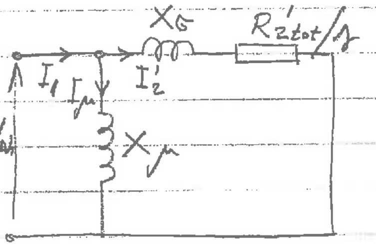
Question 1

A 3-phase, 4-pole, 50Hz, 415V, star-connected slip-ring induction motor has the rated speed of 1450rev/min. The motor can be represented by an approximate per-phase equivalent circuit in which the magnetising branch is transferred to the supply terminals and the only significant loss is in the rotor winding. The resistance of the rotor phase winding is 0.15Ω, and the total per-phase leakage reactance is 2.2 Ω. The load torque is constant and equal to the motor rated torque.

What value of resistance must be added in each rotor phase circuit in order to reduce the steady-state speed to one half of the synchronous speed ? Estimate the efficiency at this operating state.

Solution of Q1

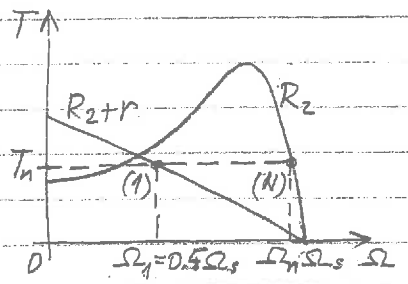


Based on assumptions stated in the question, the torque can be expressed as

$$T = \frac{P_{cu2}/s}{\Omega_s} = \frac{3 I_2'^2 R'_{2tot}/s}{\Omega_s}$$

$$= \frac{3V_{ph}^2}{\Omega_s} \times \frac{R'_{2tot}/s}{(R'_{2tot}/s)^2 + X_\sigma^2} \qquad (*)$$

In a steady state the motor torque matches load torque, and as the load torque is constant ($=T_n$) the ratio R'_{2tot}/s in (*) must remain unchanged for operating points (N) & (1), i.e.



$$\frac{R_2'}{s_n} = \frac{R_2'+r'}{s_1}$$

The rated slip: $s_n = \frac{n_s - n_n}{n} = \frac{1500-1450}{1500} = 0.033 \quad \left(n_s = \frac{60f}{p} = \frac{60 \times 50}{4/2} = 1500\,\frac{rev}{min}\right)$

The slip at $n = 0.5 n_s$: $s_1 = \frac{n_s - 0.5 n_s}{n_s} = 0.5$

Hence:

$$\frac{R_2'+r'}{0.5} = \frac{R_2'}{0.033} \implies r' = 14R_2' \implies m^2 r = 14 m^2 R_2 \quad \left(\text{where } m = \frac{N_{1eff}}{N_{2eff}}\right)$$

Hence, $r = 14 R_2 \implies r = 14 \times 0.15 = 2.1\,[\Omega]$

Based on assumptions stated in the question

$P_{out} = T \times \Omega = T\Omega_s(1-s)$ (mechanical losses neglected)

$P_{in} \approx P_{em} = T \times \Omega_s$ (stator losses neglected)

Hence

$$\eta = \frac{P_{out}}{P_{in}} \approx 1 - s = 1 - 0.5 = 0.5 \quad (\eta\% = 50\%)$$

Question 2

The rating plate of a 3-phase, 4-pole, 400 V, 50 Hz, delta-connected induction motor specifies the rated speed as 1450 rev/min. The motor can be represented by an approximate per-phase equivalent circuit where stator resistance, mechanical and iron losses are neglected and magnetising reactance is transferred to the supply terminals.

(a) If the motor is to be supplied from a 3-phase sinusoidal inverter, derive the condition for the relationship between the voltage and frequency applied to the motor so that the maximum (stall) torque remains unchanged when the frequency is reduced below 50 Hz.

Sketch in a single diagram the torque/speed characteristics of the motor operating under the above condition at frequencies of 50 Hz and 25 Hz, and estimate the speed (in rev/min) at rated torque and frequency of 25 Hz.

[Assume that at rated frequency (50 Hz) the inverter's output line-to-line voltage is 400 V (rms).]

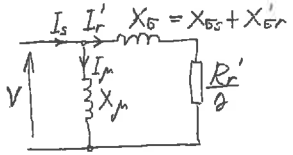
(b) The available 3-phase sinusoidal inverter produces 400 V (rms) at 50 Hz, and at 25 Hz the output voltage is 165 V. If the motor is supplied from this inverter and the operating frequency is 25 Hz, calculate:

(i) the speed (in rev/min) at which the rotor current would be at its rated value, and
(ii) the percentage of rated torque which would then be expected.

[Assume that the torque/speed characteristic of the motor has a linear form in the region around the operating point (i.e. $R_2'/s \gg X_\sigma$ in expressions for the rotor current and the torque) for both frequencies (50Hz and 25Hz).]

Solution of Q2

a)



The referred rotor current

$$I_r' = \frac{V}{\sqrt{\left(\frac{R_r'}{s}\right)^2 + X_\sigma^2}} \qquad (1)$$

The torque: $T = \frac{P_{em}}{\Omega_s} = \frac{3 I_r'^2 R_r'/s}{\Omega_s} = \frac{3V^2}{\Omega_s} \times \frac{R_r'/s}{\left(\frac{R_r'}{s}\right)^2 + X_\sigma^2}$ $\qquad (2)$

Maximum (stall) torque

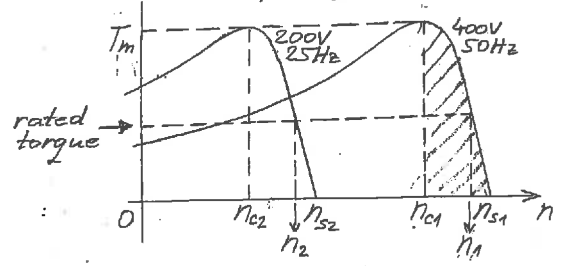
$$T_m = \frac{3V^2}{\Omega_s} \times \frac{1}{2X_\sigma} \qquad \left(\text{derived from } \frac{dT}{ds} = 0\right)$$

where $\Omega_s = 2\pi f/p$ (p – the pole pair number)

$X_\sigma = 2\pi f L_\sigma$

$$\therefore \; T_m = \frac{3p}{2L_\sigma} \times \left(\frac{V}{2\pi f}\right)^2$$

(*) Hence, if the max. torque should remain unchanged the ratio V/f ought to be kept constant.



$\left(\Omega = \frac{2\pi n}{60}\right)$

$n_{s1} = \frac{60 f_1}{p} = 1500 \frac{rev}{min}$

$n_{s2} = \frac{60 f_2}{p} = 750 \frac{rev}{min}$

The critical slip speed which corresponds to the max torque

$$\Delta\Omega_m = s_m \times \Omega_s = \frac{R_r'}{X_\sigma} \times \Omega_s = \frac{R_r'}{2\pi f L_\sigma} \times \frac{2\pi f}{p} = \frac{R_r'}{p L_\sigma}$$

is independent of frequency.

If 'V/f' ratio is kept constant, then the shaded area remains unchanged when the frequency is varied below the rated value, and therefore the 'gradient' of the operating part of the torque/speed characteristics remains unchanged.

Hence, the slip speed ($\Delta\Omega = \Omega_s - \Omega$) remains also unchanged, i.e

$$\Delta n = n_{s2} - n_2 = n_{s1} - n_1$$

$\therefore \; 750 - n_2 = 1500 - 1450$

$\therefore \; n_2 = 700 \frac{rev}{min}$ (speed at 25Hz and at the rated torque)

b) (i) The motor operates on the linear part of the torque/speed characteristic where

$$\frac{R_r'}{s} \gg X_\sigma$$

i.e. the torque and the rotor current equations (2) and (1) become:

$$(3) \quad T \approx \frac{3V^2}{\Omega_s} \times \frac{s}{R_r'} \qquad \text{and} \qquad I_r' \approx \frac{V}{R_r'} \times s \quad (4)$$

As the rotor current should remain unchanged, i.e. at the rated value for both supply conditions (400 Hz, 50 Hz and 165 V, 25 Hz), then

$$(5) \quad \frac{400}{R_r'} \times s_1 = \frac{165}{R_r'} \times s_2$$

s_1 is the rated slip at 50Hz i.e.

$$s_1 = \frac{n_{s1} - n_1}{n_{s1}} = \frac{1500 - 1450}{1500} = 0.0333$$

So, the slip at 165V and 25Hz is

$$s_2 = \frac{400}{165} \times s_1 = \frac{400}{165} \times 0.0333 = 0.0808$$

and the rotor speed is

$$n_2 = (1 - s_2) n_{s2} = (1 - 0.0808) \times 750 = 689.4 \frac{rev}{min}$$

(ii) Using equation (3), the torque

$$T_1 = \frac{3 \times 400^2}{2\pi \times 50/2} \times \frac{0.0333}{R_r'} \qquad \text{(This is the rated torque.)}$$

and

$$T_2 = \frac{3 \times 165^2}{2\pi \times 25/2} \times \frac{0.0808}{R_r'} \qquad \text{(This is the new torque.)}$$

The ratio T_2/T_1 is

$$\frac{T_2}{T_1} = \left(\frac{165}{400}\right)^2 \times \frac{50}{25} \times \frac{0.0808}{0.0333} = 0.826$$

Hence, if the rotor current is kept at the rated value when the inverter supplies 25Hz and 165V, the expected motor torque is 82.6% of the rated torque.