Homework 1

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August 28, 2024

Problem 1

The fuel consumption of cars is specified in Europe in terms of liters per 100 km. Convert 30 miles per gallon to this unit. Note that 1 gallon (U.S.) = 3.79 L.

Solution

We are given $30\,\mathrm{mi/gal}$ and need to convert it to L/100km. The desired units are the reciprocal of the given units—volume of fuel per distance traveled compared to distance traveled per unit of fuel.

$$30\,\frac{\mathrm{mi}}{\mathrm{gal}} = \frac{1}{30}\,\frac{\mathrm{gal}}{\mathrm{mi}}$$

Next, we use chain multiplication to convert miles to kilometers and gallons to liters:

$$\begin{aligned}\frac{1}{30}\,\frac{\cancel{\mathrm{gal}}}{\mathrm{mi}}\cdot\frac{3.79\,\mathrm{L}}{1\,\cancel{\mathrm{gal}}} &= \frac{3.79}{30}\,\frac{\mathrm{L}}{\mathrm{mi}}\\ \frac{3.79}{30}\,\frac{\mathrm{L}}{\cancel{\mathrm{mi}}}\cdot\frac{1\,\mathrm{mi}}{1.609\,\cancel{\mathrm{km}}} &= \frac{3.79}{48.27}\,\frac{\mathrm{L}}{\mathrm{km}}\\ \frac{3.79}{48.27}\,\frac{\mathrm{L}}{\mathrm{km}} &= 0.0785\,\frac{\mathrm{L}}{\mathrm{km}} = 7.85\,\frac{\mathrm{L}}{100\,\mathrm{km}}\end{aligned}$$

Problem 2

Check the following equations for dimensional consistency where t is time (s), ν is speed ($\mathrm{m\,s^{-1}}$), a is acceleration ($\mathrm{m/s^2}$), and x is position (m):

1. $x = \frac{\nu^2}{2a}$
2. $x = \frac{1}{2}at$
3. $t = \sqrt{\frac{2x}{a}}$

Solution

Checking the dimensional consistency involves verifying that the units balance out correctly on both sides of the equations.

Equation 1: $x = \frac{\nu^2}{2a}$ The units of ν^2 and $2a$ must cancel such that we are left with just meters (m):

$$\frac{(\mathrm{m/s})^2}{\mathrm{m/s^2}} = \frac{\mathrm{m}^2}{\mathrm{s}^2}\cdot\frac{\mathrm{s}^2}{\mathrm{m}} = \mathrm{m}$$

Therefore, Equation 1 is dimensionally consistent.

Equation 2: $x = \frac{1}{2}at$ We are again looking for position in meters (m):

$$\frac{\mathrm{m}}{\mathrm{s}^2}\cdot\mathrm{s} \neq \mathrm{m}$$

Equation 2 is not dimensionally consistent.

Equation 3: $t = \sqrt{\frac{2x}{a}}$ Here, we are looking for time in seconds (s):

$$\sqrt{\frac{\text{m}}{\text{m/s}^2}} = \sqrt{\text{s}^2} = \text{s}$$

Equation 3 is dimensionally consistent.

Problem 3

A can of paint that covers $20.0\,\text{m}^2$ costs \$24.60. The walls of a room $13.0\,\text{ft} \times 18.0\,\text{ft}$ are $8.00\,\text{ft}$ high. What is the cost of paint for the walls?

Solution

We need to start by deriving an equation that yields the number of cans of paint required to cover the surface area, A, as a function of the given dimensions.

$$A = 2h(x_1 + x_2)\ \text{ft}^2$$

where x_1 and x_2 are the lengths of the respective edges and h is the height of the walls. If each can is capable of covering y m^2 then then it will cover $3.28084y$ ft^2 and so our equation for n cans becomes;

$$n = \left\lceil \frac{2h(x_1 + x_2)}{3.28084y} \right\rceil = \left\lceil \frac{2 \cdot 8.00(13.0 + 18.0)}{3.28084 \cdot 20} \right\rceil = 3$$

3 cans at \$24.60 per can gives us a total cost of \$73.80 to paint the walls.

Problem 4

Consider a race car on a 5.00 km track. Car A finishes the race in 4.00 h and is 1.50 laps ahead of B at this time. What is B's time for the race?

Solution

We determine Car B's race time by finding how far it has traveled in the time it took Car A to finish the race and then determine the time it would have taken Car B to complete the full distance. Since Car B was 7.5 km behind Car A, Car B traveled 292.5 km in 4 hours. We can then solve for the time t_B it took Car B to finish the full 300 km:

$$\frac{292.5\,\text{km}}{4\,\text{h}} = \frac{300\,\text{km}}{t_B\,\text{h}}$$

$$t_B = \frac{300\,\text{km} \cdot 4\,\text{h}}{292.5\,\text{km}} \approx 4.10\,\text{h}$$

Problem 5

On its maiden voyage in July 1952, the liner *United States* won the coveted Blue Ribbon for the fastest crossing of the Atlantic from New York to Cornwall, U.K. The trip took 3 days 10 hours 40 min at an average speed of 34.5 knots (65.5 km/h). This was 10 hours 2 minutes less than the 14-year-old record held by the *Queen Mary.* What was the average speed of the *Queen Mary*?

Solution

The distance across the Atlantic is the same for both ships, so we have:

$$x = v_1 \cdot t_1 = v_2 \cdot t_2$$

Solving for v_2:

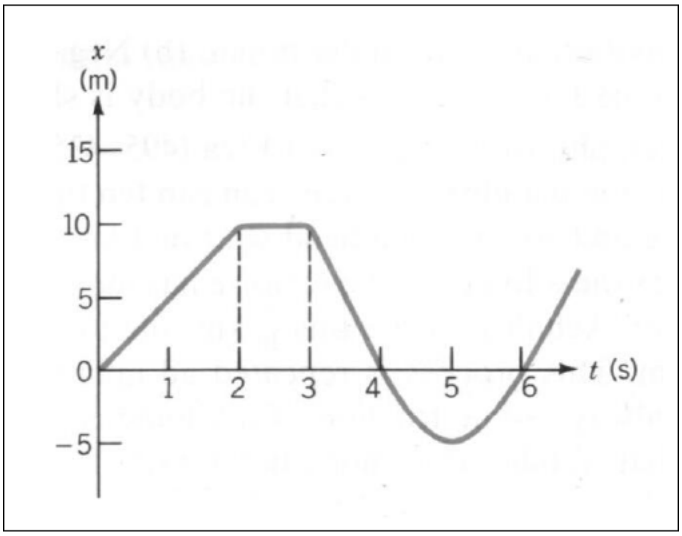
$$v_2 = \frac{v_1 \cdot t_1}{t_2}$$

Substituting the given values:

$$v_2 = \frac{65.5\,\mathrm{km\,h^{-1}} \cdot 82.67\,\mathrm{h}}{92.70\,\mathrm{h}} \approx 58.41\,\mathrm{km\,h^{-1}}$$

Problem 6

Based on the graph below, estimate the instantaneous velocity at the following times: (a) 1.0 s; (b) 2.5 s; (c) 3.5 s; (d) 4.5 s; (e) 5.0 s;



Solution

(a) $5\,\mathrm{m\,s^{-1}}$

(b) $0\,\mathrm{m\,s^{-1}}$

(c) $-10\,\mathrm{m\,s^{-1}}$

(d) $-3\,\mathrm{m\,s^{-1}}$

(e) $0\,\mathrm{m\,s^{-1}}$

Problem 7

At $t = 2.25\,\mathrm{s}$, a particle is at $x = 7.00\,\mathrm{m}$ and has velocity $v = 3.50\,\mathrm{m\,s^{-1}}$. At $t = 7.00\,\mathrm{s}$, it is at $x = -5.10\,\mathrm{m}$ and has velocity $v = 6.00\,\mathrm{m\,s^{-1}}$. Find: (a) its average velocity; (b) its average acceleration.

Solution

The average velocity is defined as:

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

Substituting the values:

$$v_{\text{avg}} = \frac{-5.10\,\mathrm{m} - 7.00\,\mathrm{m}}{7.00\,\mathrm{s} - 2.25\,\mathrm{s}} = \frac{-12.10\,\mathrm{m}}{4.75\,\mathrm{s}} \approx -2.55\,\mathrm{m\,s^{-1}}$$

The average acceleration is defined as:

$$a_{\text{avg}} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$$

Substituting the values:

$$a_{\text{avg}} = \frac{6.00\,\text{m}\,\text{s}^{-1} - 3.50\,\text{m}\,\text{s}^{-1}}{7.00\,\text{s} - 2.25\,\text{s}} = \frac{2.50\,\text{m}\,\text{s}^{-1}}{4.75\,\text{s}} \approx 0.53\,\text{m/s}^2$$

Problem 8

The position of a particle is given by $x = 4.50e^{-0.30t}$, where x has units of meters. What is (a) the average velocity between 2.00 and 3.00 s; (b) the instantaneous velocity at 2.00 s; (c) the instantaneous acceleration at 2.00 s.

Solution

Part a

The average velocity is the change in position over the change in time. First, find the positions at $t = 2.00\,\text{s}$ and $t = 3.00\,\text{s}$:

$$x(3) = 4.50e^{-0.9}, \quad x(2) = 4.50e^{-0.6}$$

The average velocity is then:

$$v_{\text{avg}} = \frac{4.50e^{-0.9} - 4.50e^{-0.6}}{3 - 2} \approx -0.64\,\text{m}\,\text{s}^{-1}$$

Part b

The instantaneous velocity is the derivative of the position function:

$$\begin{aligned} v(t) &= \frac{d}{dt}\left(4.50e^{-0.30t}\right) \\ &= -0.30 \cdot 4.50e^{-0.30t} \\ &= -1.35e^{-0.30t} \end{aligned}$$

Evaluating at $t = 2.00\,\text{s}$:

$$v(2) = -1.35e^{-0.6} \approx -0.74\,\text{m}\,\text{s}^{-1}$$

Part c

The instantaneous acceleration is the derivative of the velocity function:

$$a(t) = \frac{d}{dt}\left(-1.35e^{-0.30t}\right) = 0.405e^{-0.30t}$$

Evaluating at $t = 2.00\,\text{s}$:

$$a(2) = 0.405e^{-0.6} \approx 0.25\,\text{m/s}^2$$