Numerics workshop. Part I

Vladyslav Pushenko

Technische Universität Ilmenau

15-25 of August,2023

FRONTIERS OF MIXING

This project has received funding from the European Union's Horizon 2020 research and innovation program under the Marie Skłodowska-Curie grant agreement N°956457.

Define flow - NARMA model

Flow velocity

2D velocity field is defined as

$$u_x(x, y, t) = A(t) \cos \omega_1 y \tag{1a}$$

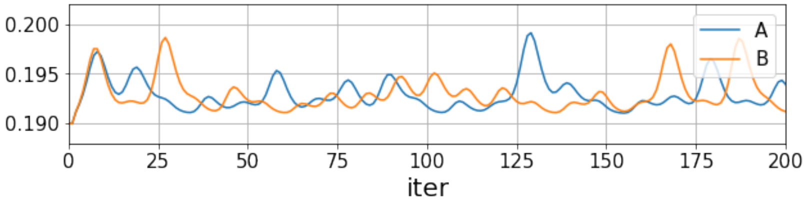
$$u_y(x, y, t) = B(t) \sin \omega_2 x \tag{1b}$$

Here $A(t)$ and $B(t)$ for k-th iteration:

$$A(t_{k+1}) = 0.4A(t_k) + 0.4A(t_k)A(t_{k-1}) + 0.6g_k^3 + 0.1 \tag{2a}$$

$$B(t_{k+1}) = 0.4B(t_k) + 0.4B(t_k)B(t_{k-1}) + 0.6g_k^3 + 0.1 \tag{2b}$$

$$g(t_k) = 0.1\,(sin(2\pi\alpha k)sin(2\pi\beta k)sin(2\pi\gamma k) + 1) \tag{2c}$$



Particle tracking

Lagrangian tracers

Evolution of particle trajectories:

$$\frac{dx_p}{dt} = u_x(x_p, y_p, t) \tag{3a}$$

$$\frac{dy_p}{dt} = u_y(x_p, y_p, t) \tag{3b}$$

To implement

- Bilinear interpolation;
- Ghost cells;
- Periodical boundary conditions.

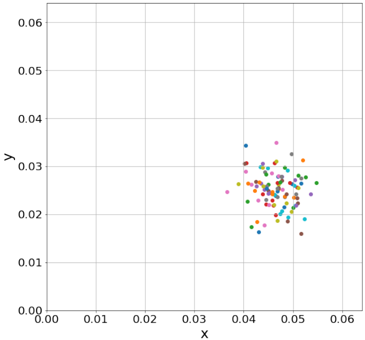


Figure: Initialization of particles

Bilinear interpolation

$$f(x,y) = f(Q_{11})w_1 + f(Q_{21})w_2 f(Q_{12})w_3 + f(Q_{22})w_4,$$

where

- $w_1 = \frac{(x_2 - x)(y_2 - y)}{(x_2 - x_1)(y_2 - y_1)}$;
- $w_2 = \frac{(x - x_1)(y_2 - y)}{(x_2 - x_1)(y_2 - y_1)}$;
- $w_3 = \frac{(x_2 - x)(y - y_1)}{(x_2 - x_1)(y_2 - y_1)}$;
- $w_4 = \frac{(x - x_1)(y - y_1)}{(x_2 - x_1)(y_2 - y_1)}$.

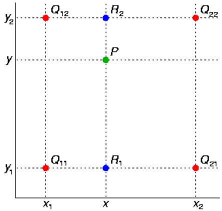


Figure: Linear interpolation. Source: Wikipedia.

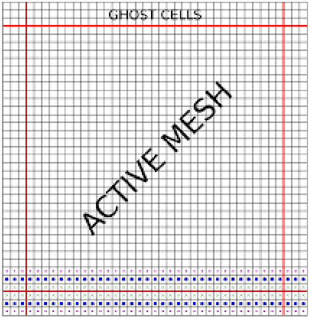


Figure: Ghost cells. Sourse: Google

Analysis to do

- Single dispersion;
- Double dispersion.