The Total Area Statistic for Dyck Paths

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1 Preliminaries

$$c_n = \frac{1}{n+1}\binom{2n}{n}, \quad C = C(x) = \sum_{n\geq 0} c_n x^n = 1 + xC^2 = \frac{1-\sqrt{1-4x}}{2x}.$$

2 Counting Dyck Paths

3 Computing the Total Area

We follow the approaches in [CEF07] and [MSV96]

Theorem 3.0.1. *Let A_n be the total area of all of the c_n Catalan paths of length n. Then*

$$A_n = 4^n - \binom{2n+1}{n}$$

References

[CEF07] Szu-En Cheng, Sen-Peng Eu, and Tung-Shan Fu. "Area of Catalan paths on a checkerboard". In: *European Journal of Combinatorics* 28.4 (2007), pp. 1331–1344.

[MSV96] Donatella Merlini, Renzo Sprugnoli, and M. Cecilia Verri. "The area determined by underdiagonal lattice paths". In: *Trees in Algebra and Programming — CAAP '96*. Ed. by Hélène Kirchner. Berlin, Heidelberg: Springer Berlin Heidelberg, 1996, pp. 59–71.