

## Homework 3

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September 19, 2024

### 7.1 Question 15

Evaluate the integral.

$$\int (\ln x)^2 dx$$

#### Solution

We can start by finding

$$\int \ln x dx$$

using integration by parts;

$$\begin{aligned} \int \ln x dx &= \int (x)' \ln x dx \\ &= x \ln x - \int x (\ln x)' dx \\ &= x \ln x - \int x \cdot \frac{1}{x} dx \\ &= x \ln x - \int 1 dx \\ &= x \ln x - x + C \end{aligned}$$

Knowing this antiderivative, we can use integration by parts on  $(\ln x)^2$ ;

$$\begin{aligned} \int (\ln x)(\ln x) dx &= \int (x \ln x - x)' (\ln x) dx \\ &= (x \ln x - x) \ln x - \int (x \ln x - x) (\ln x)' dx \\ &= (x \ln x - x) \ln x - \int (x \ln x - x) \frac{1}{x} dx \\ &= (x \ln x - x) \ln x - \int x (\ln x - 1) \frac{1}{x} dx \\ &= (x \ln x - x) \ln x - \int (\ln x - 1) dx \\ &= (x \ln x - x) \ln x - \left( \int \ln x dx - \int 1 dx \right) \\ &= (x \ln x - x) \ln x - [(x \ln x - x) - x] + C \\ &= x \ln^2 x - x \ln x - x \ln x + x + x + C \\ &= \boxed{x \ln^2 x - 2x \ln x + 2x + C} \end{aligned}$$