Homework 13

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10.2 Question 5

Find an equation of the tangent to the curve at the point corresponding to the given value of the parameter.

$$x = t \cos t, \quad y = t \sin t; \quad t = \pi$$

Solution

We start by finding the coordinate point;

$$x = \pi \cos \pi = -\pi$$

and

$$y = \pi \sin \pi = 0$$

so we have the point $(-\pi, 0)$. Now we find $\frac{dx}{dt}$ and $\frac{dy}{dt}$ to solve for $\frac{dy}{dx}$;

$$\frac{dx}{dt} = \frac{d}{dt}[t \cos t] = \cos t - t \sin t$$

and

$$\frac{dy}{dt} = \frac{d}{dt}[t \sin t] = \sin t + t \cos$$

then $\frac{dy}{dx}$ is given by;

$$\begin{aligned}
\frac{dy}{dx} &= \frac{dy/dt}{dx/dt} \\
&= \frac{\sin t + t \cos}{\cos t - t \sin t} \\
\frac{dy}{dx}\bigg|_{t=\pi} &= \frac{\sin \pi + \pi \cos \pi}{\cos \pi - \pi \sin \pi} \\
&= \frac{0 + -\pi}{-1 + 0} \\
&= \pi
\end{aligned}$$

Now that we have our slope, and our point, our tangent line is given by;

$$\begin{aligned}
y - y_1 &= m(x - x_1) \\
y - 0 &= \pi(x - (-\pi)) \\
y &= \pi x + \pi^2
\end{aligned}$$

Note: I wish I had more time to mess around with this problem, this is Archimedes Spiral and there are a lot of interesting maths here.