Homework 11

Aaron W. Tarajos
November 6, 2024

Problem 1

A neutron at rest decays into a proton, an electron, and a neutrino. If the proton's momentum is 3.00×10^{-24} kg m/s in the direction 37° N of E and the electron's momentum is 4.00×10^{-24} kg m/s in the direction 53° S of W, what is the momentum of the neutrino?

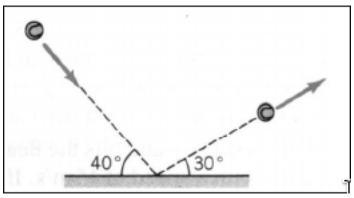
Solution

Since the neutrino is at rest we just need to find the value of the neutrino momentum vector such that the resultant of all three particles is zero.

$$\begin{aligned}
0 &= \vec{p}_\mathrm{p} + \vec{p}_\mathrm{e} + \vec{p}_\mathrm{n} \\
-\vec{p}_\mathrm{p} &= \vec{p}_\mathrm{e} + \vec{p}_\mathrm{n} \\
-\vec{p}_\mathrm{p} &= 3.00 \times 10^{-24} \cos(37)\,\hat{\mathbf{i}} + 3.00 \times 10^{-24} \sin(37)\,\hat{\mathbf{j}} \\
&\quad + 4.00 \times 10^{-24} \cos(233)\,\hat{\mathbf{i}} + 4.00 \times 10^{-24} \sin(233)\,\hat{\mathbf{j}} \\
-\vec{p}_\mathrm{p} &= -1.135 \times 10^{-26}\,\hat{\mathbf{i}} - 1.389 \times 10^{-24}\,\hat{\mathbf{j}} \\
\vec{p}_\mathrm{p} &= \boxed{1.135 \times 10^{-26}\,\hat{\mathbf{i}} + 1.389 \times 10^{-24}\,\hat{\mathbf{j}}\text{ kg m/s}}
\end{aligned}$$

Problem 2

A 60.0-g tennis ball strikes the ground at 25.0 m/s at 40° to the horizontal. It bounces off at 20.0 m/s at 30° to the horizontal. (a) Find the impulse exerted on the ball. (b) If the collision lasted 5.00 ms, find the average force exerted on the ball by the court.



Solution

Part a:

The impulse is given by;

$$\vec{J} = \Delta\vec{p}$$

Therefore;

$$\begin{aligned}
\vec{J} &= [(60.0)(20)\cos(30) \; - (60)(25)\cos(-40)]\,\hat{\mathbf{i}} + [(60)(20)\sin(30) - (60)(25)\sin(-40)]\,\hat{\mathbf{j}} \\
&= \boxed{-109.836\,\hat{\mathbf{i}} + 1564.181\,\hat{\mathbf{j}}\text{ g m/s}}
\end{aligned}$$

Part b:

The average force is equal to the change in impluse over time;

$$\vec{F}_{\text{avg}} = \frac{-109.836\,\hat{\mathbf{i}}156.181\,\hat{\mathbf{j}}}{0.05} = \boxed{-2196.724\,\hat{\mathbf{i}} - 31\,283.628\,\hat{\mathbf{j}}\text{ g m/s}^2}$$

Problem 3

A 1000-kg Subaru at rest at a stoplight is struck from the rear by a 1400-kg Pontiac. They couple together and leave skid marks 4.25 m long. The coefficient of kinetic friction is 0.6. (a) What was their common speed just after the collision? (b) What was the speed of the Pontiac just prior to the collision?

Solution

Part a:

The work done by friction is $W = \vec{F}_n \cdot d$ while work is also equal to ΔK so we have;

$$\begin{aligned}
W &= \Delta K \\
\vec{F}_n \cdot d &= K_f - K_i \\
(\mu_k)(m_1 + m_2)g\cos(\theta)d &= 0 - \frac{1}{2}(m_1 + m_2)v^2 \\
v^2 &= -2g\mu_k\cos(\theta)d \\
v &= \sqrt{-2g\mu_k\cos(\theta)d} \\
v &= \sqrt{(-2)(9.81)(0.60)\cos(180)(4.25)} \\
v &= \boxed{7.07\text{ m/s}}
\end{aligned}$$

Part b:

Using conservation of momentum we can say that;

$$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$$

and then solve for v_{2i}

$$\begin{aligned}
v_{2i} &= \frac{m_1v_{1f} + m_2v_{2f} - m_1v_{1i}}{m_2} \\
&= \frac{(1000)(7.07) + (1400)(7.07) - (1000)(0)}{1400} \\
&= \boxed{12.12\text{ m/s}}
\end{aligned}$$

Problem 4

Two particles with masses m_1 and m_2 travel toward each other with velocities $v_{1,i}$ and $v_{2,i}$. They collide and stick together. Show that the loss in kinetic energy is

$$\frac{m_1m_2(v_{1,i} - v_{2,i})^2}{2(m_1 + m_2)}$$

Solution

The initial kinetic energy is

$$K_i = \frac{1}{2}m_1v_{1,i}^2 + \frac{1}{2}m_2v_{2,i}^2$$

and the final kinetic energy is

$$K_f = \frac{1}{2}(m_1 + m_2)v_f^2$$

We also know that v_f from the conservation of momentum is given by;

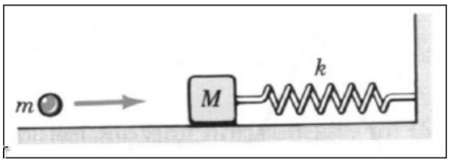
$$v_f = \frac{m_1 v_{1,i} - m_2 v_{2,i}}{m_1 + m_2}$$

and so we have

$$\begin{aligned}
\Delta K &= \frac{1}{2} m_1 v_{1,i}^2 + \frac{1}{2} m_2 v_{2,i}^2 - \frac{1}{2}(m_1 + m_2) v_f^2 \\
&= \frac{1}{2}\left[m_1 v_{1,i}^2 + m_2 v_{2,i}^2 - \frac{(m_1 v_{1,i} + m_2 v_{2,i})^2}{m_1 + m_2} \right] \\
&= \frac{1}{2}\left[\frac{(m_1 + m_2) m_1 v_{1,i}^2 + m_2 v_{2,i}^2 - (m_1 v_{1,i} + m_2 v_{2,i})^2}{m_1 + m_2} \right] \\
&= \frac{m_1 m_2 (v_{1,i-v_{2,i}})^2}{2(m_1 + m_2)}
\end{aligned}$$

Problem 5

A projectile of mass 0.25 kg moving at 24.0 m/s collides with and sticks to a 1.75-kg block that is connected to a spring for which k = 40.0 N/m, as in the figure below. The block is initially on a frictionless part of a horizontal surface but starts to slide on a rough section immediately after the collision. If the maximum compression of the spring is 0.5 m, what is the force of friction on the block?



Solution

Using the conservation of momentum to find velocity at the collision;

$$v_f = \frac{m v_i}{m + M}$$

then the kinetic energy after the collision is converted into potential energy and work done against friction;

$$\begin{aligned}
\frac{1}{2}(m + M)\left(\frac{m v_i}{m + M}\right)^2 &= \frac{1}{2} k x^2 + \vec{F}_k x \\
\vec{F}_k &= \left(\frac{1}{2}(m + M)\left(\frac{m v_i}{m + M}\right)^2 - \frac{1}{2} k x^2\right) \cdot \frac{1}{x} \\
\vec{F}_k &= \left(\frac{1}{2}(0.25 + 1.75)\left(\frac{0.2524.0}{0.25 + 1.75}\right)^2 - \frac{1}{2}(40.0)(0.5)^2\right) \cdot \frac{1}{0.5} \\
&= \boxed{8.0 \text{ N}}
\end{aligned}$$

Problem 6

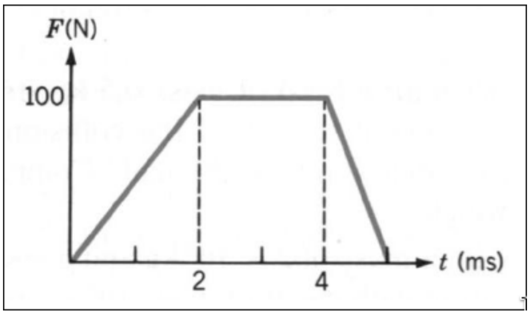
From the F versus t curve shown below, find: (a) the impulse; (b) the average force.

Solution

Part a:

The impulse is

$$\vec{J} = \int \vec{F} dt$$



for the given graph that is

$$\vec{J} = \frac{1}{2}(2)(100) + (100)(2) + \frac{1}{2}(100)(1) = \boxed{350\ \mathrm{N \cdot ms}}$$

Part b:

The average force is given by;

$$\frac{\vec{J}}{\Delta t} = \frac{350}{5} = \boxed{70\ \mathrm{N}}$$

Problem 7

A nucleus of radioactive radium (^{226}Ra), initially at rest, decays into a radon nucleus (^{222}Rn) and an α-particle (a ^{4}He nucleus). If the kinetic energy of the α-particle is 6.72×10^{-13} J, what is (a) the recoil speed of the radon nucleus, and (b) its kinetic energy? The superscripts indicate, roughly, the mass of each nucleus in unified mass units (u), where 1 u = 1.66×10^{-27} kg.

Solution

Part a:

Using conservation of momentum and solving for v_{Rn};

$$\begin{aligned} m_\alpha v_\alpha &= -m_{\mathrm{Rn}} v_{\mathrm{Rn}} \\ v_{\mathrm{Rn}} &= \frac{m_\alpha v_\alpha}{-m_{\mathrm{Rn}}} \end{aligned}$$

Then using kinetic energy to solve for velocity;

$$\begin{aligned} K_\alpha &= \frac{1}{2} m_\alpha v_\alpha^2 \\ v_\alpha &= \sqrt{\frac{2K_\alpha}{m_\alpha}} \\ v_\alpha &= \sqrt{\frac{2K_\alpha}{m_\alpha}} \end{aligned}$$

Givng us the equation;

$$\begin{aligned} v_{\mathrm{Rn}} &= \frac{(4 \times 1.66 \times 10^{-27}) \sqrt{\frac{2 \times 6.72 \times 10^{-13}}{4 \times 1.66 \times 10^{-27}}}}{222 \times 1.66 \times 10^{-27}} \\ v_{\mathrm{Rn}} &= \boxed{2.563 \times 10^5 \mathrm{m/s}} \end{aligned}$$

Part b:

The kinetic energy is given by

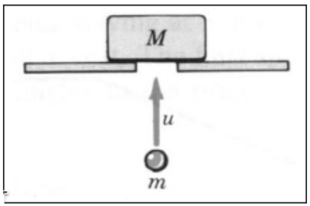
$$K = \frac{1}{2}mv^2$$

so we have

$$K = \frac{1}{2} \cdot 222 \times 1.66 \times 10^{-27} \times (2.56 \times 10^5)^2 = \boxed{1.21 \times 10^{-14}\ \text{J}}$$

Problem 8

A projectile of mass $m = 200$ g strikes a stationary block of mass $M = 1.30$ kg from below with speed $u = 30.0$ m/s as shown in the figure below. The projectile embeds in the block. (a) To what height does the block rise? (b) What is the loss in kinetic energy due to the collision?



Solution

Part a:

Using the conservation of momentum to find the velocity after the collision;

$$v = \frac{mu}{m + M}$$

Then the height is given by

$$(m + M)gh = \frac{1}{2}(m + M)v^2$$

so we have

$$h = \frac{v^2}{2g} = \frac{\left(\frac{mu}{m+M}\right)^2}{2g} = \frac{\left(\frac{200 \cdot 30.0}{1500}\right)^2}{2 \cdot 9.81} = \boxed{0.815\ \text{m}}$$

Part b:

$$\Delta K = \frac{1}{2}(1500)(4.0)^2 - \frac{1}{2}(200)(30.0)^2 = \boxed{-78.0\ \text{J}}$$