Homework 8

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8.2 Question 31

Find the solution of the differential equation that satisfies the given initial condition.

$$\frac{du}{dt} = \frac{2t + sec^2 t}{2u}\ , \quad u(0) = -5$$

Solution

We start by moving $2u$ to the other side of the equation and integration both side with respect to t;

$$\begin{aligned}
\frac{du}{dt} &= \frac{2t + sec^2 t}{2u} \\
2u\frac{du}{dt} &= 2t + \sec^2 t \\
\int 2u\frac{du}{dt}\ dt &= \int 2t + \sec^2 t\ dt \\
\int 2u\frac{du}{dt}\ dt &= \int 2t\ dt + \int \sec^2 t\ dt
\end{aligned}$$

Then by the rule of substitution we obtain an implicit solution for the differential equation;

$$\begin{aligned}
\int 2u\ du &= \int 2t\ dt + \int \sec^2 t\ dt \\
u^2 &= t^2 + \tan t + C
\end{aligned}$$

We find the unique solution by solving for C using the given constraint;

$$\begin{aligned}
u^2 &= t^2 + \tan t + c \\
(-5)^2 &= (0^2) + \tan(0) + C \\
C &= 25
\end{aligned}$$

Then

$$\boxed{u = \sqrt{t^2 + \tan t + 25}}$$