081203M04001H - Algorithm Design and Analysis

Assignment 3

November 21, 2020

Notice:

1. Please submit your answer in hard copy AND submit a digital version to UCAS website http://sep.ucas.ac.cn.
2. Hard copy should be submitted before 9.am. December 4 and digital version should be submitted before 11.pm. December 4.
3. Please choose **three** from problems 1-6, and you should do at least the following things:
   (a) Describe the basic idea of your algorithm in natural language AND pseudo-code;
   (b) Describe the greedy-choice property and optimal substructure;
   (c) Prove the correctness of your algorithm;
   (d) Analyse the complexity of your algorithm.
4. You should finish problems 7-8 on Universal Online Judge before 10 am. December 7.

1 Monkeys and Bananas

There are N Monkeys and N bananas are placed in a straight line.

Each monkey want to have a banana, if two monkeys want to own the same banana, there will be a fight! A monkey can stay at his position, move one step right from x to $x+1$, or move one step left from x to $x-1$. Any of these moves consumes 1 second. Assign monkeys to banana so that no monkey fight each other and the time when the last monkey gets a banana is minimized.

2 Job Schedule

There are n distinct jobs, labeled $J_1,\ J_2,\ \ldots, J_n$, which can be performed completely independently of one another. Each jop consists of two stages: first it needs to be preprocessed on the supercomputer, and then it needs to be finished on one of the PCs.

Let's say that job J_i needs p_i seconds of time on the supercomputer, followed by f_i seconds of time on a PC.

Since there are at least n PCs available on the premises, the finishing of the jobs can be performed on PCs at the same time. However, the supercomputer can only work on a single job a time without any interruption. For every job, as soon as the preprocessing is done on the supercomputer, it can be handed off to a PC for finishing.

Let's say that a schedule is an ordering of the jobs for the supercomputer, and the completion time of the schedule is the earlist time at which all jobs have finished processing on the PCs. Give a polynomial-time algorithm that finds a schedule with as small a completion time as possible.

3 Cross the River

Some people want to cross a river by boat. Each person has a weight, and each boat can carry an equal maximum weight limit. Each boat carries at most 2 people at the same time, provided the sum of the weight of those people is at most boat's weight limit. Return the minimum number of boats to carry every given person.

Note that it is guaranteed each person can be carried by a boat.

4 Permutation Partition

Bob is given a permutation $p_1,\ p_2, \ldots,\ p_n$ of integers from 1 to n and an integer k, such that $1 \leq k \leq n$. A permutation means that every number from 1 to n is contained exactly once.

Consider all partitions of this permutation into k disjoint segments. Formally, a partition is a set of segments $\{[s_0,\ s_1],\ [s_1+1,\ s_2], \ldots,\ [s_{k-1}+1,\ s_k]\}$, such that: $1 = s_0 < s_1 < s_2 < \ldots < s_{k-1} < s_k = n$. Two partitions are different if there exists a segment that lies in one partition but not the other.

Bob wants to calculate the partition value, defined as $\sum_{i=1}^{k} max_{s_{i-1} \leq j \leq s_i} p_j$ for all possible partitions of the permutation into k disjoint segments. Please help him find the maximum possible partition value over all such partitions, and the number of ways to make partition with this value.

5 Toy Buildings

Bob has n toy buildings in a line, the i-th from left of which has weight a_i. He wants to make these building nondecreasing in height from left to right. In one operation, he can take any contiguous subsegment of them and add 1 to each of their heights.

Help Bob find the minimum number of operations he needs to perform to make his toy buildings nondecreasing.

6 Cows cross the river

Bob is going to drive N cows across the river. It takes $t_1,\ t_2, \ldots,\ t_n$ for each cow to get cross. Only two cows can pass through the river at the same time and Bob needs to ride on one of them. Please give the shortest time for Bob to drive all the cows across the river.

7 Programming Task 1: String Bomb

Description:

Bob is playing a game. Given a string that consists of letters 'A' and 'B', he can use bombs to bomb a substring which is either "AB" or "BB". When he bombs such a substring, it is deleted from the string and the remaining parts of the string get concatenated.

For example, Bob can use two such operations: AABABBA $\rightarrow$ AABBA $\rightarrow$ AAA.

Can you help him find the length of the shortest string that he can make?

Input:

The only line of input contains a non-empty string s. It is guaranteed that all symbols of s are either 'A' or 'B' and the length of s does not exceed 200000.

Output:

For each test case, print a single integer: the length of the shortest string that Bob can make.

Sample Input:

AABBA

Sample Output:

1

8 Programming Taks 2: Triangle

Description:

Bob has n line segments, the i-th of them has length a_i. Given the lengths of the line segments, check if he can choose exactly 3 of them to form a non-degenerate triangle.

Bob should use exactly 3 line segments, he can't concatenate two line segments or change any length. A non-degenerate triangle is a triangle with positive area.

Input:

The first line contains single integer: the number of line segments n ($1 \leq$ n ≤ 100000).

The second line contains n integers: the lengths of line segments $a_1,\ a_2, \ldots,\ a_n$ ($1 \leq a_i \leq 10^{18}$).

Output:

In the only line print "YES" if he can choose exactly three line segments and form a non-degenerate triangle with them, and "NO" otherwise.

Sample Input:

4

1 2 4 6

Sample Output:

NO