Reading notes - Motion Along a Straight Line

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Problem 1

The fuel consumption of cars is specified in Europe in terms of liters per 100 km. Convert 30 miles per gallon to this unit. Note that 1 gallon (U.S.) = 3.79 L.

Solution

We are given 30 mi/gal to covert to L/100km. The desired units in some sense are the reciprocal of the given units–volume unit of fuel per distance unit traveled compared to distance unit traveled per unit of fuel.

$$30\frac{\text{mi}}{\text{gal}} = \frac{1}{30}\frac{\text{gal}}{\text{mi}}$$

Then we use chain multiplaction to convert miles to kilometers and gallons to liters;

$$\frac{1}{30}\frac{\cancel{\text{gal}}}{\text{mi}} \cdot \frac{3.79}{1}\frac{\text{L}}{\cancel{\text{gal}}} = \frac{3.79}{30}\frac{\text{L}}{\text{mi}}$$
$$\frac{3.79}{30}\frac{\text{L}}{\cancel{\text{mi}}} \cdot \frac{1}{1.609}\frac{\cancel{\text{mi}}}{\text{km}} = \frac{3.79}{48.27}\frac{\text{L}}{\text{km}}$$
$$\frac{3.79}{48.27}\frac{\text{L}}{\text{km}} = 0.0785\frac{\text{L}}{\text{km}} = 7.85\frac{\text{L}}{100\text{km}}$$

Problem 2

Check the following for dimensional consistency where t is time (s), ν is speed (m/s), a is acceleration (m/s^2), and x is position (m):

1. $x = \frac{\nu^2}{2a}$
2. $x = \frac{1}{2}at$
3. $t = \sqrt{\frac{2x}{a}}$

Solution

Checking the dimensional consistency effectively means checking the arithmetic of the units used for each quantity. Meaning that for example the first equation;

$$x = \frac{\nu^2}{2a}$$

the units of ν^2 and $2a$ must cancel such that we are left with just meters, m, and we find that they do;

$$\frac{(\text{m/s})^2}{\text{m/s}^2} = \frac{\text{m}^{\cancel{2}1}}{\cancel{\text{s}^2}} \cdot \frac{\cancel{\text{s}^2}}{\cancel{\text{m}}} = \text{m}$$

For equation two, we are again looking for position, m, and find that it is not dimensionally consistent;

$$\frac{\text{m}}{\text{s}^2} \cdot \text{s} \neq \text{m}$$

For equation three, we are looking for time in seconds, s and find that it is dimensionally consistent;

$$\sqrt{\frac{m}{\text{m/s}^2}} = \cancel{\text{m}^{1/2}} \cdot \frac{\text{s}^{2\cdot 1/2}}{\cancel{\text{m}^{1/2}}} = s$$

Problem 3

A can of paint that covers 20.0 m^2 costs \$24.60. The walls of a room 13.0 ft x 18.0 ft are 8.00 ft high. What is the cost of paint for the walls?

Solution

If the room is 13 ft x 18 ft then it is

$$13\cancel{\text{ft}} \cdot \frac{0.3048}{1} \frac{\text{m}}{\cancel{\text{ft}}} = 3.9624\text{m}$$

by 5.4864m with 2.4383m high walls and the total surface area to cover is the sum of the surface area of each wall.

$$\text{area} = 2\,(3.9624\text{m} \cdot 2.4383\text{m}) + 2\,(5.4864\text{m} \cdot 2.4383\text{m}) = 23.039\text{m}^2$$

and so we will nee to buy 2 cans of paint to cover the walls costing us \$49.20. Suppose we are lucky and the store agrees to refund the remaining paint in the partially used can, meters painted per dollar spent;

$$\frac{24.60}{20} \frac{\$}{\text{m}^2} = 1.23 \frac{\$}{\text{m}^2}$$

and with 23.039m^2 to paint we have a cost of

$$23.039\text{m}^2 \cdot 1.23 \frac{\$}{\text{m}^2} = \$28.34$$

Problem 4