081203M04001H - Algorithm Design and Analysis

Assignment 2

November 5, 2020

Notice:

1. Please submit your answer in hard copy AND submit a digital version to UCAS website http://sep.ucas.ac.cn.
2. Hard copy should be submitted before 9 am. November 20 and digital version should be submitted before 11 pm. November 20.
3. You can choose **three** from problems 1-5, and you should do at least the following things:
   (a) Describe the optimal substructure and DP equation;
   (b) Describe your algorithm in daily language or pseudo-code;
   (c) Prove the correctness of your algorithm;
   (d) Analyse the complexity of your algorithm.
4. You should finish problems 6-7 on Universal Online Judge before 10 am. November 16.

1 Money robbing

A robber is planning to rob houses along a street. Each house has a certain amount of money stashed, the only constraint stopping you from robbing each of them is that adjacent houses have security system connected and it will automatically contact the police if two adjacent houses were broken into on the same night.

1. Given a list of non-negative integers representing the amount of money of each house, determine the maximum amount of money you can rob tonight without alerting the police.
2. What if all houses are arranged in a circle?

2 Largest Divisible Subset

Given a set of distinct positive integers, find the largest subset such that every pair (S_i, S_j) of elements in this subset satisfies: $S_i\%S_j = 0$ or $S_j\%S_i = 0$.

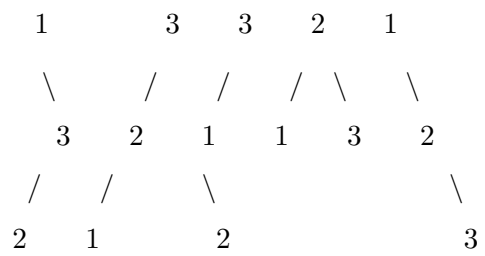
Please return the largest size of the subset.

Note: $S_i\%S_j = 0$ means that S_i is divisible by S_j.

3 Unique Binary Search Trees

Given n, how many structurally unique BST's (binary search trees) that store values 1...n?

Explanation: Given n = 3, there are a total of 5 unique BST's:



4 Coin Change

You are given coins of different denominations and a total amount of money amount. Write a function to compute **the total number of ways** to make up that amount using some of those coins.

Note: You may assume that you have an infinite number of each kind of coin.

5 The smallest difference

You are given a collection of stones, each stone has a positive integer weight.You need to divide these stones into two piles whose weights are as balanced as possible. Please give the smallest difference between the weights of the two piles.

6 Climbing broken stairs

You are climbing a stair case. It takes x steps to reach to the top. However, not every step you can step on because some steps may be broken. Given a list of steps' positions in sorted ascending order which contain all steps you can step on (named "unbroken step"), determine if you are able to reach to the top by stepping on the last unbroken step. Initially, you are on the first step and assume the first climb must be 1 step. If your last climb was k steps, then your next climb must be either k - 1, k, or k + 1 step. Note that you can only climb in the forward direction.

Note: The number of unbroken steps is ≥ 2 and is $\leq$ 1,000. Each unbroken steps position will be a non-negative integer $\leq 2^{31}$. The first unbroken step's position is always 0.

7 Maximum Subarray Sum with One Deletion

Given an array of integers, return the maximum sum for a non-empty subarray (contiguous elements) with at most one element deletion. In other words, you want to choose a subarray and optionally delete one element from it so that there is still at least one element left and the sum of the remaining elements is maximum possible.