Reading notes - Motion Along a Straight Line

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Problem 1

The fuel consumption of cars is specified in Europe in terms of liters per 100 km. Convert 30 miles per gallon to this unit. Note that 1 gallon (U.S.) = 3.79 L.

Solution

We are given 30 mi/gal to covert to L/100km. The desired units in some sense are the reciprocal of the given units–volume unit of fuel per distance unit traveled compared to distance unit traveled per unit of fuel.

$$30\frac{\text{mi}}{\text{gal}} = \frac{1}{30}\frac{\text{gal}}{\text{mi}}$$

Then we use chain multiplaction to convert miles to kilometers and gallons to liters;

$$\begin{aligned}
\frac{1}{30}\frac{\cancel{\text{gal}}}{\text{mi}} \cdot \frac{3.79}{1}\frac{\text{L}}{\cancel{\text{gal}}} &= \frac{3.79}{30}\frac{\text{L}}{\text{mi}} \\
\frac{3.79}{30}\frac{\text{L}}{\cancel{\text{mi}}} \cdot \frac{1}{1.609}\frac{\cancel{\text{mi}}}{\text{km}} &= \frac{3.79}{48.27}\frac{\text{L}}{\text{km}} \\
\frac{3.79}{48.27}\frac{\text{L}}{\text{km}} = 0.0785\frac{\text{L}}{\text{km}} &= 7.85\frac{\text{L}}{100\text{km}}
\end{aligned}$$

Problem 2

Check the following for dimensional consistency where t is time (s), ν is speed (m/s), a is acceleration (m/s^2), and x is position (m):

1. $x = \frac{\nu^2}{2a}$
2. $x = \frac{1}{2}at$
3. $t = \sqrt{\frac{2x}{a}}$

Solution

Checking the dimensional consistency effectively means checking the arithmetic of the units used for each quantity. Meaning that for example the first equation;

$$x = \frac{\nu^2}{2a}$$

the units of ν^2 and $2a$ must cancel such that we are left with just meters, m, and we find that they do;

$$\frac{(\text{m/s})^2}{\text{m/s}^2} = \frac{\text{m}^{\cancel{2}\,1}}{\cancel{\text{s}^2}} \cdot \frac{\cancel{\text{s}^2}}{\cancel{\text{m}}} = \text{m}$$

Therefore, equation 1 is dimensionally consistent. For equation two, we are again looking for position, m, and find that it is not dimensionally consistent;

$$\frac{\mathrm{m}}{\mathrm{s}^2} \cdot \mathrm{s} \neq \mathrm{m}$$

For equation three, we are looking for time in seconds, s and find that it is dimensionally consistent;

$$\sqrt{\frac{m}{\mathrm{m/s}^2}} = \cancel{\mathrm{m}^{1/2}} \cdot \frac{\mathrm{s}^{2 \cdot 1/2}}{\cancel{\mathrm{m}^{1/2}}} = s$$

Problem 3

A can of paint that covers 20.0 m^2 costs \$24.60. The walls of a room 13.0 ft x 18.0 ft are 8.00 ft high. What is the cost of paint for the walls?

Solution

If the room is 13 ft x 18 ft then it is

$$13\cancel{\mathrm{ft}} \cdot \frac{0.3048}{1} \frac{\mathrm{m}}{\cancel{\mathrm{ft}}} = 3.9624\mathrm{m}$$

by 5.4864m with 2.4383m high walls and the total surface area to cover is the sum of the surface area of each wall.

$$\text{area} = 2\left(3.9624\mathrm{m} \cdot 2.4383\mathrm{m}\right) + 2\left(5.4864\mathrm{m} \cdot 2.4383\mathrm{m}\right) = 46.078\mathrm{m}^2$$

and so we will need to cover to buy 3 cans of paint to cover the walls costing us \$73.8. Suppose we are lucky and the store agrees to refund the remaining paint in the partially used can, meters painted per dollar spent;

$$\frac{24.60}{20} \frac{\$}{\mathrm{m}^2} = 1.23 \frac{\$}{\mathrm{m}^2}$$

and with 46.078 m^2 to paint we have a cost of

$$46.078\mathrm{m}^2 \cdot 1.23 \frac{\$}{\mathrm{m}^2} = \$56.68$$

Problem 4

Consider a race car on a 5.00 km track. Car A finishes the race in 4.00h and is 1.50 laps ahead of B at this time. What is B's time for the race?

Solution

In order to find B's time for the race we need to know the velocity which we can derive from Car A's velocity given that we know how far ahead Car A was when it finished the race as well as how long it took them to finish the race.