Homework 2

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6.3 Question 41

The region bounded by the given curves is rotated about the specified axis. Find the volume of the resulting solid by any method.

$$x^2 + (y-1)^2 = 1; \quad \text{about the } y\text{-axis} \tag{1}$$

Solution

The given curve is a unit circle shifted upwards by one unit and therefore the volume of the shape rotated about the y-axis is the volume of a sphere with a radius of length 1. Let v equal the volume of a sphere as a function of the radius such that;

$$v(r) = \frac{4}{3}\pi r^3 \tag{2}$$

where r is the radius from the center of the sphere to the surface. Then the volume of sphere created by the revolution of equation (1) about the y-axis is;

$$v(1) = \frac{4}{3}\pi(1)^3 = \boxed{\frac{4\pi}{3}}$$

Proof

The equation of a circle is $(x-a)^2 + (y-b)^2 = r^2$. Given $a = 0$, $b = 1$, and $r = 1$ we obtain equation (1);

$$\begin{aligned}(x-0)^2 + (y-1)^2 &= 1^2 \\ x^2 + (y-1)^2 &= 1\end{aligned}$$

Revolve the curve described by the equation of the circle $y = \sqrt{r^2 - x^2}$(for $x \in [-r, r]$) around the y-axis. By disc integration, the equation for the volume of revolution about the y-axis is:

$$V = \pi \int_a^b \left[f(y)^2 - g(y)^2 \right] dy\ . \tag{3}$$

Therefore

$$\begin{aligned}
V &= \pi \int_{-r}^{r} \left[\sqrt{r^2 - x^2} - (0)^2 \right] dy \\
V &= \pi \left(\left[r^2(r) - \frac{r^3}{3} \right] - \left[r^2(-r) - \frac{(-r)^3}{3} \right] \right) \\
V &= \pi \left(2 \left(r^3 - \frac{r^3}{3} \right) \right) \\
V &= \pi \left(2 \cdot \frac{2r^3}{3} \right) \\
&= \frac{4}{3}\pi r^3 \quad \blacksquare
\end{aligned}$$