# Homework 6

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## 8.2 Question 31

Find the exact area of the surface obtained by rotating the curve about the x-axis.

$$x = \frac{1}{3}\left(y^2 + 2\right)^{3/2} \quad 1 \leq y \leq 2$$

### Solution

The surface area of a curve is given by

$$S = \int 2\pi y \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy \tag{1}$$

Where $\frac{dx}{dy}$ is;

$$\begin{aligned}\frac{dx}{dy} &= \frac{d}{dy}\left[\frac{1}{3}\left(y^2 + 2\right)^{3/2}\right]\\ &= \frac{1}{3} \cdot \frac{3}{2} \cdot 2y\left(y^2 + 2\right)^{1/2}\\ &= y\left(y^2 + 2\right)^{1/2}\end{aligned}$$

Then

$$\begin{aligned}S &= \int 2\pi\, y \sqrt{1 + \left(y\,(y^2 + 2)^{1/2}\right)^2}\, dy\\ &= \int 2\pi\, y \sqrt{1 + y^2\,(y^2 + 2)}\, dy\\ &= \int 2\pi\, y \sqrt{y^4 + 2y^2 + 1}\, dy\\ &= \int 2\pi\, y \sqrt{(y^2 + 1)^2}\, dy\\ &= \int 2\pi\, y\left(y^2 + 1\right)\, dy\\ &= 2\pi \int \left(y^3 + y\right)\, dy\end{aligned}$$

evaluating the integral for $1 \leq y \leq 2$ we find the surface area obtained by rotating the curve about the $x$-axis

$$\begin{aligned}S &= 2\pi \int_1^2 \left(y^3 + y\right)\, dy\\ &= 2\pi\left[\frac{y^4}{4} + \frac{y^2}{2}\right]_1^2\\ &= \boxed{2\pi\left[4 + 2 - \frac{1}{4} - \frac{1}{2}\right]}\end{aligned}$$