Intro Math Research HW1

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1 Definitions

Definition 1.0.1. Let R be a ring. The ring of *symmetric polynomials* in n variables is the subring $R_s(n) \subseteq R[x_1, \ldots, x_n]$ consisting of all polynomials f such that

$$f(\sigma(x_1), \ldots, \sigma(x_n)) = f(x_1, \ldots, x_n) \text{ for all } \sigma \in S_n.$$

3 Facts

- (Very cool fact) These polynomials arise as precisely the characters for $Irr(GL_n(\mathbb{C}))$ (finite dimensional reps only).
- These polynomials form a basis for the ring of symmetric polynomials.
- These polynomials are parameterized by partitions.