Exam 1 Notes

polar → component: $a_x = a\cos\theta$ and $a_y = a\sin\theta$
component → polar: $a = \sqrt{a_x^2 + a_y^2}$ and $\tan\theta = \frac{a_y}{a_x}$

Angular kinematics

It works the same as linear motion but with different variables.
Position is defined by θ in radians;

$$\theta = \frac{S}{r}$$

angular velocity is

$$\omega = \frac{d\theta}{dt}$$

angular acceleration is

$$\alpha = \frac{d^2\theta^2}{dt^2}$$

kinematic equations

Variable	Equation
Velocity	$v = at + v_0$
Position	$\Delta x = v_0 t + \frac{1}{2}at^2$
Missing t	$v^2 = 2a\Delta x + v_0^2$
Missing a	$\Delta x = \frac{v+v_0}{2}t$
Missing v_0	$\Delta x = vt - \frac{1}{2}at^2$

other useful stuff;

$$v_t = \omega r$$
$$a_t = \alpha r$$

for tangential acceleration and velocity.
Centripetal acceleration;

$$a_c = \omega^2 r$$

Period of revolution;

$$T = \frac{2\pi r}{v} = \frac{2\pi}{\omega}$$

Variable	Equation
Horizontal displacement	$\Delta x = v_0 \cos\theta t$
Vertical displacement	$\Delta y = v_0 \sin\theta t - \frac{1}{2}gt^2$
Vertical velocity	$v_y = v_0 \sin\theta - gt$
Trajectory	$\Delta y = \tan\theta \Delta x - \frac{g\Delta x^2}{2(v_0\cos\theta)^2}$
Range	$R = \frac{v_0^2 \sin 2\theta}{g}$

Useful units

Work: J → kg m^2/s^2
Impulse: N·s → kg m/s
Force: N → kg m/s^2

Check your algebra you fucking idiot

Electrostatic Force

Coulomb's Law

$$\vec{F} = k\frac{q_1 q_2}{r^2}$$

Fields

Electric field for a point is just force but divide out q_1.
Electric field due to a line of charge (point perpendicular to the end);

$$\begin{aligned}
\vec{E} &= \int_{-L}^{0} k\frac{dq}{r^2}\,\hat{\mathbf{r}} \\
&= \int_{-L}^{0} k\frac{dq}{x^2+z^2}\,\hat{\mathbf{r}} \\
&= \int_{-L}^{0} k\frac{\lambda dx}{x^2+z^2}\,\hat{\mathbf{r}} \\
&= \int_{-L}^{0} k\frac{\lambda dx}{x^2+z^2}\,\frac{-x\,\hat{\mathbf{i}} + z\,\hat{\mathbf{j}}}{\sqrt{x^2+z^2}} \\
&= k\lambda\left[\frac{1}{\sqrt{z^2}} - \frac{1}{\sqrt{L^2+z^2}}\right]\hat{\mathbf{i}} + \frac{k\lambda L}{z\left(L^2+z^2\right)^{1/2}}\,\hat{\mathbf{j}}
\end{aligned}$$

Electric field due to a disc;

$$\begin{aligned}
\vec{E} &= \int d\vec{E} \\
&= \int \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2+z^2}\,\hat{\mathbf{r}} \\
&= \frac{1}{4\pi\epsilon_0}\int \frac{\sigma\cdot dA}{r^2+z^2}\,\hat{\mathbf{r}} \\
&= \frac{1}{4\pi\epsilon_0}\int \frac{\sigma\cdot(2\pi r dr)}{r^2+z^2}\,\hat{\mathbf{r}} \\
&= \frac{1}{4\pi\epsilon_0}\int \frac{\sigma\cdot(2rdr)}{\left(r^2+z^2\right)^{3/2}}\,z\hat{\mathbf{j}} \\
&= \frac{\sigma z}{4\pi\epsilon_0}\int_0^R \frac{2rdr}{\left(r^2+z^2\right)^{3/2}}\hat{\mathbf{j}} \\
&= \frac{\sigma}{2\epsilon_0}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right)
\end{aligned}$$

Charge Density: $dq = \lambda ds$ for a line, $dq = \sigma dA$ for a sufrace, etc... and these are simply $\frac{\text{charge}}{\text{quantity}}$
Electric Field due to a dipole;

$$\vec{E} = \frac{2kQd}{z^3\left(1 - d^2/4z^2\right)^2}\,\hat{\mathbf{k}}$$

Dipole stuff

$$\vec{p} = Qd\hat{\mathbf{k}}$$
$$\vec{\tau} = \vec{p}\times\vec{E}$$
$$U = -\vec{p}\cdot\vec{E}$$
$$W = -\Delta U$$