Homework 6

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Problem 1

Two forces $\vec{F}_1 = 1.00\hat{\imath} + 2.00\hat{\jmath}$ N and $\vec{F}_2$ which is 4.00 N directed at 37°, measured from the positive x- axis, act on a 200-g particle. What is its acceleration?

Solution

Given

$$\vec{F}_{\text{net}} = m\vec{a};$$

and

$$\vec{F}_{\text{net}} = \left(1.00\,\hat{\imath} + 4\cos 37\,\hat{\imath}\right) + \left(2.00\,\hat{\jmath} + 4\sin 37\,\hat{\jmath}\right)$$

then

$$\vec{a} = \frac{\left(1.00\,\hat{\imath} + 4\cos 37\,\hat{\imath}\right) + \left(2.00\,\hat{\jmath} + 4\sin 37\,\hat{\jmath}\right)}{0.200} = \boxed{20.973\,\hat{\imath} + 22.036\,\hat{\jmath}\text{ m/s}^2}$$

Problem 2

A Saturn V rocket has a mass of 2.70×10^6 kg and a thrust of 3.30×10^7 N. What is its initial vertical acceleration?

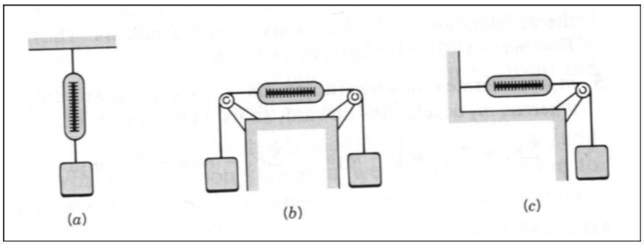
Solution

We can use the same equation but need to consider that the acceleration of gravity is going to decrese the vertical velocity generated by the thrust of the engine.

$$\vec{a} = \frac{3.30 \times 10^7 - \left(2.70 \times 10^6 \cdot 9.81\right)}{2.70 \times 10^6} = \boxed{2.412\text{ m/s}^2}$$

Problem 3

What is the reading on the spring scale for each of the situations depicted in the figure below? Each of the blocks has a mass of 5.00 kg



Solution

Part a:

There are two forces at work, the tension force from the spring and the downward acceleration from gravity. These must sum to zero because the block is at rest;

$$T - mg = 0$$

For the 5.00 kg block, that is

$$T = mg \implies 5.00 \cdot 9.81 = 49.05 \text{ N}$$

Part b:

The tension force from the block on a given side is the same as in part a therefore we have;

$$2T - 2(mg) = 0 \implies 5.00 \cdot 9.81 = 49.05 \text{ N}$$

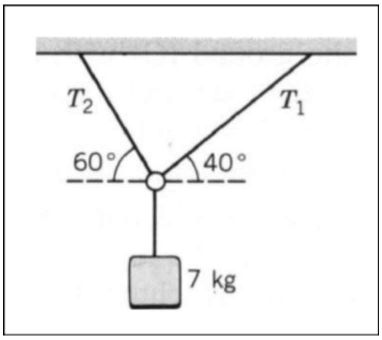
Part c:

This is no different than part a in terms of the forces, therefore, the reading is;

$$T = mg = 5.00 \cdot 9.81 = 49.05 \text{ N}$$

Problem 4

A 7.00-kg block is suspended with two ropes, as shown in the figure. Find the tension in each rope.



Solution

Let T be the force of the vertical rope that the block is hanging from. Then;

$$T_x = 0 \implies T_2 \cos\theta_{T_2} = T_1 \cos\theta_{T_1} \tag{1}$$

and

$$T_y = mg \implies T_2 \sin\theta_{T_2} + T_1 \sin\theta_{T_1} = mg \tag{2}$$

Using (1), we obtain

$$T_2 = T_1 \frac{\cos\theta_{T_1}}{\cos\theta_{T_2}}$$

and substituting into (2) we have;

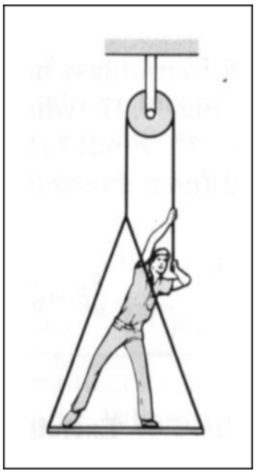
$$mg = T_1 \frac{\cos\theta_{T_1}}{\cos\theta_{T_2}} \sin\theta_{T_2} + T_1 \sin\theta_{T_1}$$
$$mg = T_1 \left(\frac{\cos\theta_{T_1}}{\cos\theta_{T_2}} \sin\theta_{T_2} + \sin\theta_{T_1} \right)$$
$$T_1 = \frac{mg}{\left(\frac{\cos\theta_{T_1}}{\cos\theta_{T_2}} \sin\theta_{T_2} + \sin\theta_{T_1} \right)}$$
$$T_1 = \frac{7.0 \cdot 9.81}{\left(\left(\frac{\cos 40}{\cos 60} \sin 60 \right) + \sin 40 \right)} = \boxed{34.865 \text{ N}}$$

then we use T_1 to solve for T_2 using any of the equations with T_2

$$T_2 = \left(\frac{7.0 \cdot 9.81}{\left(\left(\frac{\cos 40}{\cos 60} \sin 60 \right) + \sin 40 \right)} \right) \left(\frac{\cos 40}{\cos 60} \right) = \boxed{53.416 \text{ N}}$$

Problem 5

A painter of mass $M = 75.0$ kg stands on a platform of mass $m = 15.0$ kg. He pulls on a rope that passes around a pulley, as shown in the figure. Find the tension in the rope given that
(a) he is at rest, or
(b) he accelerates upward at 0.400 m/s2.
(c) If the maximum tension the rope can withstand is 700 N, what happens when he ties the rope to a hook on the wall?



Solution

Part a:

Let T represent the tension in the rope. Then

$$2T - (M + m)\,g = 0$$

because there is the tension force from the painter to the rope and from the platform to the rope, therefore, the tension is

$$T = \frac{(75.0 + 15.0)\,9.81}{2} = \boxed{441.45 \text{ N}}$$

Part b:

We add the additional force vector of

$$\vec{F}_a = M\vec{a} + m\vec{a}$$

to the original Tension calculation to obtain

$$T = \frac{(75.0 + 15.0)\,9.81 + (75.0 + 15.0)\,0.400}{2} = \boxed{459.45\text{ N}}$$

Part c:

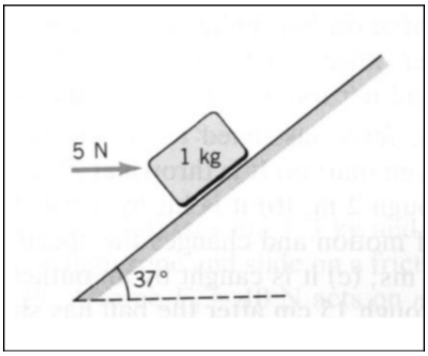
If the rope is tied to the wall then there is no longer the second tension force from the painter pulling on the rope, therefore the tension force;

$$T = (75.0 + 15.0)\,9.81 = \boxed{882.9\text{ N}}$$

exceeding the maximum load of the rope, so we would expect it to snap.

Problem 6

The figure shows a block of mass 1.00 kg is on a frictionless 37.0° incline that is subject to a horizontal force of 5.00 N.
(a) What is its acceleration?
(b) If it is initially moving up the incline at 4.00 m/s, what is its displacement along the incline in 2.10 s?



Solution

Part a:

The force of an object on an incline is given by;

$$\vec{F}_{\text{net}} = F_x \cos\theta + F_y \sin\theta \tag{3}$$

then, because $\vec{F} = m\vec{a}$

$$m\vec{a} = F_x \cos\theta - mg\sin\theta$$
$$\vec{a} = \frac{F_x \cos\theta - mg\sin\theta}{m}$$

therefore, the acceleration of the block is;

$$\vec{a} = \frac{5\cos 37 - (1)(9.81)\sin 37}{1} = \boxed{-1.911\text{ m/s}^2}$$

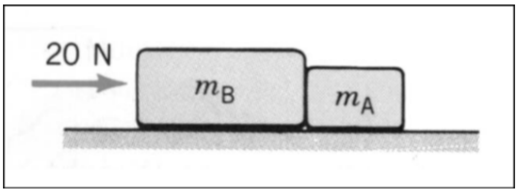
Part b:

Since we are given velocity along the incline, and we solved for acceleration along the incline, we can simply use the one dimensional kinematic equation for constant acceleration. If the dimension of the incline is r then;

$$\begin{aligned}\Delta r &= v_0 t + \frac{1}{2}at^2 \\ \Delta r &= (4.00)(2.10) + \frac{1}{2}(5\cos 37 - (1)(9.81)\sin 37)(2.10)^2 \\ &= \boxed{4.187\text{ m}}\end{aligned}$$

Problem 7

The two blocks shown in the figure below masses $m_A = 1.95$ kg and $m_B = 3.10$ kg. They are in contact and slide over a frictionless horizontal surface. A force of 20.0 N acts on B as shown. Find:
(a) the acceleration;
(b) the force on B due to A;
(c) the net force on B;
(d) the force on B due to A if the blocks are interchanged.



Solution

Part a:

$$\vec{a} = \frac{\vec{F}_x}{m_B + m_A} = \frac{20}{3.10 + 1.95} = \boxed{3.960\text{ m/s}^2}$$

Part b:

Using Newton's second law

$$\left|\vec{F}_{AB}\right| = \left|\vec{F}_{BA}\right|$$

and

$$\vec{F}_{AB} = m_A\vec{a}$$

therefore

$$\vec{F}_{BA} = 1.95 \cdot \frac{20}{3.10 + 1.95} = \boxed{7.723\text{ N}}$$

Part c:

The net force on m_B, $\vec{F}_B$ is;

$$\vec{F}_B = \vec{F}_x - \vec{F}_{AB}$$

because the force of m_A on m_B is in the opposite direction of $\vec{F}_x$.

$$\vec{F}_B = 20 - 7.723 = \boxed{12.277\text{ N}}$$

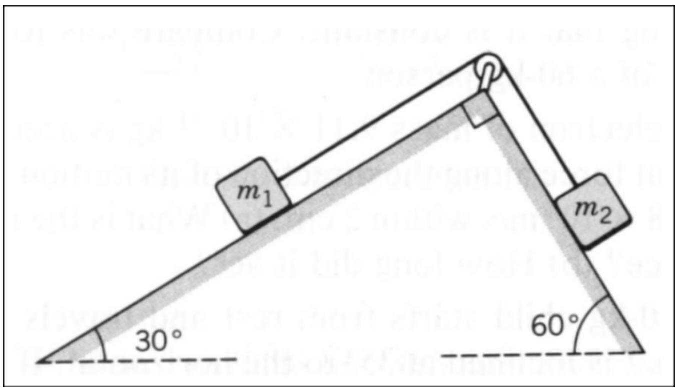
Part d:

We solve the forces the same way but with the mass of m_B instead of m_A;

$$\vec{F}_{AB} = 3.10 \cdot \frac{20}{3.10 + 1.95} = \boxed{12.277 \text{ N}}$$

Problem 8

Two blocks of masses $m_1 = 4.85$ kg and $m_2 = 5.75$ kg kg are on either side of the wedge as shown in the figure below. Find their acceleration and the tension in the rope. Ignore friction and the pulley.



Solution

We have two blocks to consider the forces for. m_1 and m_2. For m_1 we have Tension less gravity given by;

$$\vec{F}_{m1} = T - m_1 g \sin(30)$$

and for m_2 we also have gravity and Tension but the directions are the opposite direction of m_1;

$$\vec{F}_{m2} = m_2 g \sin(60) - T$$

Setting up this problem as a system of equations we have

$$\begin{aligned} m_1\vec{a} &= T - m_1 g \sin(30) \\ + \quad m_2\vec{a} &= m_2 g \sin(60) - T \end{aligned}$$

$$\begin{aligned} \vec{a}\,(m_1 + m_2) &= m_2 g \sin(60) - m_1 g \sin(30) \\ \vec{a} &= \frac{m_2 g \sin(60) - m_1 g \sin(30)}{(m_1 + m_2)} \end{aligned}$$

plugging in our values we obtain

$$\vec{a} = \frac{5.75(9.81)\sin(60) - 4.85(9.81)\sin(30)}{(4.85 + 5.75)} = \boxed{2.364 \text{ m/s}^2}$$

Now we just plugin acceleration to find the Tension

$$T = (4.85)(2.364) + (4.85)(9.81)\sin(30) = \boxed{35.255 \text{ N}}$$