HW1 001021120 Zehua Wang

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perspective projection | orthographic projection

1\. a)

$$\begin{cases} x' = \dfrac{f'x}{z} = f'\dfrac{x_0+ta}{z_0+tc} & ① \\ y' = \dfrac{f'y}{z} = f'\dfrac{y_0+tb}{z_0+tc} & ② \end{cases} \qquad \begin{cases} x' = x = x_0+ta & ③ \\ y' = y = y_0+tb & ④ \end{cases}$$

b) Yes $\dfrac{①}{②} \Rightarrow \dfrac{x'}{y'} = \dfrac{x_0+ta}{y_0+tb} \Rightarrow t = \dfrac{x_0y'-y_0x'}{bx'-ay'}$

$$\therefore x' = f'\frac{x_0 + \frac{x_0y'-y_0x'}{bx'-ay'}a}{z_0 + \frac{x_0y'-y_0x'}{bx'-ay'}c} \Rightarrow y' = \frac{1}{cx_0-az_0}\left[(y_0c-z_0)x' + (bx_0-ay_0)f'\right]$$

$\therefore$ so is a line

c) Yes ④ − ③ $\Rightarrow x'-y' = x_0-y_0+t(a-b) \Rightarrow t = \dfrac{x'-y'}{x_0-y_0}$ $\therefore x' = x_0+ta = x_0 + \dfrac{x'-y'}{x_0-y_0}$

$\therefore y' + (x_0-y_0-1)x' - x_0^2 + x_0y_0 = 0$

$\therefore$ so is a line

d) shown in zipped file

e) $x' = \dfrac{a}{c}f' + f'\dfrac{x_0-\frac{a}{c}z_0}{z_0+tc}$ $\quad t\to\infty \quad x' \to \dfrac{a}{c}f' = 0$ becomes a constant.

$y' = \dfrac{b}{c}f' + f'\dfrac{y_0-}{z_0+tc}$ $\quad t\to\infty \quad y' \to \dfrac{b}{c}f' = -1$

2\. a) perspective projection | orthographic projection

$$\begin{cases} x' = f'\dfrac{x}{z} = f'\dfrac{(x_1+ta)}{z_0} \\ y' = f'\dfrac{y}{z} = f'\dfrac{(y_1+tb)}{z_0} \end{cases} \qquad \begin{cases} x' = x = x_1+ta \\ y' = y = y_1+tb \end{cases}$$

$$\begin{cases} \hat{x}' = f'\dfrac{\hat{x}}{z} = f'\dfrac{(x_2+ta)}{z_0} \\ \hat{y}' = f'\dfrac{\hat{y}}{z} = f'\dfrac{(y_2+tb)}{z_0} \end{cases} \qquad \begin{cases} \hat{x}' = \hat{x} = x_2+ta \\ \hat{y}' = \hat{y} = y_2+tb \end{cases}$$

b) shown in files

c) PP:

eliminate t:

$$\hat{y}' = \frac{f'b}{a}x' + \frac{f'}{z_0}y_1 - \frac{f'^2 b}{az_0}x_1$$

$$\hat{y}' = \frac{f'b}{a}x' + \frac{f'}{z_0}y_2 - \frac{f'^2 b}{az_0}x_2$$

$$k_1 = k_2 = \frac{f'b}{a}$$

$\therefore$ parallel

op

eliminate t:

$$y' = bx' - bx_1 + ay_1$$

$$\hat{y} = b\hat{x}' - bx_2 + ay_2$$

$$k_1 = k_2 = b$$

$\therefore$ parallel

(d) Yes

(e) Yes. Because both perspective and othographic projections are parallel.

(f) The perspective projection will be a reversal version of othographic projection

3.

a)

perspective

$$\begin{cases} x' = f'\dfrac{x_1}{z_0 + tc} \\ y' = f'\dfrac{(y_1 + tb)}{z_0 + tc} \end{cases}$$

othographic

$$\begin{cases} \hat{x}' = \dfrac{f'x_2}{z_0 + tc} \\ \hat{y}' = \dfrac{f'(y_0 + tb)}{z_0 + tc} \end{cases}$$

othographic

$$\begin{cases} x' = x_1 \\ y' = y_0 + tb \end{cases}$$

$$\begin{cases} \hat{x}' = x_2 \\ \hat{y}' = y_0 + tb \end{cases}$$

b) shown in files

$m = \frac{f'}{-z}$.

c) for perspective, when $z \to \infty$, $m \to 0$, the distance bethen two will be small

$|| \to \Lambda$

for othographic, $m \equiv 1$ so the projections will be parallel

$|| \to ||$

d) Yes

e) No. when t changes from $-\infty$ to $+\infty$. Δz will be very large.

and orthographic projection is parallel while $\underbrace{\text{perspective}}_{\text{projection}}$ is not parallel

f)

$$\lim_{t\to\infty} x' = \lim_{t\to\infty} f' \frac{x_1}{z_0 + tc} = 0$$

$$\lim_{t\to\infty} y' = \lim_{t\to\infty} \frac{b}{c} f' + f' \frac{y_0}{z_0 + tc} = \frac{b}{c} f'$$

$\therefore (0,0)$

$(0,1)$

$(0,-1)$

$$\lim_{t\to\infty} \hat{x}' = \lim_{t\to\infty} \frac{f' x_2}{z_0 + tc} = 0$$

$$\lim_{t\to\infty} \hat{y}' = \lim_{t\to\infty} \frac{b}{c} f' + f' \frac{y_0}{z_0 + tc} = \frac{b}{c} f'$$

$\therefore (0,0)$

$(0,1)$

$(0,-1)$

(images are shown below)

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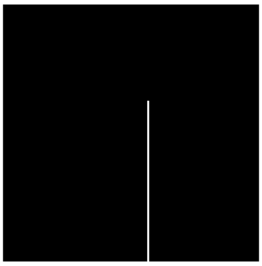
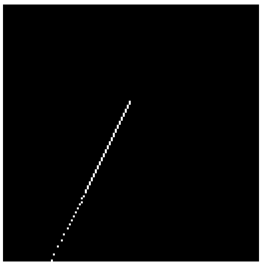
1.

d)



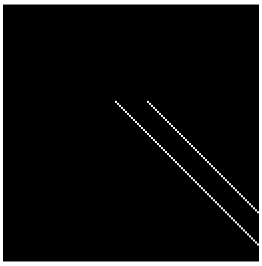
orthographic



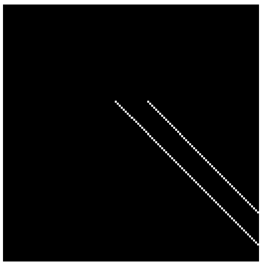
perspective

2.

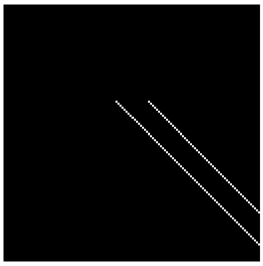
b)



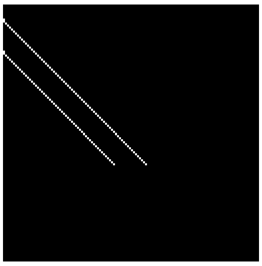
orthographic z0=-1



orthographic z0=-2



orthographic z0=-3



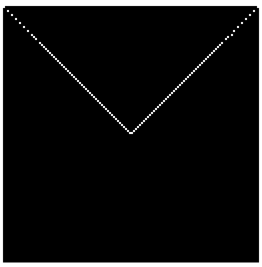
perspective z0=-1

perspective z0=-2

perspective z0=-3

3.

b) perspective

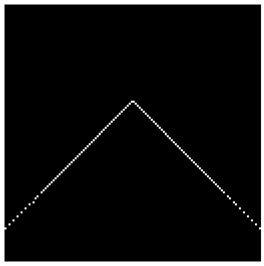


b=0 c=1

b=1 c=1

b=-1 c=1

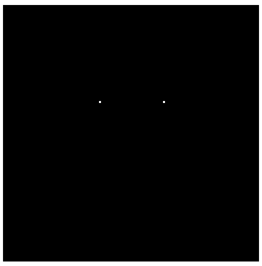
b=0 c=-1



b=1 c=-1

b=-1 c=-1

orthographic



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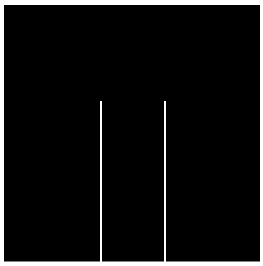
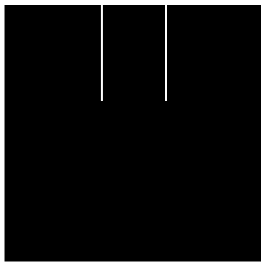
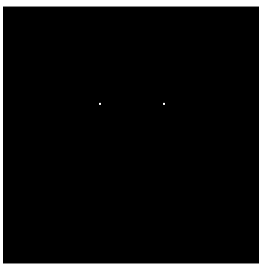


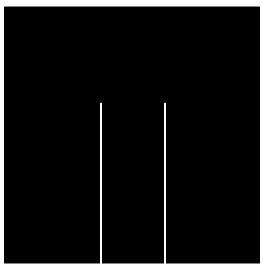
b=1 c=1



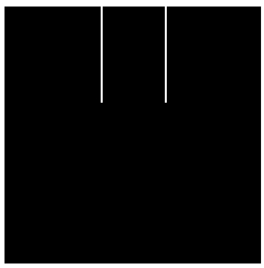
b=-1 c=1



b=0 c=-1



b=1 c=-1



b=-1 c=-1