Homework 9

Aaron W. Tarajos
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11.1 Question 17

Find a formula for the general term a_n, of the sequence, assuming that the pattern of the first few terms continues.

$$\left\{\frac{1}{2}, -\frac{4}{3}, \frac{9}{4}, -\frac{16}{5}, \frac{25}{6}, \cdots\right\}$$

Solution

For all integers $n \geq 1$, the general term a_n of the sequence $\{a_n\}$ defined by

$$\left\{\frac{1}{2}, -\frac{4}{3}, \frac{9}{4}, -\frac{16}{5}, \frac{25}{6}, \cdots\right\}$$

is given by

$$a_n = \frac{(-1)^{n+1} n^2}{n+1}. \tag{1}$$

Proof

Let the denomenator be $b_n \coloneqq n + 1$, then we have;

$$\{b_n\} = \{(1+1), (2+1), (3+1), \dots\} = \{2, 3, 4, 5, 6, \dots\}$$

then let the numerator be $c_n \coloneqq n^2$, then;

$$\{c_n\} = \{(1^2), (2^2), (3^2), \dots\} = \{1, 4, 9, 16, 25, \dots\}$$

Lastly the sign of the function is given by $s_n \coloneqq (-1)^{n+1}$;

$$\{s_n\} = \{(-1)^{1+1}, (-1)^{2+1}, (-1)^{3+1}\} = \{1, -1, 1, -1, 1, \dots\}$$

combining these sequences, the n^{th} term is given by;

$$a_n = s_n \frac{c_n}{b_n} = \frac{(-1)^{n+1} n^2}{n+1}$$