

Homework 8

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Problem 1

A 0.315-kg particle moves from an initial position $\vec{r}_1 = 2.00 \hat{i} - 1.00 \hat{j} + 3.00 \hat{k}$ m to a final position $\vec{r}_2 = 4.00 \hat{i} - 3.00 \hat{j} - 1.00 \hat{k}$ m while a force $\vec{F} = 2.00 \hat{i} - 3.00 \hat{j} + 1.00 \hat{k}$ N acts on it. What is the work done by the force on the particle?

Solution

The distance traveled by the particle, \vec{d} , is equal to the difference in final and initial position

$$\begin{aligned}\vec{d} &= \vec{r}_2 - \vec{r}_1 \\ &= (4.00 - 2.00) \hat{i} + (-3.00 + 1.00) \hat{j} + (-1.00 - 3.00) \hat{k} \\ &= 2.00 \hat{i} - 2.00 \hat{j} - 4.00 \hat{k}\end{aligned}$$

Then work is the dot product of \vec{F} and \vec{d}

$$W = \vec{F} \cdot \vec{d} = (2)(2) + (-2)(-3) + (-4)(1) = \boxed{6 \text{ J}}$$

Problem 2

Compute the kinetic energy for each of the cases below. Through what distance would a 800-N force have to act to stop each object?

- (a) A 150-g baseball moving at 40 m/s;
- (b) a 13-g bullet from a rifle moving at 635 m/s;
- (c) a 1500-kg Corvette moving at 250 km/h;
- (d) a 1.8×10^5 -kg Concorde airliner moving at 2240 km/h.

Solution

The kinetic energy is given by

$$k = \frac{1}{2}mv^2 \tag{1}$$

Then

$$k = Fd \implies d = \frac{k}{F}$$

Using these equations to solve for each part;

Part a:

$$k = \frac{1}{2}(0.150)(40)^2 = \boxed{120 \text{ J}}$$

and

$$d = \frac{120}{800} = \boxed{0.15 \text{ m}}$$

Part b:

$$k = \frac{1}{2}(0.013)(635)^2 = \boxed{2620.96 \text{ J}}$$

and

$$d = \frac{2620.96}{800} = \boxed{3.276 \text{ m}}$$

Part c:

$$k = \frac{1}{2}(1500)(69.44444)^2 = \boxed{3\,616\,897 \text{ J}}$$

and

$$d = \frac{3\,616\,897}{800} = \boxed{4.521 \text{ km}}$$

Part d:

$$k = \frac{1}{2}(1.8 \times 10^5)(622.2222)^2 = \boxed{3.484 \times 10^{10} \text{ J}}$$

and

$$d = \frac{2620.96}{800} = \boxed{43\,555.552 \text{ km}}$$

Problem 3

Compute the kinetic energies for each of the following. What force would be required to stop each object in 1.00 km?

- (a) The 8.00×10^7 -kg carrier Nimitz moving at 55 km/h;
- (b) a 3.4×10^5 -kg Boeing 747 moving at 1000 km/h;
- (c) the 270-kg Pioneer 10 spacecraft moving at 51,800 km/h.

Solution

Part a:

Kinetic energy is

$$k = \frac{1}{2}mv^2 = \frac{1}{2}(8.00 \times 10^7)(15.27778)^2 = \boxed{9.336 \times 10^9 \text{ J}}$$

and the force required to stop it is

$$F = \frac{9.336 \times 10^9 \text{ J}}{1000 \text{ m}} = \boxed{9\,336\,422 . 469 \text{ N}}$$

Part b:

Kinetic energy is

$$k = \frac{1}{2}(3.4 \times 10^5)(277.7778)^2 = \boxed{1.312 \times 10^{10} \text{ J}}$$

and the force required to stop it is

$$F = \frac{1.312 \times 10^{10} \text{ J}}{1000 \text{ m}} = \boxed{13\,117\,286 . 049 \text{ N}}$$

Part c:

Kinetic energy is

$$k = \frac{1}{2}(3.4 \times 10^5)(277.7778)^2 = \boxed{1.312 \times 10^{10} \text{ J}}$$

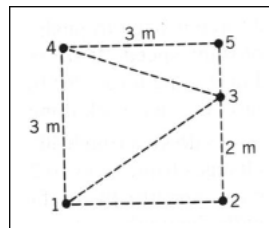
and the force required to stop it is

$$F = \frac{1.312 \times 10^{10} \text{ J}}{1000 \text{ m}} = \boxed{13\,117\,286.049 \text{ N}}$$

Problem 4

A 1.50-kg block is moved at constant speed in a vertical plane from position 1 to position 3 via several routes shown in the figure. Compute the work done by gravity on the block for each segment indicated, where W_{ab} means work done from a to b.

- (a) $W_{1\,3}$
- (b) $W_{1\,2} + W_{2\,3}$
- (c) $W_{1\,4} + W_{4\,3}$
- (d) $W_{1\,4} + W_{4\,5} + W_{5\,3}$

**Solution****Part a:**

Let \vec{d} be the path from position 1 to position 3, the angle between \vec{F}_g and \vec{d} is given by

$$\phi_g = 90 + \arctan\left(\frac{d_y}{d_x}\right)$$

then

$$W_{1\,3} = (mg)(d) \cos \phi_g$$

substituting the given values values

$$W_{1\,3} = (1.50)(9.81)(\sqrt{13}) \cos\left(90 + \arctan\left(\frac{2}{3}\right)\right) = \boxed{-29.43 \text{ J}}$$

Part b:

The distance vector \vec{d} is orthogonal to \vec{F}_g for $W_{1\,2}$, therefore that component is zero. For $W_{2\,3}$, the angle between the \vec{F}_g and \vec{d} is 180° so we have

$$W_{1\,2} + W_{2\,3} = 0 + (mg)(d)(-1) = (1.50)(9.81)(2)(-1) = \boxed{-29.43 \text{ J}}$$

Part c:

Similar to part a, we have;

$$\phi_g = 90 - \arctan\left(\frac{d_y}{d_x}\right)$$

for $W_{4\ 3}$, giving us

$$\begin{aligned} W_{1\ 4} + W_{4\ 3} &= (mg)(d_1)(-1) + (mg)(d_2)\left(\cos\left(90 - \arctan\left(\frac{d_y}{d_x}\right)\right)\right) \\ &= (1.50 \cdot 9.81)(3)(-1) + (1.50 \cdot 9.81)(\sqrt{10})\left(\cos\left(90 - \arctan\left(\frac{1}{3}\right)\right)\right) \\ &= \boxed{-29.43\text{ J}} \end{aligned}$$

Part d:

This is similar to part b, except, for the third component of work, $\phi_g = 0$.

$$W_{1\ 4} + W_{4\ 5} + W_{5\ 3} = (mg)(d)(-1) + 0 + (mg)(d)(1) = (1.5)(9.81)(3)(-1) + (1.5)(9.81)(1)(1) = \boxed{-29.43\text{ J}}$$

Problem 5

What is the work needed to lift 14.7 kg of water from a well 11.0 m deep. Assume the water has a constant upward acceleration of 0.700 m/s^2 .

Solution

Let \vec{F} be the upward force acting on the bucket

$$\vec{F} = m\vec{a}$$

then work W is given by

$$W = Fd \cos(0) = (14.7)(0.7 + 9.81)(11) = \boxed{1699.467\text{ J}}$$

Problem 6

The variation of a force with position is shown in the figure below. Find the work from (a) $x = 0$ to $x = -A$
(b) $x = +A$ to $x = 0$

Problem 7

Consider a particle on which several forces act, one of which is known to be constant in time: $\vec{F}_1 = 3.00\hat{i} + 4.00\hat{j}\text{ N}$. As a result, the particle moves along a straight path from a Cartesian coordinate of (0.00 m, 0.00 m) to (5.00 m, 6.00 m). What is the work done by \vec{F}_1 ?

Problem 8

A bungee cord exerts a nonlinear elastic force of magnitude $F(x) = k_1x + k_2x^3$, where x is the distance the cord is stretched, $k_1 = 204\text{ N/m}$ and $k_2 = -0.233\text{ N/m}^3$. How much work must be done on the cord to stretch it 16.7 m?