Math 7820, Fall 2022

Homework 3, due on Thursday, September 29, electronically on Collab.

The first two problems refer to Loring Tu's textbook.

1. Problem 9.3.

2. Problem 9.10.

3. (a) Consider the "height map" $h\colon S^2 \longrightarrow \mathbb{R}$. Here S^2 is the unit sphere in $\mathbb{R}^3$ and $h(x,y,z) = z$. Find the critical points and critical values for this map.

(b) Show that any map $f\colon S^2 \longrightarrow \mathbb{R}$ has at least two critical points. Generalize this proof from S^2 to any n-dimensional compact manifold.

4. Consider a submanifold $M^n \subset \mathbb{R}^k$, and let $TM \subset \mathbb{R}^k \times \mathbb{R}^k$ be the set of all pairs (x,v) where x is a point in M and $v \in T_x M$. Show that TM is a smooth $2n$-dimensional submanifold of R^{2k}.