Chapter 3 - Vectors

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3.1 Vectors and Their Components

key ideas

- Scalars consist of only magnitude and are subject to the ordinary rules of algebra. Vectors consist of magnitude and direction and are subject to the rules of vector algebra.
- Two vectors $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ can be added geometrically by drawing them at common scales and place them head to tail. The vector $\vec{\mathbf{s}}$ that connects the head and the tail is the summed vector. To subtract a vector switch the direction of the one you are subtracting and then proceed as if you were adding them.
- The scalar components, $(a_x, a_y, a_z, \dots)$, of a vector $\vec{\mathbf{a}}$ are found by dropping perpendicular lines from the ends of $\vec{\mathbf{a}}$ onto the coordinate axes. In two dimensions, the components are given by;

$$a_x = a\cos\theta \quad \text{and} \quad a_y = a\sin\theta$$

and the magnitude and orientation by;

$$a = \sqrt{a_x^2 + a_y^2} \quad \text{and} \quad \tan(\theta) = \frac{a_y}{a_x}$$

3.1.1 Vectors and Scalars

Motion: A particle moving along a straight line can move in two directions, which can be represented as positive or negative motion. For a particle moving in three dimensions, using just a plus or minus sign is insufficient; vectors are needed to represent both direction and magnitude.

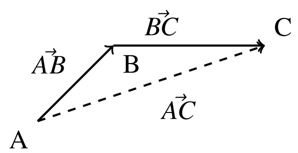
Vectors: A vector has both magnitude and direction and follows specific rules for combination. Vector quantities: These are physical quantities that have both magnitude and direction, such as displacement, velocity, and acceleration.

Scalars: Physical quantities that do not have a direction, like temperature, pressure, energy, mass, and time. Scalars are handled using regular algebra, and a single value (with a sign) specifies them.

Displacement: The simplest vector quantity, which represents a change in position. A vector representing displacement is called a displacement vector. The arrow from point A to B shows displacement, and vectors with the same magnitude and direction (even if shifted) represent the same displacement.

3.1.2 Adding Vectors Geometrically

Given a particle that moves from A to B and then B to C. We represent the overall displacement, regardless of the path taken, as two displacement vectors AB and BC. The net displacement is the **vector sum** AC.



The order of the addition does not matter. This is the commutative law;

$$\vec{\mathbf{a}} + \vec{\mathbf{b}} = \vec{\mathbf{b}} + \vec{\mathbf{a}}$$

While adding two vectors works the way someone may intuitively guess, subtracting is a little different than scalar subtraction.

$$\vec{\mathbf{d}} = \vec{\mathbf{a}} - \vec{\mathbf{b}} = \vec{\mathbf{a}} + (-\vec{\mathbf{b}})$$

The vector being subtracted is changed to the opposite direction and then added.

3.1.3 Component Vectors

Vector Components

A *component* of a vector is the projection of the vector on an axis. To find the projection of a vector along an axis, we draw perpendicular lines from the two ends of the vector to the axis. The projection of a vector on the x axis is its x component, and similarly the projection on the y axis is the y component. This process of finding the components of a vector is called *resolving* the vector. A component of a vector has the same direction (along an axis) as the vector. If we were to reverse vector $\vec{a}$, then both components would be negative, and their arrowheads would point toward the negative x and y directions.

Finding the Components

We can find the components of $\vec{a}$ geometrically from the right triangle:

$$a_x = a\cos\theta \quad \text{and} \quad a_y = a\sin\theta$$

where θ is the angle that vector $\vec{a}$ makes with the positive x axis, and a is the magnitude of $\vec{a}$. We arrange those components head to tail, and then complete a right triangle with the vector forming the hypotenuse, from the tail of one component to the head of the other.

Component Notation vs Magnitude-Angle Notation

Once a vector has been resolved into its components along a set of axes, the components themselves can be used in place of the vector. It can also be given by its components a_x and a_y. Both pairs of values contain the same information. If we know a vector in component notation (a_x and a_y) and want it in magnitude-angle notation (a and θ), we can use the equations:

$$a = \sqrt{a_x^2 + a_y^2} \quad \text{and} \quad \tan\theta = \frac{a_y}{a_x}$$

Three-Dimensional Vectors

In the more general three-dimensional case, we need a magnitude and two angles (say, a, θ, and ϕ) or three components (a_x, a_y, and a_z) to fully specify a vector.

3.2 Unit Vectors, Adding Vectors By Components

key ideas

- Unit vectors $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$ are the base unit vectors with a magnitude of 1 and directed in the positive direction of the x, y, and z axes respectively.
- We write a vector $\vec{\mathbf{a}}$ as

$$\vec{\mathbf{a}} = a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}} + a_z\hat{\mathbf{k}}$$