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# Derivation of equations for constant acceleration

Aaron W. Tarajos

*August 27, 2024*

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## Velocity at time $t$

The change in velocity,  $d\nu$ , is equal to the product of acceleration,  $a$ , and the change in time  $dt$ .

$$d\nu = a dt$$

and we integrate to find the equation to solve for velocity;

$$\begin{aligned}d\nu &= a dt \\ \int d\nu &= \int a dt \\ \int d\nu &= a \int dt \\ \nu &= at + c \\ \nu_0 &= a(0) + c = c \\ \nu &= at + \nu_0\end{aligned}$$

Note that there is no constant  $c$  for the integration of  $d\nu$  because it is more a symbolic recovering of the velocity function and any constant would be inherently included in the velocity function.

## Position at time $t$

Similar to acceleration we integrate velocity to find the equation to solve for position.

$$\begin{aligned}dx &= \nu dt \\ \int dx &= \int \nu dt \\ &= \int (\nu_0 + at) dt \\ &= \nu_0 \int dt + a \int t dt \\ x &= \nu_0 t + \frac{1}{2}at^2 + c \\ x - x_0 &= \nu_0 t + \frac{1}{2}at^2\end{aligned}$$

## An equation without time

We can solve problems for a various circumstances where one of these variables are missing from the problem entirely using the two previous derivations, starting with an equation without time. Given

$$x - x_0 = \nu_0 t + \frac{1}{2}at^2$$

and

$$\nu = \nu_0 + at$$

We start with the second equation to solve for time as a function of initial velocity, velocity and acceleration;

$$\frac{\nu - \nu_0}{a} = t$$

Furthermore, because acceleration is constant we know that average velocity,  $\bar{\nu}$ , is

$$\bar{\nu} = \frac{\nu + \nu_0}{2}$$

as well as

$$x = \bar{\nu}t + x_0$$

substituting our equations for time and average velocity;

$$\begin{aligned} \left(\frac{\nu + \nu_0}{2}\right) \left(\frac{\nu - \nu_0}{a}\right) + x_0 &= x \\ \frac{\nu^2 - \nu_0^2}{2a} + x_0 &= x \\ \nu^2 &= 2a(x - x_0) + \nu_0^2 \end{aligned}$$

## An equation without acceleration

Given

$$x = x_0 + \bar{\nu}t$$

and

$$\bar{\nu} = \frac{\nu + \nu_0}{2}$$

we obtain

$$x - x_0 = \frac{\nu + \nu_0}{2}t$$

## An equation without initial velocity

Given

$$\nu_0 = \nu - at$$

We substitute this into the equation for change in position

$$\begin{aligned} x - x_0 &= \frac{(\nu + \nu - at)t}{2} \\ x - x_0 &= \frac{2\nu t - at^2}{2} \\ x - x_0 &= \nu t - \frac{at^2}{2} \end{aligned}$$