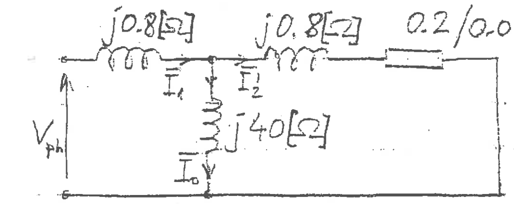
Example

A 3-phase, 415V, 50Hz, 4-pole, star-connected induction motor has the following per-phase equivalent circuit parameters: stator impedance $Z_1=(0+j0.8)\ \Omega$, referred rotor impedance $Z'_2=(0.2+j0.8)\ \Omega$ and magnetising impedance $Z_m=(0+j40)\ \Omega$.

(a) Using the exact equivalent circuit, calculate the torque when the slip is 0.05;

(b) Calculate the torque for the same slip using the approximate equivalent circuit where the magnetising branch is transferred to the supply terminals. Compare this value with the one calculated in (a).

(a)



$s = 0.05$

$$T = \frac{P_{em}}{\Omega_0} = \frac{3 I_2'^2 R_2'}{s\,\Omega_0} \qquad \text{where } \Omega_0 = \frac{\omega}{p} = \frac{2\pi f}{p}\,; \quad n_0 = \frac{60 f}{p}$$

Find I_2 from equiv. circuit.

$$\bar{Z}_e = \bar{Z}_1 + \frac{\bar{Z}_\mu \cdot \bar{Z}_2'}{\bar{Z}_\mu + \bar{Z}_2'} = j0.8 + \frac{j40(4+j0.8)}{j40+4+j0.8} = j0.8 + \frac{-32+j160}{4+j40.8} =$$

$$= j0.8 + \frac{163 \cdot e^{j101.3^\circ}}{41 \cdot e^{j84.4}} = j0.8 + 3.97\, e^{j16.9^\circ} =$$

$$= j0.8 + 3.8 + j1.15 = 3.8 + j1.95 = 4.27 \cdot e^{j27.2^\circ}$$

$$\bar{I}_1 = \frac{\bar{V}_{ph}}{\bar{Z}_e} = \frac{\frac{415}{\sqrt{3}} e^{j0}}{4.27 \cdot e^{j27.2^\circ}} = 56.1\, e^{-j27.2^\circ} = 49.9 - j25.6$$

$$\bar{E} = \bar{V}_{ph} - \bar{I}_1 \bar{Z}_1 = 239.6 - j0.8(49.9 - j25.6) = 239.6 - 20.5 - j39.9$$

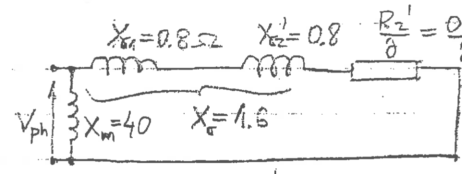
$$= 219.12 - j39.9 = 222.7 \cdot e^{-j10.3^\circ}$$

$$\bar{I}_2' = \frac{\bar{E}}{\bar{Z}_2'} = \frac{222.7\, e^{-j10.3^\circ}}{4+j0.8} = \frac{222.7 \cdot e^{-j10.3^\circ}}{4.08\, e^{j11.3^\circ}} = 54.6\, e^{-j21.6^\circ}$$

$I_2' = 54.6\ [A]$ (magnitude)

$$T = \frac{3 \times 54.6^2 \times 0.2}{0.05 \times 1500 \times \frac{2\pi}{60}} = \frac{35774}{157} = 227.9\ [Nm]$$

(b)



$$I_2' = \frac{V_{ph}}{\sqrt{\left(\frac{R_2'}{s}\right)^2 + X_\sigma^2}} = \frac{V_{ph} s}{\sqrt{R_2'^2 + (s X_\sigma)^2}}$$

$$T = \frac{3 I_2'^2 R_2'/s}{\Omega_0} = \frac{3 V_{ph}^2}{\Omega_0} \times \frac{s R_2'}{R_2'^2 + s^2 X_\sigma^2}$$

$$T = \frac{3 \times (415/\sqrt{3})^2}{1500 \times \frac{2\pi}{60}} \times \frac{0.05 \times 0.2}{0.2^2 + (0.05 \times 1.6)^2} = 1097 \times \frac{1}{4.64} = 236.4\ [Nm]$$

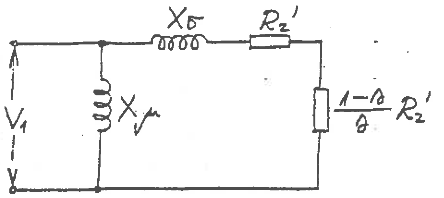
The percentage difference: $\frac{|T_{(a)} - T_{(b)}|}{T_{(a)}} \times 100\% = \frac{|227.9 - 236.4|}{227.9} \times 100\% = 3.7\%$

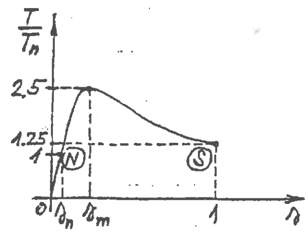
This justifies the use of approximate equiv. circuit for simpler calculation.

Example

A 3-phase, 50 Hz, 4-pole induction motor can be represented by simplified equivalent circuit where the stator and mechanical losses are neglected, and the magnetizing reactance is transferred to the supply terminals. The motor has the maximum / rated torque ratio $T_m/T_n = 2.5$ and the starting / rated torque ratio $T_{st}/T_n = 1.25$.

Calculate the slip in the steady-state operation when the motor is loaded with rated torque.





Based on approximate equivalent circuit, Kloss' equations applied at points N and S of torque/speed curve, respectively, are

$$\frac{T_m}{T_n} = \frac{\frac{s_m}{s_n} + \frac{s_n}{s_m}}{2} \qquad (N)$$

$$\frac{T_m}{T_{st}} = \frac{\frac{s_m}{1} + \frac{1}{s_m}}{2} \qquad (S)$$

$\frac{T_m}{T_n}$ is given, and knowing $\frac{T_{st}}{T_n}$ the ratio $\frac{T_m}{T_s}$ can be found as

$$\frac{T_m}{T_{st}} = \frac{\frac{T_m}{T_n}}{\frac{T_{st}}{T_n}} = \frac{2.5}{1.25} = 2$$

Hence

$$\frac{\frac{s_m}{s_n} + \frac{s_n}{s_m}}{2} = 2.5 \Longrightarrow \frac{s_m}{s_n} + \frac{s_n}{s_m} = 5 \qquad (1)$$

$$\frac{s_m + \frac{1}{s_m}}{2} = 2 \Longrightarrow s_m + \frac{1}{s_m} = 4 \qquad (2)$$

From (2) $\quad s_m^2 - 4 s_m + 1 = 0 \Rightarrow (s_m)_{1,2} = 2 \pm \sqrt{4-1}$

$s_m = 0.267$ (The value $s_m = 3.73$ is not acceptable, since machine operates as a motor.)

Substituting this value for s_m into (1) gives

$$\frac{0.267}{s_n} + \frac{s_n}{0.267} = 5$$

i.e. $\quad s_n^2 - 1.335 s_n + 0.0713 = 0 \Rightarrow s_n = \frac{1.335 \pm \sqrt{1.335^2 - 4 \times 0.0713}}{2}$

$s_n = 0.0557$ (The value $s_n = 1.279$ is not acceptable as the machine operates as motor and hence $0 < s_n < s_m$)

Example

A 3-phase, 415V, 50Hz, 4-pole delta-connected cage induction motor has the rated speed of 1425rev/min at full (rated) load. The starting current is six times larger than the rated current.

(Assume that the motor can be represented by an approximate per-phase equivalent circuit in which the magnetising branch is transferred to the supply terminals and the only significant loss is in the rotor cage.)

Determine:

(a) the starting torque developed by the motor as percentage of the full (rated) load torque;
(b) the per unit slip and speed in rev/min at which the motor develops max (stall) torque;
(c) the motor maximum (stall) torque as percentage of the full (rated) load torque.

a) The synchronous speed is

$$n_0 = \frac{60f}{p} = \frac{60 \times 50}{4/2} = 1500 \left[\frac{rev}{min}\right]$$

The full load slip is

$$s_n = \frac{1500 - 1425}{1500} = 0.05$$

The relationship between torque, rotor current and slip

$$T = \frac{P_a}{\Omega_0} = \frac{P_{cu2}/s}{\Omega_0} = \frac{3 I_2^2 R_2 / s}{\Omega_0}$$

At starting: $s_{st} = 1$ and $T_{st} = \frac{3 I_{2(st)}^2 R_2}{\Omega_0}$

At full load: $s_n = 0.05$ and $T_n = \frac{3 I_{2(n)}^2 R_2}{\Omega_0} \times \frac{1}{0.05}$

Hence,

$$\frac{T_{st}}{T_n} = \left(\frac{I_{2(st)}}{I_{2(n)}}\right)^2 \times 0.05$$

$$T_{st} = 6^2 \times 0.05 \times T_n = 1.8 T_n$$

b) From Kloss' equation

$$① \quad \frac{T_{st}}{T_m} = \frac{2}{\frac{s_m}{1} + \frac{1}{s_m}}$$

where T_m is max torque and s_m the corresponding (critical) slip

and

$$② \quad \frac{T_n}{T_m} = \frac{2}{\frac{s_m}{0.05} + \frac{0.05}{s_m}}$$

From ① and ②

$$\frac{T_{st}}{T_n} = \frac{\frac{s_m^2 + 0.05^2}{0.05 s_m}}{\frac{s_m^2 + 1}{s_m}}$$

i.e.

$$1.8 = \frac{s_m^2 + 0.05^2}{0.05(s_m^2 + 1)}$$

$$s_m^2 + 0.05^2 - 0.09 s_m^2 - 0.09 = 0$$

$$\therefore \quad s_m = \sqrt{\frac{0.0875}{0.91}} = 0.31$$

(+ve sign for motoring operation)

Speed at max. torque

$$n_m = (1 - s_m)\, n_0 = (1 - 0.31) \times 1500 = 1035 \left[\frac{rev}{min}\right]$$

c) From Eq. 2: $\frac{T_m}{T_n} = \left(\frac{s_m}{0.05} + \frac{0.05}{s_m}\right)/2$

$$\therefore \quad T_m = \frac{1}{2}\left(\frac{0.31}{0.05} + \frac{0.05}{0.31}\right) T_n = 3.18 T_n = 318\% \, T_n$$