EECS101: HOMEWORK #1 SOLUTION

1.

a) Perspective projection:

$$x' = \frac{f'x}{z} = \frac{f'(x_0 + ta)}{z_0 + tc} = -\frac{1}{2t}$$

$$y' = \frac{f'y}{z} = \frac{f'(y_0 + tb)}{z_0 + tc} = \frac{1-t}{t}$$

Orthographic projection

$$x' = x$$

$$y' = y$$

b) Yes. The projection of the line is a line for both perspective and orthographic projection.

Let $p_1 : (x_1, y_1, z_1)$, $p_2 : (x_2, y_2, z_2)$ be two points on the line and $p_1' : (x_1', y_1', z_1')$, $p_2' : (x_2', y_2', z_2')$ the projections. The parametric equations are as follows:

$$\begin{cases} x_1 = x_0 + t_1 a \\ y_1 = y_0 + t_1 b \\ z_1 = z_0 + t_1 c \end{cases} \qquad \begin{cases} x_1' = \dfrac{f'(x_0 + t_1 a)}{z_0 + t_1 c} \\ y_1' = \dfrac{f'(y_0 + t_1 b)}{z_0 + t_1 c} \\ z_1' = f' \end{cases}$$

$$\begin{cases} x_2 = x_0 + t_2 a \\ y_2 = y_0 + t_2 b \\ z_2 = z_0 + t_2 c \end{cases} \qquad \begin{cases} x_2' = \dfrac{f'(x_0 + t_2 a)}{z_0 + t_2 c} \\ y_2' = \dfrac{f'(y_0 + t_1 b)}{z_0 + t_2 c} \\ z_2' = f' \end{cases}$$

The slope of the line $p_1' p_2'$ is

$$k = \frac{y_2' - y_1'}{x_2' - x_1'} = \frac{y_0 c - z_0 b}{x_0 c - z_0 a}$$

The intercept on Y axis is

$$b = y' - x'k = \frac{f'(x_0 b - y_0 a)}{x_0 c - z_0 a}$$

k and b are independent of t. Therefore, projection of any point on the line has the same parameter and the projection of the line is still a line.

The same applies to orthographic projection in which k and b become:

$$k = \frac{y_2' - y_1'}{x_2' - x_1'} = \frac{b}{a}$$

$$b = y' - x'k = \frac{f'(y_0 a - x_0 b)}{z_{avg} a}$$

Perspective projection is shown is Figure 1(a) and orthographic projection is shown in Figure 1(b). The upper left corner corresponds to (-4, -4) and the lower right corner corresponds to (4, 4). The horizontal direction is the x axis.

c) For perspective projection, as t goes to ∞, the projection is getting closer to point (0,-1).

$$\lim_{t\to\infty} x' = \lim_{t\to\infty} \frac{f'(\frac{x_0}{t} + a)}{\frac{z_0}{t} + c} = \frac{f'a}{c} = 0$$

$$\lim_{t\to\infty} y' = \lim_{t\to\infty} \frac{f'(\frac{y_0}{t} + b)}{\frac{z_0}{t} + c} = \frac{f'b}{c} = -1$$

In figure 1(a), the brighter the line is, the larger t is. It is consistent with the limit.

(a) Perspective (b) Orthographic

Figure 1

2.

a)

z_0	Line 1		Line 2	
	Perspective	Orthographic	Perspective	Orthographic
-1	$x = -0.5 - t$ $y = 1 - t$	$x = 0.5 + t$ $y = -1 + t$	$x = 0.5 - t$ $y = 1 - t$	$x = -0.5 + t$ $y = -1 + t$
-2	$x = \frac{-0.5 - t}{2}$ $y = \frac{1 - t}{2}$	$x = 0.5 + t$ $y = -1 + t$	$x = \frac{0.5 - t}{2}$ $y = \frac{1 - t}{2}$	$x = 0.5 + t$ $y = -1 + t$
-3	$x = \frac{-0.5 - t}{3}$ $y = \frac{1 - t}{3}$	$x = 0.5 + t$ $y = -1 + t$	$x = \frac{0.5 - t}{3}$ $y = \frac{1 - t}{3}$	$x = 0.5 + t$ $y = -1 + t$

b) Yes. The projections of the lines are parallel for both perspective and orthographic projection with the constant settings. In perspective projection, the slope for line 1 projection is

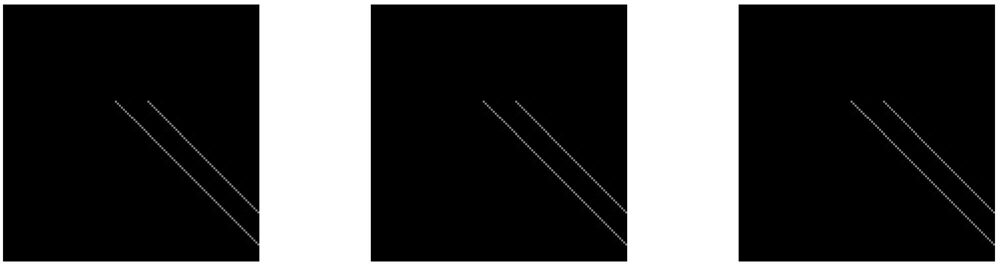
$$k_1 = \frac{y_1 c - z_0 b}{x_1 c - z_0 a} = 1$$

The slope for line 2 projection is

$$k_2 = \frac{y_2 c - z_0 b}{x_2 c - z_0 a} = 1 = k_1$$

In orthographic projection, the slopes for both line projections are $\frac{b}{a}$.They are parallel. They are consistent with the images shown in Figure 2(a)-(g).

(a) Perspective $z_0 = -1$ (b) Perspective $z_1 = -2$ (c) Perspective $z_3 = -3$



(d) Orthographic $z_0 = -1$ (e) Orthographic $z_1 = -2$ (f) Orthographic $z_3 = -3$

Figure 2

c) In this case, orthographic projection is a good approximation since for perspective projection, x and y are equally magnified. When $z_0 = f'$, there is no magnification effect and projection of the two lines are the two lines.

3.

a)

c	b	Line 1		Line 2	
		Perspective	Orthographic	Perspective	Orthographic
1	0	$x = -\frac{1}{t}$ $y = -\frac{1}{t}$	$x = -1$ $y = -1$	$x = \frac{1}{t}$ $y = -\frac{1}{t}$	$x = 1$ $y = -1$

	1	$x=-\frac{1}{t}$ $y=\frac{-1+t}{t}$	$x=-1$ $y=-1+t$	$x=\frac{1}{t}$ $y=\frac{-1+t}{t}$	$x=1$ $y=-1+t$
	-1	$x=-\frac{1}{t}$ $y=\frac{-1-t}{t}$	$x=-1$ $y=-1-t$	$x=\frac{1}{t}$ $y=\frac{-1-t}{t}$	$x=1$ $y=-1-t$
-1	0	$x=\frac{1}{t}$ $y=\frac{1}{t}$	$x=-1$ $y=-1$	$x=-\frac{1}{t}$ $y=\frac{1}{t}$	$x=1$ $y=-1$
	1	$x=\frac{1}{t}$ $y=\frac{1-t}{t}$	$x=-1$ $y=-1+t$	$x=-\frac{1}{t}$ $y=\frac{1-t}{t}$	$x=1$ $y=-1+t$
	-1	$x=\frac{1}{t}$ $y=\frac{1+t}{t}$	$x=-1$ $y=-1-t$	$x=-\frac{1}{t}$ $y=\frac{1+t}{t}$	$x=1$ $y=-1-t$

b) The perspective projections of the lines are not parallel with the constant settings.

The slope for line 1 projection is

$$k_1 = \frac{y_0 c - z_0 b}{x_1 c - z_0 a} = 1$$

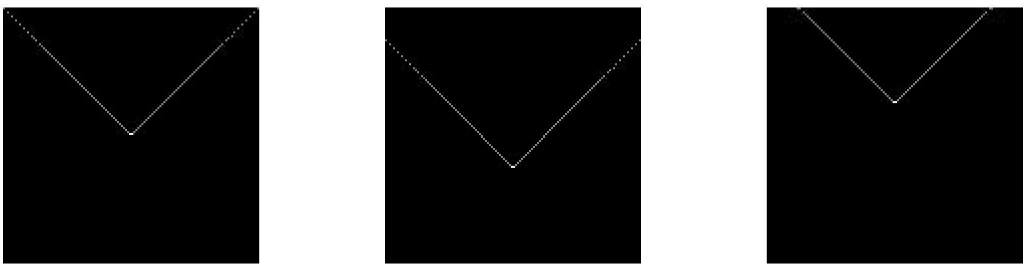
The slope for line 2 projection is

$$k_2 = \frac{y_0 c - z_0 b}{x_2 c - z_0 a} = -1 = -k_1$$

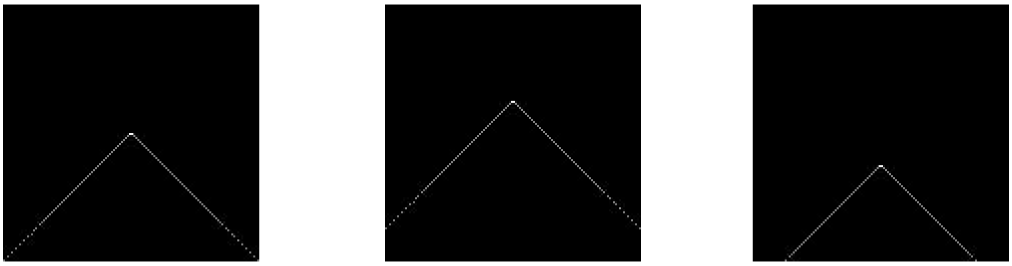
The projections are perpendicular to each other.

In orthographic projection, the slopes for both line projections are $\frac{b}{a}$.They are parallel.

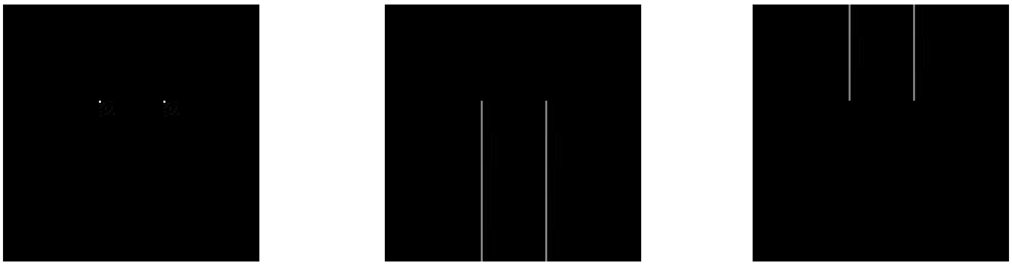
The projections are shown in Figure 3(a)-(l).



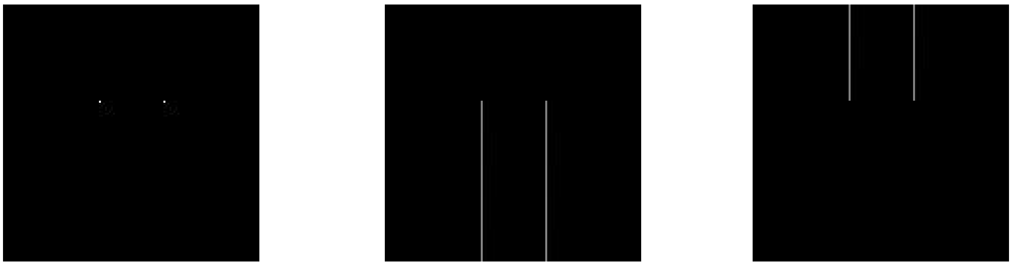
(a) Perspective $b=0, c=1$ (b) Perspective $b=1, c=1$ (c) Perspective $b=-1, c=1$



(d) Perspective $b=0, c=-1$ (e) Perspective $b=1, c=-1$ (f) Perspective $b=-1, c=-1$



(g) Orthographic $b=0, c=1$ (h) Orthographic $b=1, c=1$ (i) Orthographic $b=-1, c=1$



(j) Orthographic $b=0, c=-1$ (k) Orthographic $b=1, c=-1$ (l) Orthographic $b=-1, c=-1$

Figure 3

c) Orthographic projection is not a good approximation to perspective projection.

d) In the perspective projection, as t goes to ∞, the projection gets closer to point $(0, \frac{f'b}{c})$.

$$\lim_{t\to\infty} x' = \lim_{t\to\infty} \frac{f' x_1}{z_0 + tc} = 0$$

$$\lim_{t\to\infty} y' = \lim_{t\to\infty} \frac{f'(y_0 + tb)}{z_0 + tc} = \frac{f'b}{c} = (0,1,-1,0,-1,1)$$

for $(c,b): (1,0), (1,1), (1,-1), (-1,0), (-1,1), (-1,-1)$

In Figure 3(a)-(f), the brighter the line is, the larger t is. They are consistent with the limit.

4. Write a description explaining how hw1-m.c works. Specifically, explain what each statement in the main function does and how it achieves the effect by examining its arguments, return value and functionality.

The purpose of the program is to read in a raw image file and saves the image to an ras file. The main function first declares some variables. Among them, there is an image buffer which is represented as an unsigned char array. It then assigns values to the two filename variables, ifile and ofile. Next, it clears the buffer by calling the clear() function. The function accepts an array name as its argument and puts zero in the array. Next, it opens the input file by calling fopen(). This function has two arguments. The first one is the filename to be opened and the second one is the mode as to how to open it. In this case, it will be opened for reading, i.e., “rb”. The function returns a file handle to be used later. If opening succeeds, a for loop structure is used to traverse each row of the image. In each iteration, fread() function is used to read the whole row. fread() has four arguments. The first one is a buffer to hold the content to be read in. The second one is the unit size. In this case, it is 1 meaning 1 byte. The third argument is how many units to read. The last argument is a file handle which is obtained when it’s opened. If reading succeeds, the program then closes the file. If anything goes wrong, an error message will be given and the program will exit. If everything goes well, the program proceeds to write the buffer to another file. It does this by first opening the output file using write mode “wb”. Before it writes the buffer to the file, it prepares a 32-byte array to hold the header information by calling header(). The function has three arguments. The first two are the size of the image and the last one is the header array. It then writes the header to the file followed by the content stored in the buffer. Finally, it closes the file.