CMND 2022 Problem Set 1

James Harbour

May 30, 2022

Section 1

Problem 2

Proof. Suppose that $X \subseteq Y$ and let $f \in I(Y)$. Then $f(P) = 0$ for all $P \in Y$, so *a fortiori* $f(P) = 0$ for all $P \in X$.

Suppose that $I \subseteq J$ and let $P \in V(J)$. Then $f(P) = 0$ for all $f \in J$, whence again we have that $f(P) = 0$ for all $f \in I$.

As $a^1 \in I$ for all $a \in I$, it is clear that $I \subseteq \sqrt{I}$. Hence, by the previous part $V(\sqrt{I}) \subseteq V(I)$. Now suppose that $P \in V(I)$ and let $f \in \sqrt{I}$. Then there exists some $n \in \mathbb{N}$ such that $f^n \in I$, so $(f(P))^n = 0$ whence $f(P) = 0$ as k is a field (and thus an integral domain).

For notational clarity, let $\mathfrak{a}$ be an ideal and suppose that $\mathfrak{p} \supset \mathfrak{A}$ be a prime ideal. Suppose that $f \in I(V(\mathfrak{a}))$. Then $f(P) = 0$ for all $P \in V(\mathfrak{a})$, whence $f \in$.

$$I(V(\mathfrak{a})) = \bigcap_{P \in V(I)} I(P)$$

Suffices to show

$$I(V(\mathfrak{a})) = \bigcup_{\mathfrak{p} \supset \mathfrak{a}} \mathfrak{p}$$

$\square$