

Advanced Macroeconomics
Assignment 1
2021-2022 SEMESTER 2
Submit your work in class on 18 March 2022

Consider a model with a unit mass of identical, infinitely-lived agents and an initial stock of capital $k_0 > 0$. The technology for production transforms labour n_t and capital k_t into a final good as follows:

$$y_t = Ak_t^\alpha n_t^{1-\alpha}$$

where $A > 0$ is total factor productivity and $\alpha \in (0,1)$ is the income-share parameter of capital. The profit function is defined as $Ak_t^\alpha n_t^{1-\alpha} - r_t k_t - w_t n_t$ where r_t is the rental price of capital and w_t is the wage rate.

The representative agent has one unit of time endowment in each period. The representative agent chooses the sequences of consumption $\{c_t\}_{t=0}^\infty$ and capital $\{a_{t+1}\}_{t=0}^\infty$ to maximise lifetime utility:

$$U = \sum_{t=0}^\infty \beta^t \ln c_t,$$

where $\beta \in (0,1)$, subject to a budget constraint $c_t + a_{t+1} \leq w_t + R_t a_t$ and solvency

$$\lim_{t \rightarrow \infty} \frac{a_{t+1}}{\prod_{s=1}^t R_s} \geq 0,$$

given an initial asset level $a_0 > 0$. Capital depreciation is complete in each period $\delta = 1$ and $R_t = 1 + r_t - \delta$ is the gross return to assets.

Please answer the following questions:

- a) Set up the problem of firms' maximising profits, taking prices as given. Derive and explain the optimal conditions for labour and capital. [30 marks]
- b) Set up the problem of consumers' maximising utility over an infinite horizon, taking prices as given. Derive and explain the optimal conditions. Should the solvency be binding? Please explain why or why not? [30 marks]
- c) Define the competitive equilibrium and derive the equilibrium solutions for output, labour, consumption, investment, and welfare. Is the equilibrium path of k_t convergent to any steady state level without sustainable growth? Is the equilibrium allocation as efficient as the social planner's allocation? Please explain why or why not. [40 marks]