Personal Statement
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Often in mathematics, one has a strong feeling of what should be true without necessarily seeing a method of proof. What excites me the most in mathematics is when such apparent connections in one discipline are solidified by dipping into a completely different discipline. Why do conditions that imply a family of analytic functions from a fixed region is normal always seem to imply that a given entire function is constant? Why are nonabelian groups of order a product of two prime powers never simple?

My true mathematical journey started as a junior in high school when I began dual-enrolling in three courses per semester at my local university (the University of South Florida). After completing my first "introuduction to proofs" course, I dipped my toes into analysis and algebra. I enjoyed both equally and wished to experience them at the graduate-level, so I went about convincing my county's board of education to allow me to take graduate courses for dual-enrollment. As the graduate-level analysis courses were offered in the mornings, a time during which I was required by law to attend high school every day, I was pushed towards entering the University of South Florida's graduate algebra sequence.

It was during this sequence of courses that I first encountered one of those aforementioned facts that one feels "should" be true and whose truth arises from completely different fields of mathematics. Whilst covering Sylow's theorems, I would often encounter statements of the form "a nonabelian group of order a product of two given prime powers isn't simple." The reoccurrence of these similar statements piqued my curiosity and led me to discover the existence of a general result: Burnside's pq-theorem. What intrigued me about this result the most is that its standard proof relied on concepts completely outside group theory.

This connection led me to take a course on representation theory during my first semester at the University of Virginia at the end of which we covered the proof of Burnside's pq theorem in full. There are two aspects of this proof which truly fascinated me. The first is the reduction of the original theorem statement to showing the existence of a nontrivial complex representation ρ and nonidentity group element g such that $\rho(g)$ is a scalar operator. To me, the appeal of this connection is that it circumnavigates the lack of any conceivable purely group-theoretic approaches for proving nonsimplicity by providing an intuitive way forward through searching for a candidate representation among the irreducible complex representations of the group. The second part that I found exciting is that the rest of the proof follows almost immediately after showing that, for any group element g and irreducible complex character χ, the quantity $\frac{|K(g)|}{\chi(1)}\chi(g)$ is an algebraic integer. The fact that this remaining details of the proof rest upon a foray into algebraic number theory astonished me. These two aspects of this proof exemplify where I find excitement in mathematics.

In addition to representation theory, during my first semester at the University of Virginia the mathematics department gave me the freedom to push myself right out of the gate taking almost an identical courseload to the university's first-year graduate students. My major takeaways from these courses are threefold. First, from my graduate algebra course I gained an appreciation for the numerous benefits of maintaining a categorical perspective. Whilst teaching this course, Professor Andrei Rapinchuk constantly emphasized the categorical/functorial properties of our objects of study and demonstrated the usefulness of category theory as a bookkeeping technique. Second, through my graduate complex analysis course I grew accustommed to an "in the weeds" style of doing analysis. Professor Benjamin Hayes preferred the use analytical tools of complexity approximately equal to that of the theory at hand and encouraged our adoption of this philosophy. For example, in one homework we were tasked with justifying the holomorphicity of the limit of a locally uniformly convergent sequence of holomorphic functions; however, we were not allowed to use Morera's theorem for a quick proof and had to proceed using more fundamental results such as the Cauchy integral formula.

Third and most importantly, I experienced just how invigorating collaborating (and struggling) on tough problems with a cohort of people who are just as determined and passionate about math as I am is. From the outset of the semester, I began to work on mathematics together with my new classmates, some of whom were advanced undergraduates and the rest first-year graduate students. Gradually, I became part of a tight-knit group of fellow students who, regardless of age or background, were all working through the same concepts I was and shared the same passion I had. Whether it be in the form of desparately brainstorming for ideas to tackle a problem or intently listening to each other discuss fascinating research over pizza, this past semester I have learned that there is nothing more rewarding than doing mathematics with others.

This current semester, I am continuing to push myself by again taking the standard courses for first-year graduate students: algebraic topology I, real analysis, and algebra II. [TODO expand upon what these courses will cover]

Following these courses, I plan to take both the analysis and algebra qualifying exams to both solidify my understanding of the material and provide concrete evidence of my background in the respective subjects to graduate schools. In the long term, these few semesters will provide a strong foundation for me to begin my own independent mathematics research as an undergraduate and to be able to focus heavily on research right from the start of graduate school.