Exam 2 Notes

polar → component: $a_x = a\cos\theta$ and $a_y = a\sin\theta$
component → polar: $a = \sqrt{a_x^2 + a_y^2}$ and $\tan\theta = \frac{a_y}{a_x}$

Friction

Static friction is equal to the opposing force, up to a maximum given by;

$$F_{s,\max} = \mu_s F_n$$

after the force exceeds the maximum force of static friction the object slips where kinetic friction acts on the object such that

$$F_k = \mu_k F_n$$

Uniform circular motion

$$a = \frac{v^2}{r}$$

$$F_c = \frac{mv^2}{r}$$

Work and energy

Kinetic energy;

$$K = \frac{1}{2}mv^2$$

Work;

$$W = Fd\cos\phi \quad \text{Or} \quad W = \Delta K$$

work is defined as the dot product of Force and Distance. Work done by gravity;

$$W_g = mgd\cos\phi$$

Work done by a spring;

$$W_s = -\frac{1}{2}kx^2$$

Hooke's law;

$$F_s = -kx$$

Potential energy;

$$\Delta U = -W$$

gravitational potential energy;

$$U = mgy$$

spring potential energy;

$$U = \frac{1}{2}kx^2$$

For non-linear forces;

$$W = \int_{x_1}^{x_2} F(x)\,dx$$

Power

$$P = \frac{dW}{dt} \quad \text{or} \quad P = \vec{F}\cdot\vec{v} = Fv\cos\phi$$

Mechanical Energy

$$E_{\text{mech}} = K + U$$

Unless energy enters the system energy is conserved;

$$\Delta E_{\text{mech}} = \Delta K + \Delta U = 0$$

kinetic energy can become potential energy or the other way around. Kinetic energy is never negative.
Therma energy is

$$E_{\text{th}} = F_k \cdot d$$

Work from external forces

Work can be done by external forces influencing the system;

$$W = \Delta K + \Delta U \quad \text{Or} \quad W = \Delta E$$

Center of Mass

For many particle system;

$$x_{CM} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i$$

where M is the total mass of all the particles. (Average location x weighted by mass m, i.e. expected value). For solid bodies;

$$x_c m = \frac{1}{V}\int xw(x)\,dx$$

where w is width as a function of x and V is volume. Just add an integral for each dimension.

Momentum

Linear momentum;

$$p = m\vec{v}$$

for a system of object total momentum is just the sum of p_i.
Impulse;

$$\vec{J} = \Delta\vec{p}$$

$$\frac{d\vec{p}}{dt} = \vec{F}$$

Momentum is conserved (one of the fundamental conservation laws.

Collisions

Perfectly elastic $\implies$ kinetic energy is conserved
Inelastic $\implies$ kinetic energy is not conserved
Perfectly inelastic $\implies$ kinetic energy is not conserved
Conservation of momentum;

$$m_1 v_{1,i} + m_2 v_{2,i} = m_1 v_{1,f} + m_2 v_{2,f}$$

One-dimensional inelastic collisions;

$$V = \frac{m_1}{m_1 + m_2} v_{1,i}$$

where V is the final velocity of the total system.
One-dimensional elastic collisions;

$$\frac{1}{2} m_1 v_{1,i} = \frac{1}{2} m_1 v_{1,f} + \frac{1}{2} m_2 v_{2,f}$$

Then we can solve for various quantities using;

$$v_{1,f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1,i}$$

$$v_{2,f} = \frac{2m_1}{m_1 + m_2} v_{1,i}$$

Angular kinematics

It works the same as linear motion but with different variables. Position is defined by θ in radians;

$$\theta = \frac{S}{r}$$

angular velocity is

$$\omega = \frac{d\theta}{dt}$$

angular acceleration is

$$\alpha = \frac{d^2\theta^2}{dt^2}$$

kinematic equations

Variable	Equation
Velocity	$v = at + v_0$
Position	$\Delta x = v_0 t + \frac{1}{2}at^2$
Missing t	$v^2 = 2a\Delta x + v_0^2$
Missing a	$\Delta x = \frac{v+v_0}{2} t$
Missing v_0	$\Delta x = vt - \frac{1}{2}at^2$

other useful stuff;

$$v_t = \omega r$$
$$a_t = \alpha r$$

for tangential acceleration and velocity.
Centripetal acceleration;

$$a_c = \omega^2 r$$

Period of revolution;

$$T = \frac{2\pi r}{v} = \frac{2\pi}{\omega}$$

Variable	Equation
Horizontal displacement	$\Delta x = v_0 \cos\theta t$
Vertical displacement	$\Delta y = v_0 \sin\theta t - \frac{1}{2}gt^2$
Vertical velocity	$v_y = v_0 \sin\theta - gt$
Trajectory	$\Delta y = \tan\theta \Delta x - \frac{g\Delta x^2}{2(v_0\cos\theta)^2}$
Range	$R = \frac{v_0^2 \sin 2\theta}{g}$
Rolling ball incline	$a = -\frac{g\sin\theta}{1+I_{com}/MR^2}$

Oscillations

$$x = x_m \cos(\omega t + \phi)$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$T = 2\pi\sqrt{\frac{m}{k}} \quad \text{and} \quad T = \frac{1}{f}$$

Harmonic motion

$$\tau = -\kappa\theta \implies I\alpha = -\kappa\theta$$

for a simple pendulum

$$\omega = \sqrt{\frac{g}{L}}$$

physical pendulum

$$\omega = \sqrt{\frac{mgL}{I}}$$

and

$$I = I_{\text{com}} + mR^2$$

dampening force

$$F_d = -bv \quad \text{and} \quad x(t) = e^{\frac{-bt}{2m}} \cos(\omega')t + \phi$$

where

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

Useful units

Work: J $\rightarrow$ kg m^2/s^2
Impulse: N·s $\rightarrow$ kg m/s
Force: N $\rightarrow$ kg m/s^2

Check your algebra you fucking idiot

Potential tricks

- Gravitational work require change in height i.e. satelite orbiting earth has no gravitational work.
- The location of a pendulum matters for tension because of gravity. $\frac{mv^2}{r} = F_t + mg\cos\phi$
- Double check angles for work, may need to add or subtract 180$\circ$ depending on orientation
- When in doubt use calculus and see if the result makes physical sense.
- if Velocity is constant