Math 7820, Fall 2022

Homework 6, due on Thursday, November 10, electronically on Collab.

1. Problem 17.1 in Tu's textbook.

2. Problem 17.2 in Tu's textbook.

3. Prove that a vector bundle whose fiber is an n-dimensional vector space is trivial (i.e. is isomorphic to a product bundle) if and only if it admits n sections $s_1, \ldots, s_n$ such that $s_1(x), \ldots, s_n(x)$ are linearly independent for each point x in the base.

4. Suppose $E_1 \longrightarrow B, E_2 \longrightarrow B$ are two vector bundles over the same base. It may be assumed without loss of generality that they are both trivialized over the same collection of charts $\{U_i\}$ covering B. Denote their transition maps $\phi_{ij} \colon U_i \cap U_j \longrightarrow GL_m(\mathbb{R}), \psi \colon U_i \cap U_j \longrightarrow GL_n(\mathbb{R})$. The tensor product of these two bundles is a vector bundle over the same base B, defined by taking the tensor product of the transition maps at each point, with values in $Gl_{mn}(\mathbb{R})$.

Carry out this construction in the case of two copies of the non-trivial line bundle over the circle discussed in class, and identify their tensor product.