Homework 1

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Problem 1

The fuel consumption of cars is specified in Europe in terms of liters per 100 km. Convert 30 miles per gallon to this unit. Note that 1 gallon (U.S.) = 3.79 L.

Solution

We are given 30 mi/gal and need to convert it to L/100km. The desired units are the reciprocal of the given units—volume of fuel per distance traveled compared to distance traveled per unit of fuel.

$$30\,\frac{\text{mi}}{\text{gal}} = \frac{1}{30}\,\frac{\text{gal}}{\text{mi}}$$

Next, we use chain multiplication to convert miles to kilometers and gallons to liters:

$$\begin{aligned}
\frac{1}{30}\,\frac{\cancel{\text{gal}}}{\text{mi}} \cdot \frac{3.79\,\text{L}}{1\,\cancel{\text{gal}}} &= \frac{3.79}{30}\,\frac{\text{L}}{\text{mi}} \\
\frac{3.79}{30}\,\frac{\text{L}}{\cancel{\text{mi}}} \cdot \frac{1\,\text{mi}}{1.609\,\cancel{\text{km}}} &= \frac{3.79}{48.27}\,\frac{\text{L}}{\text{km}} \\
\frac{3.79}{48.27}\,\frac{\text{L}}{\text{km}} &= 0.0785\,\frac{\text{L}}{\text{km}} = 7.85\,\frac{\text{L}}{100\,\text{km}}
\end{aligned}$$

Problem 2

Check the following equations for dimensional consistency where t is time (s), ν is speed ($\text{m}\,\text{s}^{-1}$), a is acceleration (m/s^2), and x is position (m):

1. $x = \frac{\nu^2}{2a}$
2. $x = \frac{1}{2}at$
3. $t = \sqrt{\frac{2x}{a}}$

Solution

Checking the dimensional consistency involves verifying that the units balance out correctly on both sides of the equations.

Equation 1: $x = \frac{\nu^2}{2a}$ The units of ν^2 and $2a$ must cancel such that we are left with just meters (m):

$$\frac{(\text{m/s})^2}{\text{m/s}^2} = \frac{\text{m}^2}{\text{s}^2} \cdot \frac{\text{s}^2}{\text{m}} = \text{m}$$

Thus, Equation 1 is dimensionally consistent.

Equation 2: $x = \frac{1}{2}at$ We are again looking for position in meters (m):

$$\frac{\text{m}}{\text{s}^2} \cdot \text{s} \neq \text{m}$$

Equation 2 is not dimensionally consistent.

Equation 3: $t = \sqrt{\frac{2x}{a}}$ Here, we are looking for time in seconds (s):

$$\sqrt{\frac{\mathrm{m}}{\mathrm{m/s^2}}} = \sqrt{\mathrm{s}^2} = \mathrm{s}$$

Equation 3 is dimensionally consistent.

Problem 3

A can of paint that covers $20.0\,\mathrm{m}^2$ costs \$24.60. The walls of a room $13.0\,\mathrm{ft} \times 18.0\,\mathrm{ft}$ are $8.00\,\mathrm{ft}$ high. What is the cost of paint for the walls?

Solution

First, convert the dimensions of the room to meters:

$$13\,\cancel{\mathrm{ft}} \cdot \frac{0.3048\,\mathrm{m}}{1\,\cancel{\mathrm{ft}}} = 3.9624\,\mathrm{m}$$
$$18\,\cancel{\mathrm{ft}} \cdot \frac{0.3048\,\mathrm{m}}{1\,\cancel{\mathrm{ft}}} = 5.4864\,\mathrm{m}$$
$$8\,\cancel{\mathrm{ft}} \cdot \frac{0.3048\,\mathrm{m}}{1\,\cancel{\mathrm{ft}}} = 2.4384\,\mathrm{m}$$

The total surface area to cover is the sum of the surface area of each wall:

$$\mathrm{Area} = 2\,(3.9624\,\mathrm{m} \cdot 2.4384\,\mathrm{m}) + 2\,(5.4864\,\mathrm{m} \cdot 2.4384\,\mathrm{m}) = 46.08\,\mathrm{m}^2$$

We will need to buy 3 cans of paint to cover the walls, costing us \$73.80. If the store agrees to refund the remaining paint in the partially used can, the cost per square meter is:

$$\frac{24.60\,\$}{20\,\mathrm{m}^2} = 1.23\,\$/\mathrm{m}^2$$

The total cost of painting $46.08\,\mathrm{m}^2$ is:

$$46.08\,\mathrm{m}^2 \cdot 1.23\,\$/\mathrm{m}^2 = \$56.68$$

Problem 4

Consider a race car on a $5.00\,\mathrm{km}$ track. Car A finishes the race in $4.00\,\mathrm{h}$ and is 1.50 laps ahead of B at this time. What is B's time for the race?

Solution

We determine Car B's race time by finding how far it has traveled in the time it took Car A to finish the race and then determine the time it would have taken Car B to complete the full distance.

Since Car B was $7.5\,\mathrm{km}$ behind Car A, Car B traveled $292.5\,\mathrm{km}$ in 4 hours. We can then solve for the time t_B it took Car B to finish the full $300\,\mathrm{km}$:

$$\frac{292.5\,\mathrm{km}}{4\,\mathrm{h}} = \frac{300\,\mathrm{km}}{t_B\,\mathrm{h}}$$

$$t_B = \frac{300\,\mathrm{km} \times 4\,\mathrm{h}}{292.5\,\mathrm{km}} \approx 4.102\,\mathrm{h}$$

Problem 5

On its maiden voyage in July 1952, the liner *United States* won the coveted Blue Ribbon for the fastest crossing of the Atlantic from New York to Cornwall, U.K. The trip took 3 days 10 hours 40 min at an average speed of 34.5 knots (65.5 km/h). This was 10 hours 2 minutes less than the 14-year-old record held by the *Queen Mary*. What was the average speed of the *Queen Mary*?

Solution

The distance across the Atlantic is the same for both ships, so we have:

$$x = v_1 \cdot t_1 = v_2 \cdot t_2$$

Solving for v_2:

$$v_2 = \frac{v_1 \cdot t_1}{t_2}$$

Substituting the given values:

$$v_2 = \frac{65.5\,\mathrm{km\,h^{-1}} \times 82.67\,\mathrm{h}}{92.70\,\mathrm{h}} \approx 58.41\,\mathrm{km\,h^{-1}}$$

Problem 6

Based on the graph below, estimate the instantaneous velocity at the following times: (a) 1.0 s; (b) 2.5 s; (c) 3.5 s; (d) 4.5 s; (e) 5.0 s;

Solution

(a) $5\,\mathrm{m\,s^{-1}}$

(b) $0\,\mathrm{m\,s^{-1}}$

(c) $-10\,\mathrm{m\,s^{-1}}$

(d) $-3\,\mathrm{m\,s^{-1}}$

(e) $0\,\mathrm{m\,s^{-1}}$

Problem 7

At $t = 2.25\,\mathrm{s}$, a particle is at $x = 7.00\,\mathrm{m}$ and has velocity $v = 3.50\,\mathrm{m\,s^{-1}}$. At $t = 7.00\,\mathrm{s}$, it is at $x = -5.10\,\mathrm{m}$ and has velocity $v = 6.00\,\mathrm{m\,s^{-1}}$. Find: (a) its average velocity; (b) its average acceleration.

Solution

The average velocity is defined as:

$$v_{\mathrm{avg}} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

Substituting the values:

$$v_{\mathrm{avg}} = \frac{-5.10\,\mathrm{m} - 7.00\,\mathrm{m}}{7.00\,\mathrm{s} - 2.25\,\mathrm{s}} = \frac{-12.10\,\mathrm{m}}{4.75\,\mathrm{s}} \approx -2.547\,\mathrm{m\,s^{-1}}$$

The average acceleration is defined as:

$$a_{\mathrm{avg}} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$$

Substituting the values:

$$a_{\mathrm{avg}} = \frac{6.00\,\mathrm{m\,s^{-1}} - 3.50\,\mathrm{m\,s^{-1}}}{7.00\,\mathrm{s} - 2.25\,\mathrm{s}} = \frac{2.50\,\mathrm{m\,s^{-1}}}{4.75\,\mathrm{s}} \approx 0.526\,\mathrm{m/s^2}$$

Problem 8

The position of a particle is given by $x = 4.50e^{-0.30t}$, where x has units of meters. What is (a) the average velocity between 2.00 and 3.00 s; (b) the instantaneous velocity at 2.00 s; (c) the instantaneous acceleration at 2.00 s.

Solution

Part a

The average velocity is the change in position over the change in time. First, find the positions at $t = 2.00\,\mathrm{s}$ and $t = 3.00\,\mathrm{s}$:

$$x(3) = 4.50e^{-0.9}, \quad x(2) = 4.50e^{-0.6}$$

The average velocity is then:

$$v_{\text{avg}} = \frac{4.50e^{-0.9} - 4.50e^{-0.6}}{3-2} \approx -0.64\,\mathrm{m\,s^{-1}}$$

Part b

The instantaneous velocity is the derivative of the position function:

$$\begin{aligned} v(t) &= \frac{d}{dt}\left(4.50e^{-0.30t}\right) \\ &= -0.30 \times 4.50e^{-0.30t} \\ &= -1.35e^{-0.30t} \end{aligned}$$

Evaluating at $t = 2.00\,\mathrm{s}$:

$$v(2) = -1.35e^{-0.6} \approx -0.74\,\mathrm{m\,s^{-1}}$$

Part c

The instantaneous acceleration is the derivative of the velocity function:

$$a(t) = \frac{d}{dt}\left(-1.35e^{-0.30t}\right) = 0.405e^{-0.30t}$$

Evaluating at $t = 2.00\,\mathrm{s}$:

$$a(2) = 0.405e^{-0.6} \approx 0.25\,\mathrm{m/s^2}$$