A Not-So-Comprehensive List of Markov chain Monte Carlo Algorithms

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1 Introduction

Sample $\theta^{(1)}, \theta^{(2)}, \ldots, \theta^{(T)}$ from a target distribution $\pi(\theta)$ for $\theta \in \Omega$.

notation : At last, we introduce the notations for clarification throughout this article. Let θ be a parameter/variable of our interest and superscript ${}^{(t)}$ is used whenever an object of our interest is specific to iteration t. For example, $\theta^{(t)}$ denotes the value of a parameter at iteration t. Ω stands for the domain on which the target distribution $\pi(\theta)$ is defined. $K(\cdot|\theta^{(t)})$ is a conditional density, also known as *proposal* or *transition/Markov* kernel, to generate next candidate at iteration t.

2 Basic Algorithms

2.1 Metropolis-Hastings Algorithm

Metropolis-Hastings (MH) algorithm [MRR+53, Has70] is a fundamental building block of MCMC computation that represents a family of *acceptance-rejection* type algorithms. Metropolis algorithm can be considered as a special case of MH method where proposal density K is symmetric. That means, when $K(x|y) = K(y|x)$, MH algorithm introduced in 2 is reduced to Algorithm 1 via cancellation of the term involving the ratio of proposal densities.

Algorithm 1 Metropolis Algorithm

Initialize:

$\theta^{(0)} \in \Omega$

for t = 1 to T **do**

sample θ' from proposal $K(\cdot|\theta^{(t)})$

compute acceptance probability $\alpha^{(t)}$ such that

$$\alpha^{(t)} = \min\left\{1, \frac{\pi(\theta')}{\pi(\theta^{(t)})}\right\}$$

sample $u^{(t)} \sim U[0, 1]$ and decide by

$$\theta^{(t+1)} = \begin{cases} \theta' & \text{if } u^{(t)} \leq \alpha^{(t)} \\ \theta^{(t)} & \text{otherwise} \end{cases}$$

end for

Algorithm 2 Metropolis-Hastings Algorithm

Initialize:

$\theta^{(0)} \in \Omega$

for t = 1 to T **do**

sample θ' from proposal $K(\cdot|\theta^{(t)})$

compute acceptance probability $\alpha^{(t)}$ such that

$$\alpha^{(t)} = \min\left\{1, \frac{\pi(\theta')}{\pi(\theta^{(t)})} \times \frac{K(\theta^{(t)}|\theta')}{K(\theta'|\theta^{(t)})}\right\}$$

sample $u^{(t)} \sim U[0, 1]$ and decide by

$$\theta^{(t+1)} = \begin{cases} \theta' & \text{if } u^{(t)} \leq \alpha^{(t)} \\ \theta^{(t)} & \text{otherwise.} \end{cases}$$

end for

2.2 Gibbs Sampler

For multiavariate functions $f, g : \mathbb{R}^n \to \mathbb{R}$ with local maximum x_0, we have $\nabla f(x_0) = 0$ and $\nabla\nabla^\top f(x_0) = H_f(x_0) < 0$ for first and second-order conditions in multivariate calculus. Similar to univariate cases, we have following approximations,

- **Version 3.**

$$ver3 \tag{1}$$

- **Version 4.**

$$ver4 \tag{2}$$

3 Sampling from Intractable Distributions

References

[Has70] W. K. Hastings. Monte Carlo Sampling Methods Using Markov Chains and Their Applications. *Biometrika*, 57(1):97, April 1970.

[MRR+53] Nicholas Metropolis, Arianna W. Rosenbluth, Marshall N. Rosenbluth, Augusta H. Teller, and Edward Teller. Equation of State Calculations by Fast Computing Machines. *The Journal of Chemical Physics*, 21(6):1087–1092, June 1953.