Topological K-Theory Talk Outline

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October 16, 2023

1 References

- Karoubi, *K-theory: an introduction*
- UChicago Reu Papers

2 Outline

- Define category $Vect_{\mathbb{F}}(X)$ for compact Hausdorff space X and $\mathbb{F} \in \{\mathbb{R}, \mathbb{C}\}$.
- Define Whitney sum on $\mathrm{Vect}_{\mathbb{F}}(X)/\sim$
- Define Grothendieck completion $Gr : \mathbf{AbMon} \to \mathbf{Ab}$

$$\mathrm{Hom}_{Ab}(Gr(M), A) \cong \mathrm{Hom}_{AbMon}(M, UA)$$

- Define $K^0(X)$ (and maybe $KO^0(X)$)
- Define $\widetilde{K}^0(X)$ for pointed X.
  - $\widetilde{K}^0(X/A) \to \widetilde{K}^0(X) \to \widetilde{K}^0(A)$
  - $K^{-n}(X) := \widetilde{K}^0(\Sigma^n X)$ for $n \in \mathbb{N}$
- Compute $K^0(\{\mathrm{pt}\})$
- Talk about Bott Periodicity
- Talk about

$$K^0(-) \simeq [-, BU \times \mathbb{Z}], \quad KO^0(-) \simeq [-, BO \times \mathbb{Z}]$$

- Talk about Serre-Swan Theorem to motivate $K^0(R)$

$$\mathrm{Vect}_{\mathbb{C}}(X) \simeq \{\text{f.g. projective } \mathbb{C}(X)\text{-modules}\}$$

- Mention that we dont have suspensions to define $K^n(R)$ so its much harder.