Homework 9

Aaron W. Tarajos
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Problem 1

A spring gun with k = 90.0 N/m is compressed by 5 cm. What is the exit speed of a 2.10-g projectile?

Solution

Let U be potential energy, then;

$$U = \frac{1}{2}kx^2$$

and K be kinetic energy, then we have;

$$\begin{aligned} U &= K \\ \frac{1}{2}kx^2 &= \frac{1}{2}mv^2 \\ v^2 &= \frac{kx^2}{m} \\ v &= \sqrt{\frac{kx^2}{m}} \end{aligned}$$

For the given values;

$$v = \sqrt{\frac{(90.0)(0.05^2)}{0.00210}} = \boxed{10.351 \text{ m/s}}$$

Problem 2

The United States, with a population of 2.2×10^8 people, consumes 5×10^{19} J per year.
(a)What is the per capita consumption in watts?
(b) The sun's radiation provides the earth with 1000 W/m^2. Assuming solar energy can be converted to electrical energy with a 20% efficiency, how much area is needed to serve the energy needs of each U.S. citizen?

Solution

Part a:

Let E_T be the total power consumed in the U.S. in Joules

$$E_T = 5 \times 10^{19} \cdot \frac{1}{365} \cdot \frac{1}{24} \cdot \frac{1}{3600} = 1.585 \times 10^{12} \text{ W}$$

then the per capita energy consumption is E_C;

$$E_C = \frac{E_T}{2.2 \times 10^8} = \boxed{7206.771 \text{ W}}$$

Part b:

Let A be the area needed to serve the energy needs of each citizen;

$$A = \frac{E_C}{1000 \cdot 0.2} = \boxed{36.034 \text{ m}^2}$$

Problem 3

A 0.595-kg object is released from a height of 3.60 m and lands on the ground. Find:
(a) the work done by gravity;
(b) the change in kinetic energy of the ball;
(c) the speed just before it lands using energy methods. Ignore air resistance.

Solution

Part a:

$$W = mgd = (0.595)(9.81)(3.60) = \boxed{21.013 \text{ J}}$$

Part b:

$$\Delta K = W = \boxed{21.013 \text{ J}}$$

Part c:

$$\begin{aligned} W &= \Delta K \\ W &= \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 \\ W &= \frac{1}{2}mv^2 - \frac{1}{2}m(0)^2 \\ mgd &= \frac{1}{2}mv^2 \\ v^2 &= 2gd \\ v &= \sqrt{2gd} = \boxed{8.404 \text{ m/s}} \end{aligned}$$

Problem 4

Two horses pull a barge along a canal at a steady 5.00 km/h, as shown in the figure. The tension in each rope is 420 N and each is at 30° to the direction of motion. What is the horsepower provided by the horses?

Solution

The horses are walking at constant velocity which means that the net force in the x direction is zero. Therefore

$$\begin{aligned} F_T \cos\theta - F_x &= 0 \\ F_T \cos\theta &= F_x \\ F_x &= 420 \cos 30 = 363.730 \text{ N} \end{aligned}$$

There are two horses so the force is 2 times that, then power is given by

$$P = F_x v = \frac{2\,(420 \cos 30) \cdot 1.3889}{745.7} = \boxed{1.355 \text{ HP}}$$

Problem 5

A pendulum bob of mass 0.710 kg is suspended by a string of length 1.50 m. The bob is released from rest when the string is at 30° to the vertical. The swing is interrupted by a peg 1.00 m vertically below the support as shown below. What is the maximum angle to the vertical made by the string after it hits the peg?

Problem 6

A 2.00-kg block slides on a frictionless horizontal surface and is connected on one side to a spring with a spring constant of 45.0 N/m) as shown the figure. The other side is connected to a 4.00-kg block that hangs vertically. The system starts from rest with the spring unextended.
(a) What is the maximum extension of the spring?
(b) What is the speed of the 4.00-kg block when the extension is 50 cm?

Problem 7

A cart with a mass of 3.20 kg, an initial speed of 5.15 m/s and an initial height of 4.00 m is moving towards a hill of height 5.00 m, as shown in the figure. On the other side of the hill is a spring with a spring constant of 125 N/m and a height of 2.00 m.
(a) Does the trolley reach the spring?
(b) If so, what is the maximum compression? Ignore frictional losses and the rotational energy of the wheels.

Problem 8

A projectile is fired at 27.0 m/s in a direction 65° above the horizonal from a rooftop of height 40.0 m. Use energy considerations to find:
(a) the speed with which it lands on the ground;
(b) the height at which its speed is 15.0 m/s.