Williams College
SMALL Undergraduate Research Program
Application Form

You should apply online at http://www.mathprograms.org/db (below is the additional info required; you can type directly on this sheet or print it out and upload a scan). Remember to apply only to your top choice; if you apply to more your application will not be read.

Full Name: James Henry Harbour, II

Email: james.h.harbour@gmail.com

College or University: University of Virginia Expected Graduation Date: 2025

Write Current Status: first-year sophomore junior other (explain): first-year

Campus Address and Phone: 1827 University Avenue, Charlottesville, Virginia 22904

(434) 924-0311

Home Address and Phone: 229 Balz Dobie, Charlottesville, Virginia 22904

(205)-876-4085

Cell Phone Number: (205)-876-4085

How did you hear about SMALL? Via a professor mentioning the program to me.

You may rank up to three groups in which you are interested below. For updates and descriptions see http://math.williams.edu/small/ or the MathJobs page. Your first choice should be the one you are applying to.

First choice: Knot Theory

Second choice: Number Theory and Probability

Third choice: ____________________

List of math courses completed or currently being taken, with grades for those completed (if this is not enough room just add additional lines).

University of Virginia	
MATH 7752, Graduate Algebra II; *current*	MATH 7310, Graduate Real Analysis; *current*
MATH 7800, Graduate Algebraic Topology I; *current*	MATH 7751, Graduate Algebra I; *A+*
MATH 7340, Graduate Complex Analysis; *A+*	MATH 5657, Bilinear Forms and Representation Theory; *A*

University of South Florida (Dual enrollment concurrent with high school)	
MAS 6312, Graduate Algebra II, *A+*	MAS 5301, Graduate Algebra I, *A*
MAP 5345, Graduate Applied Partial Differential Equations, *A+*	MTG 4302, Introduction to Topology, *A+*
MTG 4254, Differential Geometry, *A+*	MTG 4214, Modern Geometry, *A+*
MAA 4211, Intermediate Analysis I, *A+*	MAS 4301, Elementary Abstract Algebra I, *A*
MAD 4203, Introduction to Combinatorics, *A+*	MAS 3105, Linear Algebra, *A+*
MGF 3301, Bridge to Abstract Mathematics, *A+*	MAC 2313, Calculus 3, *A+*

Also have at least one relevant faculty member upload a letter of recommendation by the deadline. (For Williams students, just give us at least one name of a math faculty member who can serve as a reference.) Please list the name and email address of your reference here.

Benjamin Hayes, brh5c@virginia.edu ; Thomas Bieske, tbieske@usf.edu

US citizenship status. (Funding is available for a limited number of students who are not US citizens or permanent residents.)

___Yes___ U.S. Citizen _________Permanent Resident ________Non-U.S. Citizen

Statement of interest (1-2 pages): *(Attach at the end of this form)*

All application materials must be received on-line by **the listed date for full consideration (see the webpage, usually the first or second Wednesday in February)**; if you hear from another program that has an earlier deadline than ours but are interested in SMALL, please let us know so we can give you an update on your status.

Personal Statement
James Harbour
Application to *SMALL* REU

In mathematics, one often has a statement which *should* be true but evades any seemingly natural methods of proof. What excites me most in mathematics is when the correct method for attacking such statements is the use of suprising reductions to facts that come from completely different disciplines. Mainstay canon such as Riemann's mapping theorem and Burnisde's pq-theorem are interesting statements on their own; however, the true meat of why I love these theorems is that their proofs both rely on an incredibly creative reduction to facts or properties from fields that seem, at least at first, entirely unrelated to the original statements.

My true mathematical journey started as a junior in high school when I began dual-enrolling in three courses per semester at my local university (the University of South Florida). After completing my first "introduction to proofs" course, I dipped my toes into analysis and algebra. I enjoyed both equally and wished to experience them at the graduate-level, so I went about convincing my county's board of education to allow me to take graduate courses for dual-enrollment. As the graduate-level analysis courses were offered in the mornings, a time during which I was required by law to attend high school every day, I was pushed towards entering the University of South Florida's graduate algebra sequence.

It was during this sequence of courses that I first encountered one of those aforementioned facts: Burnside's pq-theorem. The result was mentioned in a passing comment whilst covering Sylow's theorems, and what stuck with me was that its most straightforward proof relied upon representation theory. The fact that such a simply stated theorem in group theory required reaching far beyond the machinery my algebra course had built ignited my interest.

This connection led me to take a course on representation theory during my first semester at the University of Virginia at the end of which we covered the proof of Burnside's pq theorem in full. One aspect of the proof that really exemplifies my aforementioned mathematical taste is the reduction of the original theorem statement to showing the existence of a nontrivial complex representation ρ and nonidentity group element g such that $\rho(g)$ is a scalar operator. I was immediately attracted to this big-picture idea of taking something purely group-theoretic and turning it on its head by reaching outside of pure group theory to obtain an entirely non-obvious connection to a field whose tools make the problem feel much more approachable.

In addition to representation theory, during my first semester at the University of Virginia the mathematics department gave me the freedom to push myself right out of the gate taking almost an identical course load to the university's first-year graduate students. My major takeaways from these courses are threefold. First, from my graduate algebra course I gained an appreciation for the numerous benefits of maintaining a categorical perspective. Whilst teaching this course, Professor Andrei Rapinchuk constantly emphasized the categorical/functorial properties of our objects of study and demonstrated the usefulness of category theory as a bookkeeping technique.

Second, through my graduate complex analysis course I grew accustomed to an "in the weeds" style of doing analysis. Professor Benjamin Hayes preferred the use analytical tools of complexity approximately equal to that of the theory at hand and encouraged our adoption of this philosophy. For example, in one homework we were tasked with justifying the holomorphicity of the limit of a locally uniformly convergent sequence of holomorphic functions; however, we were not allowed to use Morera's theorem for a quick proof and had to proceed using more fundamental results such as the Cauchy integral formula. This course also solidified my mathematical taste through our development of normal families for proving Riemann's mapping theorem. Professor Hayes mentioned that this statement was one of the main goals of the course; however, for a large portion of the course I could not see any way forward with the tools we had developed. What I loved about normal families is that they provided a concrete framework for proving statements such as the Riemann mapping theorem by borrowing from functional analysis, an entirely different field, in an ingenious manner.

Third and most importantly, I experienced just how invigorating collaborating (and struggling) on tough problems with a cohort of people who are just as determined and passionate about math as I am is. From the outset of the semester, I began to work on mathematics together with my new classmates, some of whom were advanced undergraduates and the rest first-year graduate students. Gradually, I became part of a tight-knit group of fellow students who, regardless of age or background, were all working through the same concepts I was and shared the same passion I had. Whether it be in the form of desperately brainstorming for ideas to tackle a problem or intently listening to each other discuss fascinating research over pizza, this past semester I have learned that there is nothing more rewarding than doing mathematics with others.

The SMALL REU would provide an ideal extension of this experience to working on topics at the frontiers of research. Moreover, due to its high caliber, the experience would foster my mathematical growth by immensely. My interest towards Knot Theory project began during the previous semester when I attended an incredibly fascinating talk on knot Floer homology and was excited by the subject's use of encoding purely topological data using clever combinatorial manipulations and algorithms. I would be delighted to pursue research in the subject at this program.

In the long term, the SMALL REU would provide a strong foundation for me to eventually begin my own independent mathematics research as an undergraduate and to be able to focus heavily on research right from the start of graduate school. Thank you for your consideration.

Regards,

James Harbour