Time-domain methods for quantifying dynamic cerebral blood flow autoregulation: Review and recommendations. A white paper from the Cerebrovascular Research Network (CARNet)

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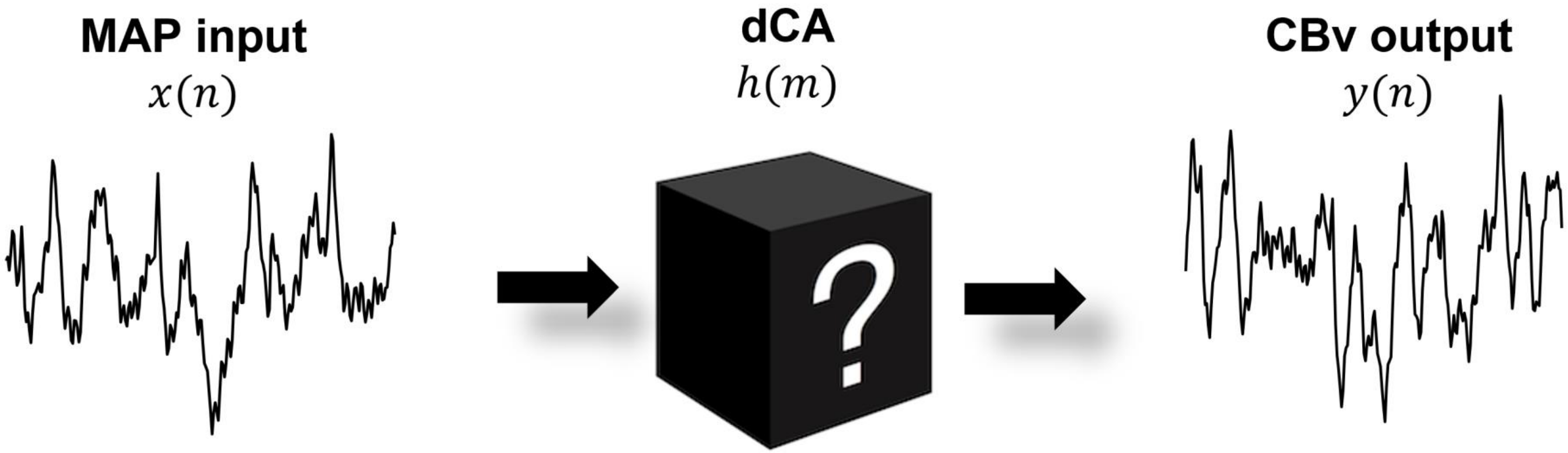

ARX model

Time domain

ARX

$$y(n) = \sum_{m=1}^{n_a} a_m y(n-m) + \sum_{m=0}^{n_b} \beta_m x(n-m) + \varepsilon(n) \Rightarrow \quad y(n) = \boldsymbol{w}^T(n)\boldsymbol{\varphi}(n) + \varepsilon(n)$$

$\boldsymbol{x}$: MABP
$\boldsymbol{y}$: CBv

Time-varying Identification - Recursive approaches

- **Kalman Filter (KF)**

Model coefficients: $\boldsymbol{w}(n) = \boldsymbol{w}(n-1) + u(n), \qquad \boldsymbol{u} \sim N(0, R1)$

Predicted CBv: $y(n) = \boldsymbol{w}^T(n)\boldsymbol{\varphi}(n) + \varepsilon(n)\,, \qquad \boldsymbol{\varepsilon} \sim N(0, R2)$

$$\hat{\varepsilon}(n) = y(n) - \boldsymbol{\varphi}^{\boldsymbol{T}}(n)\widehat{\boldsymbol{w}}(n-1)$$

$$\boldsymbol{K}(n) = \frac{\boldsymbol{P}(n-1)\boldsymbol{\varphi}(n)}{R2 + \boldsymbol{\varphi}^{\boldsymbol{T}}(n)\boldsymbol{P}(n-1)\boldsymbol{\varphi}(n)}$$

$$\boldsymbol{P}(n) = \boldsymbol{P}(n-1) + R1\boldsymbol{I} - \boldsymbol{K}(n)\boldsymbol{\varphi}^{\boldsymbol{T}}(n)\boldsymbol{P}(n-1)$$

$$\widehat{\boldsymbol{w}}(n) = \widehat{\boldsymbol{w}}(n-1) + \boldsymbol{K}(n)\hat{\varepsilon}(n)$$

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Optimized based on training data using a Genetic Algorithm (GA).

The ARX model order (n_a, n_b) is also optimized using the GA

The GA searches for the set of hyperparameters that minimize the Akaike Information Criterion (AIC) between the actual and predicted output, ensuring an optimal balance between model complexity and accuracy.

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Apply to unseen data to monitor dCA