Homework 3

Aaron W. Tarajos
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7.8 Question 31

Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

$$\int_{-2}^{3} \frac{1}{x^4} dx$$

Solution

Let's consider the region to the right of zero. Given the definition of improper integrals we can say that an integral is convergent if

$$\int_0^a \frac{1}{x^4}\, dx = \lim_{t\to 0} \int_t^a \frac{1}{x^4}\, dx$$

where $\lim_{t\to 0}$ is a finite number. So we have

$$\begin{aligned}
\lim_{t\to 0} \int_t^a \frac{1}{x^4}\, dx &= \left[-\frac{1}{3x^3}\right]_t^a \\
&= \left[\lim_{t\to 0} -\frac{1}{3t^3} + \frac{1}{3a^3}\right] \\
&= -\infty + \frac{1}{3a^3}
\end{aligned}$$

because we have an infiinity,

$$\int_0^a \frac{1}{x^4}\, dx$$

is divergent. Then, because $0 \in [-2, 3]$ we can say that

$$\int_{-2}^{3} \frac{1}{x^4} dx$$

is divergent.