{Y}}_{t+h|t} \sim (\boldsymbol{\mu}, \boldsymbol{\Sigma}_{\hat{\mathbf{Y}}})$ and $\text{Cov}(Y_{t+h}, \hat{\mathbf{Y}}_{t+h|t}) = \boldsymbol{\sigma}_{y\hat{\mathbf{Y}}}$, where for simplicity we omit time and horizon subscripts on the moments. Forecast combination methods entail finding a vector of weights, $\boldsymbol{\omega}$, that minimize the mean squared error (MSE):

$$E[\mathbf{e}_{t+h|t}'\mathbf{e}_{t+h|t}] = (\mu_y - \boldsymbol{\omega}'\boldsymbol{\mu})^2 + \sigma_y^2 + \boldsymbol{\omega}'\boldsymbol{\Sigma}_{\hat{\mathbf{Y}}}\boldsymbol{\omega} - 2\boldsymbol{\omega}'\boldsymbol{\sigma}_{y\hat{\mathbf{Y}}}. \tag{1}$$

Selection and combination methods differ in how they estimate the moments in (1) and which restrictions they impose on the weights.

Previous Best Forecast

Rather than obtaining $\boldsymbol{\omega}$ through estimation, one can simply pick the best forecast based on past forecasting performance, that is, $\hat{Y}_{t+h|t}^* = \hat{Y}_{t+h|t}^{i^*}$, where,

$$i_t^* = \arg \min_{i=1, \dots, N_t} (t-h)^{-1} \sum_{\tau=h+1}^t (Y_\tau - \hat{Y}_{\tau|t-h}^i)^2. \tag{2}$$

This corresponds to setting a single element of $\boldsymbol{\omega}$ equal to one and the remaining elements to zero. Note that the ranking of the various forecasts follows a stochastic process that may lead to shifts in the selected forecast as new data emerges.

Equal-Weighted Average

By far the most common combination approach is to use the equal-weighted forecast

$$\bar{Y}_{t+h|t} = N_t^{-1} \sum_{i=1}^{N_t} \hat{Y}_{t+h|t}^i, \tag{3}$$

which is simple to compute even for unbalanced panels of forecasts and has proven surprisingly difficult to outperform (Clemen 1989; Stock and Watson 2001, 2004; Timmermann 2006). Intuition for why this approach works well is that, in the presence of a common factor in the forecasts, the first principal component can be approximated by the $1/N$ combination. In

population, the equal-weighted average is optimal in the sense that it minimizes (1) when the forecast error variances are the same and forecast errors have identical pairwise correlations, that is, in the absence of heterogeneity among individual forecasters, but it will generally be suboptimal under heterogeneity. Importantly, optimality of the $1/N$ weights also requires that the forecasts be unbiased ($\boldsymbol{\mu} = \mu_y \mathbf{1}$) and the additional constraint $\boldsymbol{\omega}'\mathbf{1} = 1$.

Least Squares Estimates of Combination Weights

A simple way to obtain estimates of the combination weights is to perform a least squares regression of the outcome variable, Y_t , on a constant, ω_0 , and the vector of forecasts, $\hat{\mathbf{Y}}_{t|t-h} = (\hat{Y}_{t|t-h}^1, \dots, \hat{Y}_{t|t-h}^{N_t})'$. Population optimal values of the intercept and slope parameters in this regression are:

$$\begin{aligned} \omega_0^* &= \mu_y - \boldsymbol{\omega}^{*'}\boldsymbol{\mu}, \\ \boldsymbol{\omega}^* &= \boldsymbol{\Sigma}_{\hat{\mathbf{Y}}}^{-1}\boldsymbol{\sigma}_{y\hat{\mathbf{Y}}}. \end{aligned} \tag{4}$$

Granger and Ramanathan (1984) consider three versions of the least squares regression:

- (i) $Y_t = \omega_0 + \boldsymbol{\omega}'\hat{\mathbf{Y}}_{t|t-h} + \varepsilon_t$,
- (ii) $Y_t = \boldsymbol{\omega}'\hat{\mathbf{Y}}_{t|t-h} + \varepsilon_t$,
- (iii) $Y_t = \boldsymbol{\omega}'\hat{\mathbf{Y}}_{t|t-h} + \varepsilon_t, \quad \text{s.t. } \boldsymbol{\omega}'\mathbf{1} = 1$.

The first and second of these regressions can be estimated by standard OLS, the only difference being that the second equation omits an intercept term. The third regression omits an intercept and can be estimated through constrained least squares. The first regression does not require the individual forecasts to be unbiased since any bias is absorbed through the intercept, ω_0 . In contrast, the third regression is motivated by an assumption of unbiasedness of the individual forecasts. Imposing that the weights sum to one then guarantees that the combined forecast is also unbiased.

The main advantage of this approach is that it applies under general correlation patterns, including in the presence of strong heterogeneity among forecasters. On the other hand, estimation error is known to be a major problem when N is large or T is small. Another problem is that the approach is poor at handling unbalanced datasets for which the full covariance matrix cannot be estimated. In such cases, minimum data requirements must be imposed and the set of forecasts trimmed. For example, one can require that forecasts from a certain minimum number of (not necessarily contiguous) common periods be available.

Inverse MSE

To reduce the effect of parameter estimation errors, we apply a weighting scheme which, for forecasters with a sufficiently long track record, uses weights that are inversely proportional to their historical MSE values, while using equal-weights for the remaining forecasters (normalized so the weights sum to one).

Shrinkage

Stock and Watson (2004) propose shrinkage towards the arithmetic average of forecasts. Let $\hat{\omega}_t^i$ be the least-squares estimator of the weight on the i th model in the forecast combination obtained, for example, from one of the regressions in (5). The combination weights considered by Stock and Watson take the form

$$\begin{aligned}\omega_t^i &= \psi_t \hat{\omega}_t^i + (1 - \psi_t)(1/N_t), \\ \psi_t &= \max(0, 1 - \kappa N_t/(T - 1 - N_t - 1)).\end{aligned}\tag{6}$$

Larger values of κ imply a lower ψ_t and thus a greater degree of shrinkage towards equal weights. As the sample size, T , rises relative to the number of forecasts, N_t , the least squares estimate gets a larger weight, thus putting more weight on the data in large samples.

Odds Matrix Approach

The odds matrix approach (Gupta and Wilton 1988) computes the combination of forecasts as a weighted average of the individual forecasts where the weights are derived from a matrix of pair-wise odds ratios. The odds matrix, \mathbf{O} , contains the pair-wise probabilities π_{ij} that the i th forecast will outperform the j th forecast in the next realization, that is, $\pi_{ij} = \frac{\pi_{ij}}{\pi_{ji}}$. We estimate π_{ij} from $\pi_{ij} = \frac{a_{ij}}{(a_{ij}+a_{ji})}$, where a_{ij} is the number of times forecast i had a smaller absolute error than forecast j in the historical sample. The weight vector, ω , is obtained from the solution to $(\mathbf{O} - N_t \mathbf{I})\omega = \mathbf{0}$, where \mathbf{I} is the identity matrix. We follow Gupta and Wilton and use the normalized eigenvector associated with the largest eigenvalue of \mathbf{O} .

Bias-Adjusted Mean

As noted in the Introduction, there is strong empirical evidence that individual survey participants’ forecasts are biased. Moreover, equal-weighted averages of biased forecasts will themselves in general be biased. To deal with this, we propose a simple bias adjustment of the equal-weighted forecast, $\bar{Y}_{t+h|t}$:

$$\bar{Y}_{t+h|t} = \alpha + \beta \bar{Y}_{t+h|t}.\tag{7}$$

To intuitively motivate the slope coefficient in (7), β , notice that there always exists a scaling factor such that, at a given point in time, the product of this and the average forecast is unbiased. To ensure that on average (or unconditionally) the combined forecast is unbiased, we further include a constant in (7).

This extension of the equal-weighted combination uses the full set of individual forecasts (incorporated in the equal-weighted average) and only requires estimating two parameters, α and β , which can be done through least squares regression. The method is therefore highly parsimonious and so is less affected by estimation error than regression-based methods such as (5), particularly when N is large. For balanced panels the method can be viewed as a special case of the general Granger–Ramanathan regression (5), part (i), although this does not hold with missing observations.

To see more rigorously when the bias-adjustment method is optimal, notice that under homogeneity in the covariance structure of $(Y_{t+h}, \bar{Y}_{t+h|t})$, we can write (by theorem 8.3.4 in

Graybill 1983) $\Sigma_{\hat{y}\hat{y}}^{-1} = c_1 \mathbf{I}_N + c_2 \mathbf{u}\mathbf{u}'$, $\sigma_{y\hat{y}} = c_3 \mathbf{u}$, where c_i are constant scalars and \mathbf{I}_N is the $N \times N$ identity matrix, so that from (4), $\omega^* = c_3(c_1 + Nc_2)\mathbf{u}$, while $\omega_0^* = \mu_y - \omega^{*\prime}\mu$. Setting $\beta = c_3N(c_1 + Nc_2)$ and $\alpha = \mu_y - c_3(c_1 + Nc_2)\mathbf{u}'\mu$, we see that the optimal combination takes the form in (7).

The bias-adjustment method will thus be optimal if the only source of heterogeneity is individual-specific biases, although it will not be (asymptotically) optimal in the presence of heterogeneity in the variance or covariance of the forecast errors. The reason is that the intercept term, α , corrects for arbitrary forms of biases, independent of their heterogeneity. This is empirically important; for example, Elliott, Komunjer, and Timmermann (2008) find evidence of considerable heterogeneity in individual-specific biases.

To see when a slope coefficient, β , different from one may occur, suppose that both the outcome and the forecasts are driven by a single common factor, F , but that, due to misspecification, the individual forecasts contain an extraneous source of uncorrelated noise, $\varepsilon_i : Y = F + \varepsilon_Y$, $\hat{Y}_i = F + \varepsilon_i$, where $\text{Var}(F) = \sigma_F^2$, $\text{Var}(\varepsilon_i) = \sigma_\varepsilon^2$, $\text{Cov}(\varepsilon_i, \varepsilon_j) = 0$ for all $i \neq j$, and $\text{Cov}(\varepsilon_i, \varepsilon_Y) = 0$. Then one can show that $\omega^* = \mathbf{u}\sigma_F^2/(\sigma_\varepsilon^2 + N\sigma_F^2)$. The optimal weights are thus only equal to $(1/N)\mathbf{u}$ in the special case where $\sigma_\varepsilon^2 = 0$, that is, if there is no misspecification in the individual forecasts. By including a slope coefficient, $\beta = N\sigma_F^2/(\sigma_\varepsilon^2 + N\sigma_F^2)$, Equation (7) can handle this type of (homogenous) misspecification.

Model Selection Approach (SIC)

Following Swanson and Zeng (2001), this approach uses the Schwarz Information Criterion (SIC: Schwarz 1978) to select among the simple and bias-adjusted equal-weighted average. The bias-adjusted forecast requires estimating two additional parameters and so only gets selected provided that its fit improves sufficiently on the equal-weighted forecast.

EM Algorithm

Suppose that the individual forecasts follow a local level model

$$\begin{aligned}\hat{Y}_{t+h|t}^i &= x_t^i + \xi_t^i, & \xi_t^i &\sim \mathcal{N}(0, \sigma_{\xi^i}^2), \\ x_{t+1}^i &= x_t^i + \eta_t^i, & \eta_t^i &\sim \mathcal{N}(0, \sigma_{\eta^i}^2),\end{aligned}\tag{8}$$

where ξ_t^i and η_t^i are mutually independent, cross-sectionally independent as well as independent of x_1 . We use the Expectation Maximization (EM) algorithm (Watson and Engle 1983; Koopman 1993) to recursively estimate the two variances $\sigma_{\xi^i}^2$ and $\sigma_{\eta^i}^2$. This approach uses the smoothed state to back-fill missing forecasts whenever a survey participant has left the survey but rejoins at a later stage. The least squares combination approach in (5), part (i), is then used on the reconstructed panel of forecasts.

It is difficult to obtain analytical results for the forecasting performance of the above methods. The forecasts are likely to reflect past values of the predicted series and so cannot be considered strictly exogenous, making it very hard to characterize the finite sample distribution of the mean squared forecast errors. The unbalanced panel structure of the surveys further complicates attempts at analytical results. For this reason we next turn to simulations and empirical applications to study the performance of the selection and combination approaches.

3. MONTE CARLO SIMULATIONS

To analyze the determinants of the performance of the various forecast combination methods, we conduct a series of Monte Carlo experiments in the context of a simple two-factor model:

$$\begin{aligned} Y_{t+1} &= \mu_y + \beta_{y1}F_{1t+1} + \beta_{y2}F_{2t+1} + \varepsilon_{yt+1}, \\ \varepsilon_{yt+1} &\sim N(0, \sigma_{\varepsilon_y}^2), \\ \hat{Y}_{t+1|t}^i &= \mu_i + \beta_{i1}F_{1t+1} + \beta_{i2}F_{2t+1} + \varepsilon_{it+1}, \\ \varepsilon_{it+1} &\sim N(0, \sigma_{\varepsilon_i}^2), \quad i = 1, \dots, N, \end{aligned} \tag{9}$$

where we assume that $E[\varepsilon_{it+1}\varepsilon_{jt+1}] = 0$ if $i \neq j$, and $E[\varepsilon_{it+1}\varepsilon_{yt+1}] = 0$ for $i = 1, \dots, N$. Dynamics is introduced by letting the factors $(F_{1t+1} F_{2t+1})'$ follow an AR(1) process with diagonal autoregressive matrix B_F and uncorrelated innovations, $\varepsilon_{F,t+1} \sim N(0, \sigma_{\varepsilon_F}^2)$. This model is simply a different representation of a setup where at time t forecasters receive noisy signals that are imperfectly correlated with the future factor realizations, F_{1t+1} and F_{2t+1} .

We let the sample size, T , vary from 50 to 100 and 200 and let the number of forecasts (N) assume values of 4, 10, and 20. This covers situations with large N relative to the sample size T (e.g., $N = 20, T = 50$) as well as situations with plenty of data points relative to the number of estimated parameters (e.g., $N = 4, T = 200$). All forecasts are one step ahead, simulated out of sample, and are computed based on recursive parameter estimates using only information available at the time of the forecast.

The first experiments assume that the individual forecasts are unbiased and set $\mu_y = \mu_i = 0$ ($i = 1, \dots, N$). In all experiments but number 4 we assume that $\beta_{i1} = \beta_{i2} = 0.5$ ($i = 1, \dots, N$). In experiment 1 all parameters (except for B_F which equals zero) are set equal to one, and we solve for $\sigma_{\varepsilon_i}^2$ (identical for all i) so the optimal weights are identical and sum to unity. In experiments 2–7 we set $\sigma_{\varepsilon_i}^2$ so that the regression coefficient of Y_{t+1} on the individual forecasts $\hat{Y}_{t+1|t}^i$ is unity. Factor dynamics is introduced in experiment 3 by letting $B_F = 0.9 \times I$. Heterogeneity in the individual forecasters' ability is introduced in two ways: first, by drawing the factor loadings, β_{if} , from a Beta(1, 1) distribution centered on 0.5 (experiment 4) and, second, by drawing the inverse of the variance of the idiosyncratic errors, $\sigma_{\varepsilon_i}^{-2}$, from a Gamma(5, 5) distribution (experiment 5). To allow for the possibility that different forecasts capture different predictable components (thus enhancing the role of forecast combinations over the individual models), experiment 6 considers a scenario where different groups of forecasts load on different factors. Finally, forecast biases are introduced in experiment 7 by allowing for a nonzero intercept. Additional details are provided in Table 1.

3.1 Balanced Panel of Forecasters

The first set of results assumes a balanced panel of forecasts. Panel A of Table 1 reports simulated out-of-sample MSE values computed relative to the MSE value associated with the equal-weighted forecast (which is thus always equal to unity).

In the first experiment, the simple equal-weighted forecast performs best—imposing a true constraint ensures efficiency gains. For the same reason, the combination that excludes an intercept and constrains the weights to sum to unity is best among the regression-based methods. The improvement over the most general least squares regression (GR1) tends, however, to be marginal. Conversely, when the true weights do not sum to unity, as in the second experiment, the most constrained schemes such as equal weights or GR3 produce MSE values that are worse than the less constrained methods (GR1 and GR2). Constraining the intercept to be zero (GR2) leads to marginally better performance than under the unconstrained least squares model (GR1) when this constraint holds as in experiments 2–6, but leads to inferior performance when the underlying forecasts are in fact biased (experiment 7).

The shrinkage forecasts generally improve on the benchmark equal-weighted combination's performance. In most cases the shrinkage approach does as well as or slightly better than the best least squares approach. When the sample size is small, the model with the largest degree of shrinkage ($\kappa = 1$) does best. However, using a smaller degree of shrinkage ($\kappa = 0.25$) becomes better as the sample size, T , is raised (for fixed N). The benefit from shrinkage is particularly sizeable when the number of models is large. Although the differences in MSE values are small, the odds matrix and inverse MSE approaches generally dominate using equal weights.

Factor dynamics—introduced in the third Monte Carlo experiment—leads to deteriorating forecasting performance across all combination schemes. Interestingly, it also has the effect of improving the relative performance of the most general least squares methods (GR1 and GR2), shrinkage, and the bias-adjusted average.

Heterogeneity in the factor loadings of the various forecasts—introduced by drawing these from a beta distribution—means that the true performance differs across forecasting models. Models with larger factor loadings have a higher R^2 than models with small factor loadings. As a result, the equal-weighted average performs worse. Heterogeneity in factor loadings (experiment 4) leads to poor performance of the simple equal-weighted forecast. Conversely, the forecasting performance of the previous best model improves as the heterogeneity gets stronger and the best single forecast gets more clearly defined. Heterogeneity in the precision of individual forecasts (experiment 5) leads to relatively good performance for the least constrained OLS schemes along with the bias adjusted mean. Of course, this result depends on the assumed degree of heterogeneity among forecasters with increases (decreases) in the heterogeneity leading to deteriorating (improving) performance for the bias adjusted mean. For example, drawing $\sigma_{\varepsilon_i}^{-2}$ from a more disperse Gamma(1, 1) distribution means that the bias-adjusted mean is dominated by the best least squares method for most combinations of N and T . Conversely, drawing $\sigma_{\varepsilon_i}^{-2}$ from a less disperse Gamma(10, 10) distribution improves the relative performance of the bias-adjusted mean.

When half of the forecasts track factor one while the remaining half track factor two (experiment 6), the benefits from combining over using the single best model (which can only track one factor at a time) tend to be particularly large. Moreover,

Table 1. Simulation results from forecast combinations under factor structure

A) Full data												
# of forecasts	Sample size	EW	BAM	SIC	GR1	GR2	GR3	S1	S2	Odds	Previous best	Inverse MSE
Experiment 1: Equal weights summing to one												
4	50	1.00	1.04	1.00	1.10	1.08	1.06	1.08	1.08	0.98	1.44	0.97
4	100	1.00	1.02	1.00	1.05	1.05	1.04	1.05	1.04	0.99	1.54	0.99
4	200	1.00	1.01	1.00	1.02	1.02	1.01	1.02	1.02	1.00	1.58	0.99
10	50	1.00	1.05	1.00	1.30	1.27	1.24	1.23	1.20	0.97	2.36	0.96
10	100	1.00	1.02	1.00	1.13	1.12	1.11	1.12	1.10	0.99	2.62	0.98
10	200	1.00	1.01	1.00	1.06	1.06	1.05	1.05	1.05	0.99	2.91	0.99
20	50	1.00	1.04	1.00	1.79	1.72	1.67	1.50	1.32	0.97	3.79	0.95
20	100	1.00	1.02	1.00	1.25	1.24	1.22	1.21	1.13	0.98	4.36	0.97
20	200	1.00	1.01	1.00	1.13	1.12	1.11	1.11	1.11	0.99	4.84	0.99
Experiment 2: Equal weights												
4	50	1.00	0.89	0.96	0.95	0.93	1.06	0.93	0.92	0.99	1.06	0.99
4	100	1.00	0.88	0.91	0.90	0.90	1.04	0.90	0.90	1.00	1.13	0.99
4	200	1.00	0.87	0.87	0.88	0.88	1.01	0.88	0.88	1.00	1.15	1.00
10	50	1.00	0.81	0.85	1.00	0.97	1.24	0.95	0.93	0.98	1.06	0.98
10	100	1.00	0.80	0.80	0.87	0.86	1.11	0.85	0.84	0.99	1.11	0.99
10	200	1.00	0.79	0.79	0.83	0.82	1.05	0.82	0.82	1.00	1.19	1.00
20	50	1.00	0.75	0.78	1.25	1.21	1.66	1.07	0.97	0.98	1.01	0.98
20	100	1.00	0.74	0.74	0.91	0.89	1.23	0.87	0.83	0.99	1.08	0.99
20	200	1.00	0.74	0.74	0.81	0.81	1.11	0.80	0.80	1.00	1.15	1.00
Experiment 3: Factor dynamics												
4	50	1.00	0.75	0.79	0.80	0.78	1.06	0.78	0.78	0.98	1.21	0.98
4	100	1.00	0.74	0.75	0.76	0.75	1.03	0.75	0.75	0.99	1.28	0.99
4	200	1.00	0.73	0.73	0.74	0.74	1.02	0.74	0.74	1.00	1.35	0.99
10	50	1.00	0.53	0.54	0.66	0.64	1.24	0.63	0.62	0.98	1.25	0.97
10	100	1.00	0.52	0.52	0.57	0.56	1.10	0.56	0.56	0.99	1.34	0.99
10	200	1.00	0.50	0.50	0.53	0.52	1.05	0.52	0.52	0.99	1.43	0.99
20	50	1.00	0.43	0.43	0.72	0.70	1.69	0.63	0.61	0.98	1.21	0.97
20	100	1.00	0.42	0.42	0.50	0.49	1.22	0.48	0.49	0.99	1.30	0.98
20	200	1.00	0.40	0.40	0.45	0.45	1.11	0.45	0.44	0.99	1.39	0.99
Experiment 4: Heterogeneity in factor loadings												
4	50	1.00	0.87	0.93	0.89	0.87	0.93	0.87	0.86	0.95	0.94	0.95
4	100	1.00	0.86	0.88	0.85	0.84	0.90	0.84	0.84	0.96	0.94	0.96
4	200	1.00	0.85	0.85	0.83	0.82	0.89	0.82	0.82	0.96	0.96	0.96
10	50	1.00	0.79	0.83	0.94	0.91	0.94	0.89	0.87	0.94	0.85	0.94
10	100	1.00	0.78	0.79	0.82	0.81	0.86	0.80	0.79	0.95	0.88	0.95
10	200	1.00	0.77	0.77	0.78	0.78	0.82	0.78	0.77	0.95	0.90	0.95
20	50	1.00	0.74	0.77	1.21	1.17	1.17	1.03	0.94	0.94	0.79	0.93
20	100	1.00	0.72	0.73	0.87	0.85	0.87	0.83	0.80	0.94	0.81	0.94
20	200	1.00	0.73	0.73	0.78	0.78	0.80	0.77	0.77	0.95	0.84	0.94
Experiment 5: Heterogeneity in forecast precision												
4	50	1.00	0.71	0.74	0.76	0.75	1.06	0.74	0.74	1.00	0.97	1.00
4	100	1.00	0.71	0.71	0.73	0.72	1.03	0.72	0.72	1.00	0.99	1.00
4	200	1.00	0.70	0.70	0.71	0.70	1.01	0.70	0.70	1.00	1.00	1.00
10	50	1.00	0.72	0.73	0.88	0.86	1.24	0.84	0.80	0.99	0.96	1.00
10	100	1.00	0.71	0.71	0.77	0.76	1.10	0.76	0.75	1.00	0.98	1.00
10	200	1.00	0.70	0.70	0.74	0.73	1.05	0.73	0.73	1.00	1.00	1.00
20	50	1.00	0.70	0.72	1.17	1.14	1.65	1.00	0.88	0.99	0.94	1.00
20	100	1.00	0.69	0.69	0.85	0.85	1.23	0.82	0.79	1.00	0.96	1.00
20	200	1.00	0.69	0.69	0.76	0.76	1.11	0.75	0.74	1.00	0.98	1.00
Experiment 6: Block-diagonal factor structure												
4	50	1.00	0.79	0.84	0.84	0.83	1.07	0.82	0.82	0.99	1.02	0.99
4	100	1.00	0.78	0.78	0.81	0.80	1.03	0.80	0.80	1.00	1.06	1.00
4	200	1.00	0.77	0.77	0.78	0.77	1.02	0.77	0.77	1.00	1.08	1.00
10	50	1.00	0.66	0.66	0.82	0.79	1.24	0.77	0.76	0.99	0.99	0.99
10	100	1.00	0.65	0.65	0.71	0.70	1.10	0.70	0.69	0.99	1.03	0.99
10	200	1.00	0.64	0.64	0.67	0.67	1.05	0.66	0.66	1.00	1.08	1.00

Table 1. (Continued)

A) Full data												
# of forecasts	Sample size	EW	BAM	SIC	GR1	GR2	GR3	S1	S2	Odds	Previous best	Inverse MSE
Experiment 6: Block-diagonal factor structure (continued)												
20	50	1.00	0.58	0.58	0.97	0.94	1 67	0.83	0.78	0.99	0.97	0.99
20	100	1.00	0.57	0.57	0.70	0.69	1 23	0.67	0.65	0.99	1.01	0.99
20	200	1.00	0.57	0.57	0.63	0.62	1 11	0.62	0.62	1.00	1.06	1.00
Experiment 7: Bias in individual forecasts												
4	50	1.00	0.86	0.93	0.91	0.96	1 05	0.95	0.95	0.98	1.07	0.98
4	100	1.00	0.84	0.86	0.87	0.92	1 02	0.92	0.92	0.99	1.12	0.99
4	200	1.00	0.84	0.84	0.85	0.90	1 00	0.90	0.90	0.99	1.14	0.99
10	50	1.00	0.77	0.81	0.96	1.01	1 21	0.98	0.96	0.98	1.04	0.98
10	100	1.00	0.77	0.77	0.84	0.89	1 09	0.89	0.87	0.99	1.10	0.99
10	200	1.00	0.76	0.76	0.80	0.85	1 03	0.85	0.85	0.99	1.16	0.99
20	50	1.00	0.72	0.74	1.21	1.23	1 60	1.08	0.98	0.98	1.01	0.98
20	100	1.00	0.71	0.71	0.87	0.91	1 19	0.89	0.84	0.99	1.08	0.99
20	200	1.00	0.71	0.71	0.78	0.82	1 08	0.81	0.81	0.99	1.12	0.99

B) Survey-like data													
# of forecasts	Sample size	EW	BAM	SIC	GR1	GR2	GR3	S1	S2	Odds	Previous best	Inverse MSE	EM algorithm
Experiment 1: Equal weights summing to one													
20	50	1.00	1.02	1.01	1.04	1.03	1.48	1.03	1.03	1.48	1.51	1.00	1.03
20	100	1.00	1.00	1.00	1.04	1.03	1.45	1.03	1.03	1.45	1.47	1.00	1.04
20	200	1.00	0.99	1.00	1.03	1.02	1.46	1.02	1.02	1.45	1.48	1.00	1.04
Experiment 2: Equal weights													
20	50	1.00	0.60	0.61	0.99	0.99	0.98	0.99	0.99	0.98	0.98	1.00	0.99
20	100	1.00	0.58	0.58	0.99	0.99	0.98	0.99	0.99	0.98	0.98	1.00	0.99
20	200	1.00	0.57	0.57	0.99	0.98	0.98	0.98	0.98	0.98	0.98	1.00	0.99
Experiment 3: Factor dynamics													
20	50	1.00	0.38	0.38	0.96	0.97	0.97	0.97	0.97	0.97	0.97	1.00	0.96
20	100	1.00	0.36	0.36	0.97	0.97	0.97	0.97	0.97	0.97	0.97	1.00	0.97
20	200	1.00	0.34	0.34	0.96	0.96	0.97	0.96	0.96	0.97	0.97	1.00	0.97
Experiment 4: Heterogeneity in factor loadings													
20	50	1.00	0.59	0.60	0.99	0.99	0.98	0.96	0.99	0.98	0.98	1.00	0.99
20	100	1.00	0.57	0.57	0.99	0.99	0.98	0.99	0.99	0.98	0.98	1.00	0.99
20	200	1.00	0.57	0.57	0.99	0.98	0.98	0.98	0.98	0.98	0.98	1.00	0.98
Experiment 5: Heterogeneity in forecast precision													
20	50	1.00	0.52	0.52	0.95	0.94	0.96	0.94	0.94	0.96	0.96	1.00	0.95
20	100	1.00	0.49	0.49	0.94	0.94	0.96	0.94	0.94	0.96	0.96	1.00	0.94
20	200	1.00	0.49	0.49	0.94	0.94	0.95	0.94	0.94	0.95	0.95	1.00	0.94
Experiment 6: Block-diagonal factor structure													
20	50	1.00	0.77	0.79	1.01	1.00	0.99	1.00	1.00	0.99	0.99	1.00	1.00
20	100	1.00	0.76	0.76	1.00	1.00	0.99	1.00	1.00	0.99	0.99	1.00	1.00
20	200	1.00	0.75	0.75	1.00	0.99	0.99	0.99	0.99	0.99	0.99	1.00	1.00
Experiment 7: Bias in individual forecasts													
20	50	1.00	0.61	0.61	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99
20	100	1.00	0.59	0.59	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99
20	200	1.00	0.58	0.58	0.99	0.99	0.99	0.99	0.99	0.99	0.99	1.00	0.99

NOTE: Results are based on 10,000 simulations. EW: equal-weighted forecast; BAM: bias-adjusted mean. SIC: select EW if the SIC criterion of EW is smaller than the SIC criterion of BAM and select BAM otherwise; GR1: unconstrained OLS; GR2: OLS w/o constant; GR3: OLS w/o constant and weights constrained to add to unity; S1: shrinkage with $\kappa = 0.25$; S2: shrinkage with $\kappa = 1$; Odds: Odds ratio approach; Previous Best: forecast from previous best model; Inverse MSE: weights equal the inverse of the historical MSE; EM fills the missing data using the EM algorithm and then applies unconstrained OLS. The simulations are based on the two-factor model: $Y_{t+1} = \mu_y + \beta_{y1}F_{1t+1} + \beta_{y2}F_{2t+1} + \varepsilon_{yt+1}$, $\varepsilon_{yt+1} \sim N(0, \sigma_{\varepsilon y}^2)$ and $Y_{t+1|t}^i = \mu_i + \beta_{i1}F_{1t+1} + \beta_{i2}F_{2t+1} + \varepsilon_{it+1}$, $\varepsilon_{it+1} \sim N(0, \sigma_{\varepsilon i}^2)$, $i = 1, \dots, N$. The experiments are as follows: (1) Base scenario with equal weights summing to one. (2) Identical weights with weights not summing to one. (3) Factor Dynamics corresponding to an AR(1) model with persistence of 0.9. (4) Heterogeneity with $\beta_{if} \sim \text{Beta}(1, 1)$ for $f = 1, 2$. (5) Heterogeneity with $1/\sigma_{\varepsilon i}^2 \sim \text{Gamma}(5, 5)$. (6) Factor loadings in blocks where $\beta_{i1} = 1$ if $1 \leq i \leq N/2$ and $\beta_{i1} = 0$ if $N/2 < i \leq N$, also $\beta_{i2} = 0$ if $1 \leq i \leq N/2$ and $\beta_{i1} = 1$ if $N/2 < i \leq N$. (7) Biased forecasts where $\mu_i = 1/2$ if $1 \leq i \leq N/2$ and $\mu_i = 0$ if $N/2 < i \leq N$.

the bias-adjusted mean performs very well as do the least constrained OLS and shrinkage forecasts. When we let half of the forecasts be biased with a bias equal to one-half of the standard deviation parameters (experiment 7), the efficiency gain due to omitting an intercept in the least squares combination regression is now more than outdone by the resulting bias. This explains why the general regression (GR1) which includes an intercept term produces better results than the constrained regressions (GR2 and GR3). The shrinkage method pulls the least squares forecast towards the biased equal-weighted forecast and so performs worse than in the case without a bias. In contrast, the performance of the bias-adjusted mean is largely unchanged compared with the results in the second experiment since only the intercept is changed.

Overall, the best forecasting performance is produced by the bias-adjusted mean. This approach produces better results than the equal-weighted forecast in all experiments except the first one (for which a small under-performance of up to 5% is observed). Furthermore, it generally does best among all combination schemes in experiments 2–7, with slightly better results observed for the least squares and shrinkage methods in the presence of heterogeneity in factor loadings (experiment 4). Finally, the refinement that uses the SIC to select among the simple and bias-adjusted mean improves slightly upon the latter in the first experiment and produces similar forecasting performance in the other experiments.

3.2 Unbalanced Panel of Forecasters

We next perform the same set of experiments on data generated from a two-factor model that mimics the unbalanced panel structure of the Survey of Professional Forecasters data underlying Figure 1. To this end, we first group the experts into frequent and infrequent forecasters defined according to whether a forecaster participated in the survey a minimum of 75% of the time. Next, we pool observations within each of the two groups of forecasters and estimate two-state Markov transition matrices for each group, where state 1 represents participation in the survey while state 2 means absent from the survey.

Using data on inflation forecasts, the estimated “stayer” probabilities for the transition matrices conditional on frequent participation are 0.84 and 0.59, while the estimates are 0.69 and 0.97, conditional on infrequent participation. Thus, among frequent forecasters there is an 84% chance of observing a forecast next period if a forecast is reported in the current period. This probability declines to 41% if no forecast is reported in the current period. The extremely high probability (0.97) of repeated nonparticipation among the infrequent forecasters shows that this category covers forecasters who rarely participate in the survey. We set the proportion of frequent forecasters at 40%.

We use these estimates to generate a matrix of zeros and ones that tracks when a forecaster participated in the survey. We then multiply, element by element, the zero-one participation matrix with the matrix of forecasts generated from the two-factor model and apply the combination methods to the resulting (unbalanced) set of forecasts.

To apply least-squares combination methods we trim those forecasters with fewer than 20 contiguous forecasts or no prediction for the following period. Among the remaining forecasters we use the largest common data sample to estimate the

combination weights. If there are no forecasters with at least 20 contiguous observations or if there are fewer remaining forecasters than parameters to be estimated, we simply use the average forecast.

Results are reported in Panel B of Table 1 in the form of out-of-sample MSE values measured relative to the values generated by the simple average. Since unbalanced panels are more likely to occur in settings with a relatively large number of forecasters, we only report results for $N = 20$.

Compared with the earlier results in Panel A, the previous best and inverse MSE approaches perform more like the simple average in the unbalanced panel. A similar finding holds for the least squares combination and shrinkage approaches. The performance of these methods (relative to the simple equal-weighted approach) is therefore relatively worse in the unbalanced panel. Two reasons explain this finding. First, in about one-third of the periods the regression methods revert to using the equal-weighted forecast because a balanced subset of forecasters with a sufficiently long track record cannot be found. Second, since the regression methods trim the set of forecasters to obtain a balanced subset of forecasts, they discard potentially valuable information. Consequently, these methods perform worse than the equal-weighted average when conditions are in place for the latter to work well and only outperform by a small margin otherwise.

The combinations that backfill missing observations using the EM algorithm generally offer consistent, though fairly modest, improvements over the equal-weighted average in all but the first experiment.

Overall, the bias-adjusted mean along with the SIC refinement that chooses between this method and equal weights continues to perform better than the other approaches even with an unbalanced panel of forecasts. Moreover, while the bias adjustment method can be used even with balanced panels, the relative performance of this approach generally improves as a result of forecasters’ entry and exit. The only experiment where a worse (relative) performance is observed is in the sixth experiment with a block diagonal factor structure. Even in this case the bias-adjusted mean remains the best overall.

We conclude that the bias-adjusted mean is better than the other methods that can be used when estimation of the full covariance matrix of the forecast errors is not feasible (equal weights, odds matrix, or previous best forecast). This approach also performs better than the regression and shrinkage approaches modified so they can be used on a balanced subset of forecasters.

4. EMPIRICAL APPLICATION

To illustrate the empirical performance of the selection and combination methods, we study one through four-step-ahead survey forecasts of 14 variables that have data going back to 1981 and are covered by the Survey of Professional Forecasters. A brief description of these variables is provided in Table 2.

4.1 Forecasting Performance

The first 30 forecasts are used to estimate the initial combination weights. The estimation window is then recursively expanded up to the end of the sample which gives us 77 forecasts for $h = 1$ and 74 forecasts for $h = 4$. The regression-

Table 2. Variable descriptions

Name	Description	Transformation	Sample
CPI	Forecasts for the CPI Inflation Rate. SA, annual rate, percentage points. Quarterly forecasts are annualized quarter-over-quarter percent changes.	None	1981q3–2006q4
CPROF	Forecasts for the quarterly level of nominal corporate profits after tax excluding IVA and CCAdj. SA, annual rate, billions of dollars.	Growth Rate for quarter-over-quarter change, expressed in annualized percentage points, constructed as: $[\ln(CPROF_{t+h}/CPROF_{t+h-1})]400$	1981q3–2006q4
HOUSING	Forecasts for the quarterly average level of housing starts. SA, annual rate, millions.	Growth Rate for quarter-over-quarter change, expressed in annualized percentage points, constructed as: $[\ln(HOUSING_{t+h}/HOUSING_{t+h-1})]400$	1981q3–2006q4
INDPROD	Forecasts for the quarterly average of the index of industrial production. SA, index, base year varies.	Growth Rate for quarter-over-quarter change, expressed in annualized percentage points, constructed as: $[\ln(INDPROI_{t+h}/INDPROD_{t+h-1})]400$	1981q3–2006q4
NGDP	Forecasts for the quarterly level of nominal GDP. SA, annual rate, billions of dollars.	Growth Rate for quarter-over-quarter change, expressed in annualized percentage points, constructed as: $[\ln(NGDP_{t+h}/NGDP_{t+h-1})]400$	1981q3–2006q4
PGDP	Forecasts for the quarterly level of the GDP price index. SA, index, base year varies.	Growth Rate for quarter-over-quarter change, expressed in annualized percentage points, constructed as: $[\ln(PGDP_{t+h}/PGDP_{t+h-1})]400$	1981q3–2006q4
RCBI	Forecasts for the quarterly level of real change in private inventories. SA, annual rate, base year varies.	None	1981q3–2006q4
RCONSUM	Forecasts for the quarterly level of real personal consumption expenditures. SA, annual rate, base year varies.	Growth Rate for quarter-over-quarter change, expressed in annualized percentage points, constructed as: $[\ln(RCONSUM_{t+h}/RCONSUM_{t+h-1})]400$	1981q3–2006q4
RFEDGOV	Forecasts for the quarterly level of real federal government consumption and gross investment. SA, annual rate, base year varies.	Growth Rate for quarter-over-quarter change, expressed in annualized percentage points, constructed as: $[\ln(RFEDGOV_{t+h}/RFEDGOV_{t+h-1})]400$	1981q3–2006q4
RGDP	Forecasts for the quarterly level of real GDP. SA, annual rate, base year varies.	Growth Rate for quarter-over-quarter change, expressed in annualized percentage points, constructed as: $[\ln(RGDP_{t+h}/RGDP_{t+h-1})]400$	1981q3–2006q4
RRESINV	Forecasts for the quarterly level of real residential fixed investment. SA, annual rate, base year varies.	Growth Rate for quarter-over-quarter change, expressed in annualized percentage points, constructed as: $[\ln(RRESINV_{t+h}/RRESINV_{t+h-1})]400$	1981q3–2006q4
RSLGOV	Forecasts for the quarterly level of real state and local government consumption and gross investment. SA, annual rate, base year varies.	Growth Rate for quarter-over-quarter change, expressed in annualized percentage points, constructed as: $[\ln(RSLGOV_{t+h}/RSLGOV_{t+h-1})]400$	1981q3–2006q4
TBILL	Forecasts for the quarterly average three-month Treasury bill rate. Percentage points.	None	1982q1–2006q4
UNEMP	Forecasts for the quarterly average unemployment rate. SA, percentage points.	None	1981q3–2006q4

based methods estimate the combination weights on the largest common sample and require forecasters to have a minimum of 10 common contiguous observations, although the results are robust to using 20 contiguous observations instead. For the real-

time realized value we follow Corradi, Fernandez, and Swanson (2007) and use first release data.

Empirical results in the form of (pseudo real-time) root mean squared error (RMSE) values are presented in Table 3. Again

Table 3. Empirical application to forecasts from the survey of professional forecasters

RMSE ratio:	CPI				CPROF				HOUSING				INDPROD			
	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
EW	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
BAM	1.01	0.94	0.95	0.95	1.04	1.05	1.04	1.02	1.01	1.00	1.01	1.00	1.03	0.99	0.97	0.97
SIC	1.00	0.94	0.97	0.95	1.04	1.05	1.03	1.00	1.01	0.99	1.10	1.06	1.03	0.99	0.97	0.97
GR1	1.15	1.10	1.15	1.17	1.22	1.10	1.11	1.17	1.13	4.28	1.23	1.09	1.39	1.43	1.43	1.17
GR2	1.09	1.03	1.13	1.21	1.15	1.07	1.03	1.18	1.12	1.04	1.12	0.97	1.34	1.28	1.19	1.12
GR3	1.08	0.99	1.07	1.11	1.07	1.07	1.01	1.02	1.09	1.24	1.08	0.95	1.31	1.18	1.11	1.14
Shrink 1	1.07	1.01	1.07	1.11	1.08	1.03	1.00	1.04	1.09	1.01	1.04	0.96	1.25	1.19	1.09	1.03
Shrink 2	1.05	0.99	1.03	1.06	1.02	0.98	0.98	0.98	1.02	0.99	1.01	0.97	1.11	1.09	1.04	0.99
Odds	1.05	0.99	1.03	1.02	1.03	1.01	0.98	0.98	1.01	1.22	1.03	0.98	1.16	1.07	1.12	0.98
Previous best	1.12	0.98	1.06	1.07	1.01	1.04	0.95	1.04	1.05	1.23	1.05	0.97	1.17	1.11	1.12	0.98
Inverse of MSE	1.00	0.99	1.01	0.99	1.02	1.03	1.03	1.02	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99
EM	1.03	1.06	1.06	1.07	1.02	1.01	0.99	1.00	1.02	1.10	1.32	1.25	1.02	1.02	0.98	1.01
RMSE ratio:	NGDP				PGDP				RCBI				RCONSUM			
	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
EW	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
BAM	1.01	1.00	0.99	1.00	0.97	0.98	0.93	0.90	1.02	1.01	0.99	0.98	0.98	1.02	1.01	1.05
SIC	0.99	1.00	1.00	1.00	0.97	0.98	0.93	0.89	1.01	1.01	1.00	0.97	0.99	1.03	1.01	1.06
GR1	0.99	0.96	1.48	1.07	1.47	2.50	4.46	1.14	1.13	1.08	1.16	1.36	2.81	6.15	1.16	1.36
GR2	1.00	1.09	1.64	1.11	1.31	1.87	2.98	0.95	1.05	1.10	1.13	1.02	1.30	2.44	1.17	1.26
GR3	1.15	1.24	1.21	0.99	1.15	1.13	1.04	1.00	1.07	1.03	1.10	1.00	1.21	1.11	1.10	1.14
Shrink 1	0.98	1.08	1.42	1.04	1.20	1.79	2.58	1.02	1.02	1.01	1.08	0.98	1.22	1.94	1.07	1.15
Shrink 2	1.00	1.09	1.22	1.08	1.10	1.64	1.55	1.07	1.00	0.97	1.03	1.01	1.08	1.03	0.98	1.01
Odds	1.06	1.16	1.17	1.02	1.03	1.15	1.22	1.10	0.97	1.02	0.98	0.98	1.05	1.03	0.96	1.04
Previous best	1.08	1.55	1.58	0.99	1.03	1.12	2.15	1.06	1.13	1.11	1.00	0.97	1.06	1.12	0.95	1.07
Inverse of MSE	1.03	1.01	1.03	1.01	0.98	0.95	0.91	0.92	1.00	1.00	0.99	0.98	1.01	1.00	0.99	1.01
EM	1.45	1.12	1.33	1.04	1.06	1.25	1.11	0.96	1.03	1.05	1.01	1.02	1.06	1.03	1.02	1.05
RMSE ratio:	RFEDGOV				RGDP				RRESINV				RSLGOV			
	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
EW	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
BAM	1.03	1.07	1.04	1.03	1.01	1.02	1.04	1.03	0.97	0.98	0.96	0.96	1.01	1.02	1.04	1.02
SIC	0.99	1.01	1.01	1.00	1.02	1.02	1.01	1.01	0.98	0.96	1.01	1.04	1.01	1.02	1.10	1.02
GR1	1.23	1.28	2.07	1.28	1.31	1.82	1.29	1.95	1.26	1.10	1.15	1.37	1.47	1.24	1.72	1.20
GR2	1.13	1.20	1.59	1.11	1.17	1.29	1.25	1.23	1.03	1.07	0.99	1.02	1.22	1.20	1.25	1.06
GR3	1.04	1.04	1.13	1.23	1.17	1.04	1.15	1.07	1.00	1.02	0.88	1.06	1.14	1.11	1.11	1.07
Shrink 1	1.05	1.12	1.15	1.04	1.11	1.18	1.14	1.02	1.02	1.03	0.95	1.00	1.16	1.15	1.13	1.05
Shrink 2	1.01	1.03	1.03	0.99	1.02	1.03	1.01	1.00	1.02	1.07	1.01	0.95	1.04	1.07	1.03	1.00
Odds	1.03	1.05	1.02	1.01	1.03	1.04	0.99	1.01	1.01	1.03	0.90	0.99	1.02	1.02	1.03	1.00
Previous best	1.10	1.11	1.04	1.04	1.08	1.06	0.96	0.96	1.00	1.08	0.92	1.05	1.07	1.06	1.08	1.03
Inverse of MSE	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.01	1.00	1.00	1.00
EM	1.06	1.06	1.10	1.18	1.03	1.19	1.05	1.04	1.01	1.04	1.03	1.07	1.35	1.09	1.02	1.05
RMSE ratio:	TBILL				UNEMP											
	1	2	3	4	1	2	3	4								
EW	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00								
BAM	1.12	1.21	1.12	1.04	0.97	1.00	1.02	1.02								
SIC	1.01	0.96	0.98	0.96	0.99	1.00	0.99	0.93								
GR1	1.30	1.12	1.18	1.17	1.13	1.18	1.14	1.17								
GR2	1.15	1.05	0.99	1.12	1.16	1.27	1.15	1.25								
GR3	1.09	1.02	0.98	1.04	1.14	1.29	1.22	1.24								
Shrink 1	1.14	1.04	0.99	1.06	1.10	1.17	1.06	1.17								
Shrink 2	1.10	1.06	1.02	1.09	1.03	1.08	1.05	1.12								
Odds	1.09	1.03	1.03	1.03	1.07	1.05	1.04	1.05								
Previous best	1.14	1.05	1.03	1.04	1.19	1.00	1.12	1.10								
Inverse of MSE	1.25	1.01	0.96	0.91	0.95	0.94	0.97	1.11								
EM	1.01	1.00	0.94	0.93	1.06	1.07	1.09	1.04								

NOTE: Root mean squared error (RMSE) ratios are computed using the RMSE of the equal-weighted model (EW) in the denominator. Integers in the table header (1, 2, 3, 4) refer to the forecast horizon in quarters. See Table 2 for a definition of the series.

we have normalized by the RMSE for the equal-weighted average. In some cases the underlying combination models are nested while in other cases they are not. This makes it difficult to compare the statistical significance of the performance measures which we therefore do not pursue any further.

In common with empirical findings in the literature, the simple equal-weighted forecast turns out to be extraordinarily difficult to beat. For example, the Granger-Ramanathan combinations underperform across the board, in some cases by a wide margin. The shrinkage schemes mostly improve upon the least squares combination methods but continue to underperform against the equal-weighted combination. The same conclusion is true for the odds ratio and EM approaches.

Only two methods seem capable of producing better average forecasting performance than the equal-weighted average, namely the bias-adjusted/*SIC* method and the inverse of the MSE. The *SIC* method that selects between the simple and bias-adjusted equal-weighted average does particularly well, producing lower RMSE-values than the equal-weighted forecast for close to half of the series including the consumer price index, industrial production, nominal GDP growth, the GDP price deflator, changes in private inventories or residential fixed investments, T-bill rates, and the unemployment rate.

4.2 Bias and Heterogeneity

Our earlier analysis suggests that the performance of the equal-weighted mean deteriorates as a result of cross-sectional heterogeneity in forecast precision. To see if this helps explain our empirical findings, Table 4 reports measures of cross-sectional heterogeneity in the biases along with standard deviations of the forecast errors. More specifically, as a measure of

heterogeneity in the biases we report the cross-sectional standard deviation of the forecast biases normalized by the standard deviation of the outcome variable. To measure heterogeneity in the variance of the forecast errors, we report the cross-sectional standard deviation of forecast error variances, normalized by the variance of the outcome variable. Interestingly, cross-sectional heterogeneity in the bias is quite strong for variables such as the CPI and GDP price index where we find that the bias-adjusted mean improves over the simple mean. Similarly, heterogeneity in the variances is strong for variables such as the T-bill rate and the GDP price index, where the bias-adjusted mean again performs well.

Table 4 also shows estimates of α and β from the bias-adjustment regression (7) applied to the full sample. For a majority of the variables, there is strong evidence of biases as $\alpha \neq 0$ or $\beta \neq 1$. This is clearly the case for variables such as the consumer and GDP price indexes, nominal GDP growth, residential fixed investment, and the T-bill rate where significant biases coincide with good performance of the bias-adjustment method.

4.3 Attrition of Forecasters

Understanding the process whereby forecasters exit from the sample is important. Suppose, for example, that there is a systematic tendency for forecasters who previously produced relatively poor forecasts to leave the sample so that only the best forecasters remain. This would suggest focusing mainly on the forecasters with the longest track record. Conversely, if there is only scant evidence that past forecasting performance is related to attrition, then the number of forecasts reported by individual

Table 4. Measures of heterogeneity across forecasters

Measures:	CPI				CPROF				HOUSING			
	1	2	3	4	1	2	3	4	1	2	3	4
Nfore	99	98	96	91	76	74	72	69	84	85	80	71
Bias_het	0.36	0.41	0.42	0.47	0.34	0.30	0.27	0.25	0.30	0.27	0.22	0.26
Stdev_het	0.45	0.53	0.39	0.38	0.29	0.29	0.27	0.28	0.64	0.29	0.32	0.39
alpha	1.22***	1.54***	1.39***	1.84***	2.40	2.72	0.58	0.93	1.86	2.30	2.56	1.90
beta	0.58***	0.46***	0.48***	0.34***	0.85	0.82	1.17	1.27	0.79	0.53**	0.72	0.91
Measures:	INDPROD				NGDP				PGDP			
	1	2	3	4	1	2	3	4	1	2	3	4
Nfore	93	93	89	86	93	93	91	86	97	97	95	89
Bias_het	0.32	0.35	0.37	0.36	0.29	0.31	0.29	0.28	0.35	0.54	0.52	0.62
Stdev_het	0.87	1.09	0.54	1.09	0.37	0.44	0.57	0.96	3.44	2.93	2.04	3.12
alpha	-1.79**	0.72	1.81	2.32**	2.60**	4.71***	4.34***	4.54***	0.34*	0.67***	0.62***	0.66***
beta	1.42*	0.67	0.39**	0.27**	0.59**	0.22***	0.30***	0.26***	0.78***	0.63***	0.62***	0.59***
Measures:	RCBI				RCONSUM				RFEDGOV			
	1	2	3	4	1	2	3	4	1	2	3	4
Nfore	88	88	86	84	88	90	88	84	78	80	77	72
Bias het	0.25	0.32	0.37	0.39	0.22	0.29	0.30	0.30	0.16	0.20	0.21	0.20
Stdev het	0.31	0.31	0.27	0.37	0.82	0.72	0.53	0.59	0.63	0.40	0.19	0.21
alpha	-4.04	-0.49	7.63	16.81**	0.02	1.16	3.85***	2.82**	-1.31*	-0.82	-0.42	0.09
beta	1.07	0.85	0.49**	0.12***	1.23	0.81	-0.19***	0.20*	1.89***	1.76***	1.33	1.69**

Table 4. (Continued)

Measures:	RGDP				RRESINV				RSLGOV			
	1	2	3	4	1	2	3	4	1	2	3	4
Nfore	96	97	95	89	85	86	83	82	81	81	79	78
Bias_het	0.28	0.30	0.31	0.31	0.30	0.25	0.26	0.31	0.20	0.19	0.23	1.50
Stdev_het	0.33	0.67	0.49	0.68	0.60	0.65	0.53	0.54	0.54	0.99	0.58	180.15
alpha	−0.76	1.16	2.89***	2.21**	1.56*	2.99***	3.33***	2.98***	−0.11	0.64	1.18	2.42***
beta	1.34*	0.64	0.07***	0.32**	1.36**	0.80	0.70*	0.82	1.34	0.87	0.67	0.04***

Measures:	TBILL				UNEMP			
	1	2	3	4	1	2	3	4
Nfore	90	90	88	83	98	98	96	89
Bias_het	0.07	0.11	0.14	0.19	0.07	0.11	0.16	0.18
Stdev_het	0.39	1.35	2.01	1.42	0.11	0.16	0.23	0.29
alpha	−0.02	0.28	0.38	0.41	−0.17**	−0.20	−0.05	0.26
beta	1.00	0.91***	0.87***	0.84***	1.02	1.02	1.00	0.94

NOTE: Nfore is the number of forecasters with more than 10 forecasts (per variable-horizon). Bias_het is variation in the bias of the Nfore forecasters, computed as the standard deviation in the bias across forecasters divided by the variance of the outcome variable (for the whole sample). Stdev_het is the heterogeneity in the variances, computed as the standard deviation across the Nfore forecasters of the variance (per forecaster) of the forecast error divided by the variance of the outcome variable (for the same periods of the forecasts). Alpha and beta are the parameter estimates of the bias-adjusted mean based on full-sample information. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$ with alpha equal to zero and beta equal to one under the null hypothesis.

survey participants—or their past forecasting performance—is unlikely to be a good indicator of future performance.

from the survey and poor past forecasting performance is very weak.

To briefly address this issue we select survey participants with a minimum of 10 reported forecasts. For these we code continued participation as zero and exit as one. For each of the variables in the survey we pool the data across survey participants and estimate probit models using the previous number of forecasts and relative RMSE as regressors. Table 5 shows when the regressors were significant along with the sign of the coefficients in these cases. In many cases the number of previous forecasts is significantly negatively related to the probability of an exit, suggesting that forecasters who have participated in the survey for a long time are less likely to exit. The relative RMSE is significant for only a few of the variables—in many cases with the wrong sign—so the evidence of a link between exits

5. CONCLUSION

Real-time combination of survey forecasts requires trading off biases induced by using restricted and suboptimal combination weights against the effect of parameter estimation error arising from the use of less restricted combination methods. This tradeoff changes with the number of forecasts and so helps explain why the entry and exit of forecasters is important to the performance of different combination methods. Attrition in forecast surveys means that the performance of combination methods that require estimating the covariance between the individual forecasts deteriorates relative to that of more robust

Table 5. Results from probit estimation

Variable	No. of previous forecasts				Relative RMSE			
CPI	1(−)**	2(−)**	3(−)**	4(−)*				
CPROF								
HOUSING	1(+)*							
INDPROD		2(−)*						
NGDP								
PGDP			3(−)*		1(+)*			
RCBI			3(−)**					4(+)*
RCONSUM					2(−)*			4(−)*
RFEDGOV								
RGDP		2(−)*	3(−)*					
RRESINV					2(−)*	3(−)**	4(−)*	
RSLGOV				4(−)*				
TBILL	1(−)*	2(−)*	3(−)**	4(−)				
UNEMP	1(−)*	2(−)**	3(−)**	4(−)*				

NOTE: This table indicates significance and signs of coefficients in a probit regression of forecasters’ exit from the survey using the survey participants’ previous number of reported forecasts and their RMSE computed relative to the average RMSE as regressors. * indicates significance at the 10% level; ** indicates significance at the 5% level. (−) represents a negative, while (+) is a positive coefficient estimate.

methods such as equal weighting. We find empirical evidence, however, that the equal-weighted forecast is strongly biased and that a simple bias-adjustment method seems to work well in practice.

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