Homework 3

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7.8 Question 31

Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

$$\int_{-2}^{3} \frac{1}{x^4} dx$$

Solution

Let's consider the region to the right of zero. Given the definition of improper integrals we can say that an integral is convergent if

$$\int_{0^+}^{a} \frac{1}{x^4}\, dx = \lim_{t\to 0} \int_{t}^{a} \frac{1}{x^4}\, dx$$

where $\lim_{t\to 0^+}$ is a finite number. So we have

$$\begin{aligned}
\lim_{t\to 0^+} \int_{t}^{a} \frac{1}{x^4}\, dx &= \lim_{t\to 0^+} \left[-\frac{1}{3x^3}\right]_t^a \\
&= \lim_{t\to 0^+} -\frac{1}{3t^3} + \frac{1}{3a^3} \\
&= -\infty + \frac{1}{3a^3}
\end{aligned}$$

because we have an infiinity, it is divergent. Similarly for $\lim_{t\to 0^-}$, we have

$$\begin{aligned}
\lim_{t\to 0^-} \int_{t}^{a} \frac{1}{x^4}\, dx &= \lim t \to 0^- \left[-\frac{1}{3x^3}\right]_t^a \\
&= \lim_{t\to 0^-} -\frac{1}{3t^3} + \frac{1}{3a^3} \\
&= \infty + \frac{1}{3a^3}
\end{aligned}$$

because the integral is divergent near zero from both sides

$$\int_{-2}^{3} \frac{1}{x^4} dx$$

is divergent.