# 第四章作业

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1. 设有如下三类模式样本集ω1，ω2和ω3，其先验概率相等，求Sw和Sb

ω1：{(1 0)T, (2 0)T, (1 1)T}

ω2：{(-1 0)T, (0 1)T, (-1 1)T}

ω3：{(-1 -1)T, (0 -1)T, (0 -2)T}

解：$S_w = \sum_{i=1}^{3} P(\omega_i) E\{(x-m_i)(x-m_i)^T | \omega_i\}$

$P(\omega_1) = P(\omega_2) = P(\omega_3) = \frac{1}{3}$

$m_1 = (\frac{4}{3}\ \frac{1}{3})^T \quad m_2 = (-\frac{2}{3}\ \frac{2}{3})^T \quad m_3 = (-\frac{1}{3}\ -\frac{4}{3})^T \quad m_0 = (\frac{1}{9}\ -\frac{1}{9})^T$

$$S_w = \sum_{i=1}^{3} P(\omega_i) E\{(x-m_i)(x-m_i)^T | \omega_i\} = \frac{1}{3}\times(\frac{2}{9}\ \frac{2}{9})^T + \frac{1}{3}\times(\frac{2}{3}\ \frac{2}{9})^T + \frac{1}{3}\times(\frac{2}{9}\ \frac{2}{9})^T = (\frac{8}{27}\ \frac{2}{9})^T$$

$$= \frac{1}{3}\times\begin{pmatrix} \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} \end{pmatrix} + \frac{1}{3}\times\begin{pmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix} + \frac{1}{6}\times\begin{pmatrix} \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{6} & \frac{2}{3} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{2}{3} & -\frac{1}{9} \\ -\frac{1}{9} & \frac{2}{3} \end{pmatrix}$$

$$S_b = \sum_{i=1}^{3} P(\omega_i)(m_i - m_0)(m_i - m_0)^T$$

$$= \frac{1}{3}\times\begin{pmatrix} \frac{121}{81} & \frac{44}{81} \\ \frac{44}{81} & \frac{16}{81} \end{pmatrix} + \frac{1}{3}\times\begin{pmatrix} \frac{49}{81} & -\frac{49}{81} \\ -\frac{49}{81} & \frac{49}{81} \end{pmatrix} + \frac{1}{3}\times\begin{pmatrix} \frac{16}{81} & \frac{44}{9} \\ \frac{44}{9} & \frac{121}{9} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{62}{81} & \frac{13}{81} \\ \frac{13}{81} & \frac{62}{81} \end{pmatrix}$$

2. 设有如下两类样本集，其出现的概率相等：

ω1：{(0 0 0)T, (1 0 0)T, (1 0 1)T , (1 1 0)T}　　ω2：{(0 0 1)T, (0 1 0)T, (0 1 1)T , (1 1 1)T}

用K-L变换，分别把特征空间维数降到二维和一 维，并画出样本在该空间中的位置。

解：$P(\omega_1) = P(\omega_2) = \frac{1}{2} \qquad m = \frac{1}{2}\times\frac{1}{4}\sum_{x_i\in\omega_1} x_i + \frac{1}{2}\times\frac{1}{4}\sum_{x_i\in\omega_2} x_i = (\frac{1}{2}\ \frac{1}{2}\ \frac{1}{2})^T \neq \vec{0}$

为使 $E[X]=0$，将 m 作为新的原点，平移样本，新的样本为：

$\omega_1: \{(-\frac{1}{2}\ -\frac{1}{2}\ -\frac{1}{2})^T, (\frac{1}{2}\ -\frac{1}{2}\ -\frac{1}{2})^T, (\frac{1}{2}\ -\frac{1}{2}\ \frac{1}{2})^T, (\frac{1}{2}\ \frac{1}{2}\ -\frac{1}{2})^T\}$

$\omega_2: \{(-\frac{1}{2}\ -\frac{1}{2}\ \frac{1}{2})^T, (-\frac{1}{2}\ \frac{1}{2}\ -\frac{1}{2})^T, (-\frac{1}{2}\ \frac{1}{2}\ \frac{1}{2})^T, (\frac{1}{2}\ \frac{1}{2}\ \frac{1}{2})^T\}$

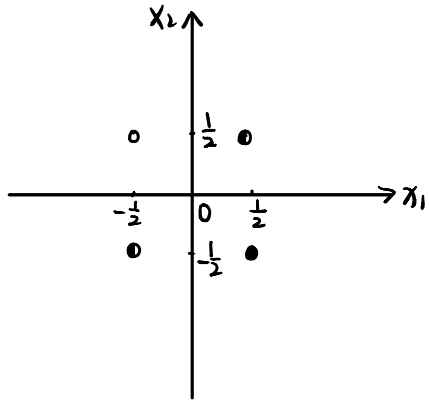
自相关矩阵 $R=\sum_{i=1}^{2}P(\omega_i)E\{xx^T\}=\frac{1}{2}\left[\frac{1}{4}\sum_{x_j\in\omega_1}x_j(x_j)^T\right]+\frac{1}{2}\left[\frac{1}{4}\sum_{x_j\in\omega_2}x_j(x_j)^T\right]=\begin{pmatrix}\frac{1}{4}&0&0\\0&\frac{1}{4}&0\\0&0&\frac{1}{4}\end{pmatrix}$

解特征值方程 $|R-\lambda I|=0 \Rightarrow \lambda_1=\lambda_2=\lambda_3=\frac{1}{4}$

对应的特征向量由 $R\varphi_i=\lambda_i\varphi_i$ 求得：$\varphi_1=\begin{pmatrix}1\\0\\0\end{pmatrix}$ $\varphi_2=\begin{pmatrix}0\\1\\0\end{pmatrix}$ $\varphi_3=\begin{pmatrix}0\\0\\1\end{pmatrix}$

把特征空间降到二维，选 $\lambda_1$、$\lambda_2$ 对应的变换向量为变换矩阵，由 $y=\Phi^Tx$ 变换后的二维模式特征为：$\omega_1:\left\{\begin{pmatrix}-\frac{1}{2}\\-\frac{1}{2}\end{pmatrix},\begin{pmatrix}\frac{1}{2}\\-\frac{1}{2}\end{pmatrix},\begin{pmatrix}\frac{1}{2}\\-\frac{1}{2}\end{pmatrix},\begin{pmatrix}\frac{1}{2}\\\frac{1}{2}\end{pmatrix}\right\}$ $\omega_2:\left\{\begin{pmatrix}-\frac{1}{2}\\-\frac{1}{2}\end{pmatrix},\begin{pmatrix}-\frac{1}{2}\\\frac{1}{2}\end{pmatrix},\begin{pmatrix}-\frac{1}{2}\\\frac{1}{2}\end{pmatrix},\begin{pmatrix}\frac{1}{2}\\\frac{1}{2}\end{pmatrix}\right\}$

在样本空间中的位置如下（其中"●"表示 $\omega_1$ 的特征，"○"表示 $\omega_2$ 的特征，"◐"表示特征既属于 $\omega_1$，又属于 $\omega_2$）：




把特征空间降到一维，选 $\lambda_1$ 对应的变换向量为变换矩阵，由 $y=\Phi^Tx$ 变换后的一维模式特征为：$\omega_1:\{-\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2}\}$ $\omega_2:\{-\frac{1}{2},-\frac{1}{2},-\frac{1}{2},\frac{1}{2}\}$.

在样本空间中的位置如下（其中"●"表示 $\omega_1$ 的特征，"○"表示 $\omega_2$ 的特征，"◐"表示特征既属于 $\omega_1$，又属于 $\omega_2$）：