Homework 7

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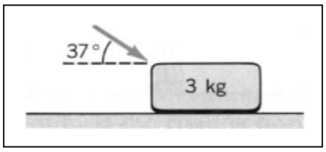
Problem 1

3.15-kg block is acted on by a 24.0-N force that acts at 37.0° below the horizontal, as shown in the figure. Take $\mu_k = 0.200$ and $\mu_s = 0.500$.
(a) Does the block move if it is initially at rest?
(b) If it is initially moving to the right, what is the blocks acceleration?



Solution

Part a:

Let F_x be the horizontal component of the force vector $\vec{F}_1$ that is acting on the block given by;

$$F_x = F_1 \cos\theta$$

If the block is at rest it will move when F_x is greater than the force of static friction, F_s, acting on the block $F_x > F_s$ where F_s is given by;

$$F_s = \mu_s F_n$$

The normal force F_n is given by

$$F_n = mg + F_1 \sin\theta$$

So we have

$$F_x = 24.00 \cos 37 = \boxed{19.167 \text{ N}}$$

and

$$F_s = 0.500\,(3.15 \cdot 9.81 + 24.00 \sin 37) = \boxed{22.673 \text{ N}}$$

Therefore, the block does not move if it is initally at rest.

Part b:

If the block is moving to the right the acceleration is equal to the difference in the horizontal component of $\vec{F}_1$ and the force of kinetic friction,F_k where;

$$F_k = \mu_k F_n = \mu_k\,(mg + F_1 \sin\theta)$$

which gives us the equation

$$\vec{a} = \frac{F_1 \cos\theta - \mu_k\,(mg + F_1 \sin\theta)}{m}$$

substituting our values;

$$\vec{a} = \frac{24.00\cos 37 - 0.200\,(3.15 \cdot 9.81 + 24.00\sin 37)}{3.15} = \boxed{3.206\ \text{m/s}^2}$$

Problem 2

A block is released at the top of a 25° incline. Determine the coefficient of kinetic friction given that it slides 2.30 m in 3.15 s.

Solution

Given the distance traveled, $\Delta r = 2.30$, and the time, $t = 3.15$, we find the coefficient of kinetic frinction, μ_k, by finding $\vec{a}$ as a function of μ_k and then solving for μ_k in the kinematic equation for distance.

$$\vec{a} = \frac{F_g - \mu_k F_n}{m}$$

where F_g is the force of gravity in the direction of the incline given by

$$F_g = mg\sin\theta$$

and F_n is the normal force that is opposing the force of gravity given by

$$F_n = mg\cos\theta$$

Therefore, $\vec{a}$ is given by

$$\begin{aligned}\vec{a} &= \frac{mg\sin\theta - \mu_k mg\cos\theta}{m} \\ &= \frac{mg\,(\sin\theta - \mu_k\cos\theta)}{m} \\ &= g\,(\sin\theta - \mu_k\cos\theta)\end{aligned}$$

then the distance function becomes

$$\begin{aligned}\Delta r &= v_0 t + \frac{1}{2}at^2 \\ \Delta r &= (0)t + \frac{1}{2}\left(g\,(\sin\theta - \mu_k\cos\theta)\right)t^2 \\ \frac{2\Delta r}{gt^2} &= \sin\theta - \mu_k\cos\theta \\ \mu_k &= \tan\theta - \frac{2\Delta r}{gt^2\cos\theta}\end{aligned}$$

substituting the given values;

$$\mu_k = \tan 25 - \frac{2 \cdot 2.30}{9.81 \cdot 3.15^2\cos 25} = \boxed{0.414}$$

Problem 3

A circular off ramp has a radius of 57.0 m and a posted speed limit of 50.0 km/h. If the road is horizontal, what is the minimum coefficient of friction required?

Solution

Centripital acceleration is given by

$$a = \frac{v^2}{r}$$

therefore, centripital force is

$$F_c = m\frac{v^2}{r}$$

The minimum coefficient of friction would be when F is equal to F_c, otherwise the centripital force would overpower the friction force and the car would fall inward towards the center.

$$\begin{aligned} F_c &= F_s \\ m\frac{v^2}{r} &= \mu F_n \\ m\frac{v^2}{r} &= \mu\, mg \\ \mu &= \frac{v^2}{gr} \end{aligned}$$

Substituting the given values we find;

$$\mu = \frac{13.8889^2}{g57.00} = \boxed{0.345}$$

Problem 4

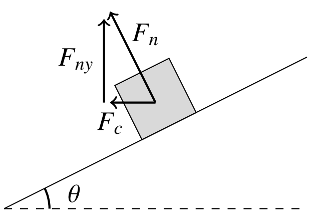
A car travels at speed v around a frictionless curve of radius r that is banked at an angle to the horizontal. Show that the proper angle of banking is given by;

$$\tan\theta = \frac{v^2}{rg}$$

(Hint, this is easier if you don't rotate the coordinate system like most other incline problems, and treat the x-axis as the horizontal direction, and the y-axis as the vertical direction. This is because the centripetal force is horizontal.)

Solution

Consider the cross section of the banked curve, the forces acting on the car are.



Notice that

$$\begin{aligned} \cos\theta &= \frac{F_{ny}}{F_n} \\ F_n\cos\theta &= F_{ny} \\ F_n\cos\theta &= mg \\ F_n &= \frac{mg}{\cos\theta} \end{aligned}$$

and

$$F_c = F_n \sin\theta$$

combining these two equations and using the equation for centripital force;

$$\begin{aligned}
F_c &= \left(\frac{mg}{\cos\theta}\right)\sin\theta \\
m\frac{v^2}{r} &= \left(\frac{mg}{\cos\theta}\right)\sin\theta \\
m\frac{v^2}{r} &= mg\tan\theta \\
\tan\theta &= \frac{v^2}{rg}
\end{aligned}$$

Problem 5

A button is at the rim of a turntable of radius 15.0 cm rotating at 45.0 rpm. What is the minimum coefficient of friction needed for it to stay on?

Solution

The velocity is given by

$$v = \frac{2\pi r}{T}$$

where T is the period of revolution. Then the minimum friction coefficient is given by the equation derived in problem 3

$$\begin{aligned}
\mu_s &= \frac{v^2}{gr} \\
&= \left(\frac{2\pi r}{T}\right)^2 \cdot \frac{1}{gr}
\end{aligned}$$

solving for the given variables;

$$\mu_s = \left(\frac{90\pi(0.15)}{60}\right)^2 \cdot \frac{1}{(9.81)(0.15)} = \boxed{0.340}$$

Problem 6

A box is dropped onto a conveyor belt moving at 3.40 m/s. If the coefficient of friction between the box and the belt is 0.270, how long will it take before the box moves without slipping?

Solution

We know that the force of static friction is given by

$$F_s = \mu_s mg$$

and using Newton'n second law

$$m\vec{a} = \mu_s mg \implies \vec{a} = \mu_s g$$

Then,

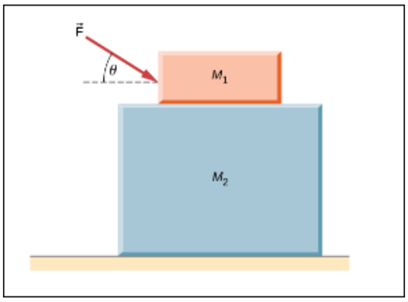
$$v = v_0 + at \implies t = \frac{v - v_0}{a}$$

giving us the equation

$$t = \frac{v - v_0}{\mu_s g} = \frac{3.40 - 0}{(0.27)(9.81)} = \boxed{1.287 \text{ s}}$$

Problem 7

Two blocks are stacked as shown below, and rest on a frictionless surface. There is friction between the two blocks with a coefficient of friction μ_s. An external force is applied to the top block at an angle θ with the horizontal. What is the maximum force F that can be applied for the two blocks to move together?



Solution

The maximum force that can be applied for the two blocks to move together exists when the horizontal component of F is equal to the static friction force that m_2 exerts on m_1.

$$F_s = F_x \implies \mu_s F_n = F\cos\theta$$

where the normal force, F_n is given by

$$F_n = m_1 g + F\sin\theta$$

Therefore the maximum force of F is

$$\begin{aligned} F\cos\theta &= \mu_s\left(m_1 g + F\sin\theta\right) \\ F\frac{cos\theta}{\mu_s} &= m_1 g + F\sin\theta \\ F\frac{\cos\theta}{\mu_s} - F\sin\theta &= m_1 g \\ F &= \frac{m_1 g}{\frac{\cos\theta}{\mu_s} - \sin\theta} \end{aligned}$$

$$\boxed{F = m_1 g\left(\frac{\mu_s}{\cos\theta} - \frac{1}{\sin\theta}\right)}$$