081203M04001H - Algorithm Design and Analysis

Assignment 4

December 3, 2020

Notice:

1. Please submit your answer in hard copy **AND** submit a digital version to UCAS website `http://sep.ucas.ac.cn`.
2. Hard copy should be submitted before 9 am. December 18 and digital version should be submitted before 11 pm. December 18.
3. You should finish **all the five problems**.
4. **Integer Linear Programming (ILP)** is different from the classic Linear Programming that some extra constraints such as

$$x_i \text{ is an integer, } \forall i = 1, 2, \cdots, n \tag{1}$$

   or

$$x_i \in 0, 1 \ \forall i = 1, 2, \cdots, n \tag{2}$$

   are added.
5. When you give the formulation of an LP or ILP, you should explain all mathematical symbols you are using if not appearing in the problem, and interpret the constraints if necessary.

1 Traveling Trump Problem

Suppose you are the U.S. presidential candidate Donald Trump, who want to hold election rallies in four swing states: Georgia(1), Pennsylvania(2), Michigan(3) and Florida(4). It is the last day before election and because of shortage of funds, you need to save money and try to travel through the shortest path, visit each state exactly once and return to the starting point. Distance between every two states can be writen as $c_{ij}, i \in [0, 3], j \in [i, 4], i, j$ is integer. Washington DC(0) is your starting point. Please formulate this problem as an ILP. (*Hint:* You can think about this problem in terms of the constraint that **visiting each state exactly once**.)

2 Profit Maximization

Your factory produces three kinds of product: A, B and C. All of them need two kinds of raw materials: nickel and aluminum. The profit and cost of each kind of product are shown in the following table.

Product	Profit($)	Nickel(kg)	Aluminum(kg)
A	10	3	4
B	5	3	2
C	15	1	8

You only have 100 kg of nickle and 200 kg of aluminum in stock. How to arrange production to maximize profits? Please formulate this problem as a LP and transform it into dual form. Then you may solve both primal and dual problems using GLPK or Gurobi or other similar tools.

3 Cutting Paper Minimization

Your factory has expanded its bussiness. Suppose you have an unlimited number of large rolls of paper, of width W meters per roll (W is a positive integer). However, different m customers demands are for smaller width of paper; in particular, customer i needs b_i rolls of paper of width w_i, $i = 1, 2, ..., m$. We assume that $w_i \leq W$ for each i, and each w_i is an integer. Smaller rolls are obtained by slicing a large roll in a certain way. You can slice one roll of paper for different customers only if their total width does not exceed W.

The goal of you is to minimize the number of large rolls used while satisfying customer demand. **Please formulate this problem as an ILP.** Assume that there is no cost for slicing.

4 Reformulation Problems with Absolute Values

Consider the problem:

$$\begin{aligned} \text{minimize} \quad & 2\,|x_1| + x_2 \\ \text{subject to} \quad & x_1 + x_2 \geq 4 \end{aligned} \tag{3}$$

Please reformulate this problem as a LP without absolute values.

5 Lawyer Recruitment for Trump

The U.S. presidential election is over, but Donald Trump refuses to accept defeat. Suppose you are Trump's election campaign manager and you are asked to recruit a group of lawyers for him to initiate litigation against the results. It is estimated that litigation will be initiated in N states, and the $i(th)$ state needs at least L_i lawyers. The number of law firms is F. Lawyers from the $j(th)$ law firm can offer legal services in several states S_j and the recruitment fee for one lawyer from the $j(th)$ law firm is C_j. Note that S_j is a subset of $N = \{1, 2, \cdots, n\}$ and the union of S_j equals to N.

Your boss wants you to save money so your need to formulate this problem as an ILP and your goal is minimizing the recruitment fee of enough lawyers.