Superrigidity Notes

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1 Preliminaries

1.1 Group Measure Space Construction

Let $\Gamma \curvearrowright (X, \mu)$ be a p.m.p. action of a countable group Γ on a probability space (X, μ). The action is

- *ergodic* if any Γ-invariante measureable subset $A \subseteq X$ must satisfy $\mu(A) \in \{0, 1\}$,
- *(essentially) free* if $\{x : gx = x\}$ is null for all $g \in \Gamma \setminus \{e\}$.

If the action is essentially free and ergodic, then $L^\infty(X) \rtimes \Gamma$ is a factor.

1.2 Examples of free, ergodic, p.m.p. actions

1.3 Cartan Subalgebras

Let M be a II_1 factor.

2 Cocycle (Super)Rigidity

3 Superrigid Groups

4 Superrigid Actions