Homework 8

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Chapter 3 Problem 73

Two vectors are given by $\vec{a} = 3.0\hat{i} + 5.0\hat{j}$ and $\vec{b} = 2.0\hat{i} + 4.0\hat{j}$. Find (a) $\vec{a} \times \vec{b}$, (b) $\vec{a} \cdot \vec{b}$, (c) $(\vec{a} + \vec{b}) \cdot \vec{b}$, and (d) the component of $\vec{a}$ along the direction of $\vec{b}$.

Solution

Part a:

$$\begin{aligned}
\vec{a} \times \vec{b} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3.0 & 5.0 & 0 \\ 2.0 & 4.0 & 0 \end{vmatrix} \\
&= \hat{k}(3.0 \cdot 4.0 - 5.0 \cdot 2.0) \\
&= \hat{k}(12.0 - 10.0) \\
&= \boxed{2.0\hat{k}}
\end{aligned}$$

Part b:

$$\begin{aligned}
\vec{a} \cdot \vec{b} &= 3.0 \cdot 2.0 + 5.0 \cdot 4.0 \\
&= 6.0 + 20.0 \\
&= \boxed{26.0}
\end{aligned}$$

Part c:

$$\begin{aligned}
(\vec{a} + \vec{b}) \cdot \vec{b} &= (3.0\hat{i} + 5.0\hat{j} + 2.0\hat{i} + 4.0\hat{j}) \cdot (2.0\hat{i} + 4.0\hat{j}) \\
&= (5.0\hat{i} + 9.0\hat{j}) \cdot (2.0\hat{i} + 4.0\hat{j}) \\
&= 10.0 + 36.0 \\
&= \boxed{46.0}
\end{aligned}$$

Part d:

$$\begin{aligned}
\text{comp}_a\vec{b} &= \frac{\vec{a} \cdot \vec{b}}{b} \\
&= \frac{26.0}{\sqrt{2.0^2 + 4.0^2}} \\
&= \boxed{\frac{26.0}{\sqrt{20.0}}}
\end{aligned}$$

Chapter 21 Problem 21

A nonconducting spherical shell, with an inner radius of 4.0 cm and an outer radius of 6.0 cm, has charge spread nonuniformly through its volume between its inner and outer surfaces. The volume charge density ρ is the charge per unit volume, with the unit coulomb per cubic meter. For this shell $\rho = b/r$, where r is the distance in meters from the center of the shell and $b = 3.0\mu C/m^2$. What is the net charge in the shell?

Solution

$$
\begin{aligned}
\rho &= \frac{dQ}{dV} \\
dQ &= \rho dV \\
\int dQ &= \int \rho dV \\
Q &= \int \frac{b}{r} dV \\
Q &= \int \frac{b}{r} 4\pi r^2 dr \\
&= 4\pi b \int_4^6 r dr \\
&= 4\pi b \left[\frac{r^2}{2}\right]_4^6 \\
&= 4\pi b \left(\frac{36}{2} - \frac{16}{2}\right) \\
&= 4\pi b\,(18 - 8) \\
&= 4\pi b \cdot 10 \\
&= 40\pi b \\
&= 40\pi(3.0 \times 10^{-6}) \\
&= \boxed{120\pi\ \mu\text{C}}
\end{aligned}
$$

Chapter 22 Problem 30

Figure 22-53 shows two concentric rings, of radii R and $R' = 3.00R$, that lie on the same plane. Point P lies on the central axis, at distance $D = 2.00R$ from the center of the rings. The smaller ring has uniformly distributed charge Q. In terms of Q, what is the uniformly distributed charge on the larger ring if the net electric field at P is zero?

Solution

$$\vec{E}_{net} = \vec{E}_R + \vec{E}'_R = 0$$

where $\vec{E}_R$ is the electric field due to the smaller ring and $\vec{E}'_R$ is the electric field due to the larger ring given by;

$$E_R = \frac{kQ}{R^2 + D^2}$$

and

$$E'_R = \frac{kQ'}{R'^2 + D^2}$$

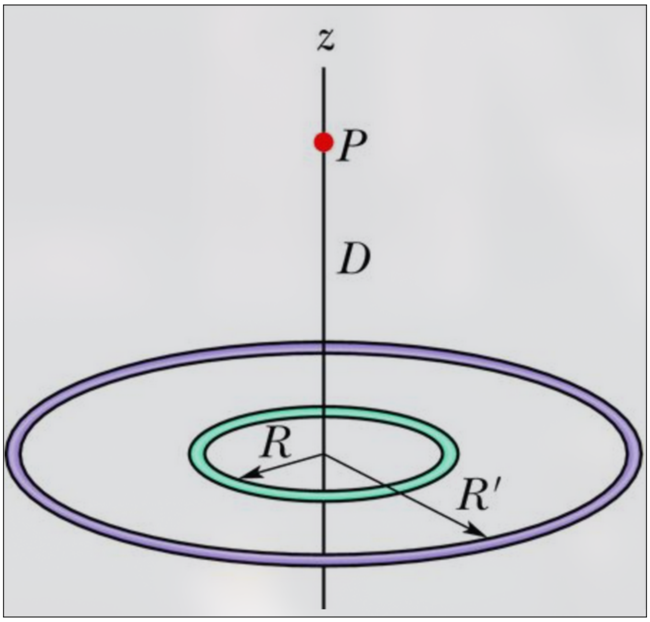

Figure 1: Figure 22-53

so we have;

$$
\begin{aligned}
\frac{kQ}{R^2 + D^2} &= -\frac{kQ'}{R'^2 + D^2} \\
Q' &= -Q\frac{R^2 + D^2}{R'^2 + D^2} \\
&= -Q\frac{R^2 + (2.00R)^2}{(3.00R)^2 + (2.00R)^2} \\
&= -Q\frac{R^2 + 4.00R^2}{9.00R^2 + 4.00R^2} \\
&= -Q\frac{5.00R^2}{13.00R^2} \\
&= -\frac{5.00}{13.00}Q \\
&= \boxed{-\frac{5}{13}Q}
\end{aligned}
$$

Chapter 23 Problem 42

Two large metal plates of area 1.0 m2 face each other, 5.0 cm apart, with equal charge magnitudes $|q|$ but opposite signs. The field magnitude E between them (neglect fringing) is 55 N/C. Find $|q|$.

Solution

The field from one of the plates is given by;

$$E = \frac{\sigma}{2\varepsilon_0}$$

and so we have;

$$\begin{aligned}
\vec{E}_{net} &= \vec{E}_1 + \vec{E}_2 \\
55 &= \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} \\
55 &= \frac{\sigma}{\varepsilon_0} \\
\sigma &= 55\varepsilon_0 \\
\frac{q}{A} &= 55\varepsilon_0 \\
q &= 55\varepsilon_0 A \\
&= 55(8.85 \times 10^{-12})(1.0) \\
&= 4.86 \times 10^{-10} C \\
&= \boxed{486 \text{ pC}}
\end{aligned}$$

Chapter 24 Problem 37

What is the magnitude of the electric field at the point $(3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}} + 4.00\hat{\mathbf{k}})$m if the electric potential in the region is given by $V = 2.00xyz^2$, where V is in volts and coordinates x, y, and z are in meters?

Solution

$$\begin{aligned}
\vec{E} &= -\nabla V \\
&= -\left(\frac{\partial V}{\partial x}\hat{\mathbf{i}} + \frac{\partial V}{\partial y}\hat{\mathbf{j}} + \frac{\partial V}{\partial z}\hat{\mathbf{k}}\right) \\
&= -\left(\frac{\partial}{\partial x}\left[2.00xyz^2\right]\hat{\mathbf{i}} + \frac{\partial}{\partial y}\left[2.00xyz^2\right]\hat{\mathbf{j}} + \frac{\partial}{\partial z}\left[2.00xyz^2\right]\hat{\mathbf{k}}\right) \\
&= -2.00yz^2\,\hat{\mathbf{i}} - 2.00xz^2\,\hat{\mathbf{j}} - 4.00xyz\,\hat{\mathbf{k}} \\
&= -2.00(-2)(4)^2\,\hat{\mathbf{i}} - 2.00(3)(4)^2\,\hat{\mathbf{j}} - 4.00(3)(-2)(-2)\,\hat{\mathbf{k}} \\
&= \boxed{64\,\hat{\mathbf{i}} - 96\,\hat{\mathbf{j}} + 48\,\hat{\mathbf{k}}}
\end{aligned}$$

Chapter 24 Problem 64

A hollow metal sphere has a potential of +400V with respect to ground (defined to be at $V = 0$) and a charge of 5.0×10^{-9} C. Find the electric potential at the center of the sphere.

Solution

Its a sphere, therefore the potential is the same everywhere.

$$\boxed{V = 400\text{V}}$$

Chapter 24 Problem 67

A metal sphere of radius 15 cm has a net charge of 3.0×10^{-8} C. (a) What is the electric field at the sphere's surface? (b) If $V = 0$ at infinity, what is the electric potential at the sphere's surface? (c) At what distance from the sphere's surface has the electric potential decreased by 500V?

Solution

Part a:

$$
\begin{aligned}
E &= \frac{kQ}{r^2} \\
&= \frac{(8.99\times10^{9})(3.0\times10^{-8})}{(0.15)^2} \\
&= \boxed{1.2\times10^{4}\ \text{N/C}}
\end{aligned}
$$

Part b:

$$
\begin{aligned}
V &= -\int_{\infty}^{r} E dr \\
&= -\int_{\infty}^{0.15} \frac{(8.99\times10^{9})(3.0\times10^{-8})}{r^2} dr \\
&= \frac{(8.99\times10^{9})(3.0\times10^{-8})}{0.15} \\
&= \boxed{1.8\times10^{3}\ \text{V}}
\end{aligned}
$$

Part c:

$$
\begin{aligned}
V &= -\int_{\infty}^{r} E dr \\
1.8\times10^{3} &= -\int_{\infty}^{r} \frac{(8.99\times10^{9})(3.0\times10^{-8})}{r^2} dr \\
&= \frac{(8.99\times10^{9})(3.0\times10^{-8})}{r} \\
&= \boxed{0.06\ \text{m}}
\end{aligned}
$$

Chapter 25 Problem 1

The two metal objects in Fig. 25-24 have net charges of +70 pC and −70 pC, which result in a 20 V potential difference between them. (a) What is the capacitance of the system? (b) If the charges are changed to +200 pC and −200 pC, what does the capacitance become? (c) What does the potential difference become?

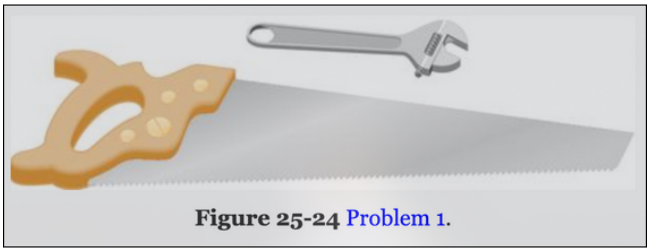

Figure 2: Figure 25-24

Solution

Part a:

$$
\begin{aligned}
C &= \frac{Q}{V} \\
&= \frac{70 \times 10^{-12}}{20} \\
&= \boxed{3.5 \times 10^{-11}\ \mathrm{F}}
\end{aligned}
$$

Part b:

$$
\begin{aligned}
C &= \frac{Q}{V} \\
&= \frac{200 \times 10^{-12}}{20} \\
&= \boxed{10 \times 10^{-11}\ \mathrm{F}}
\end{aligned}
$$

Part c:

$$
\begin{aligned}
V &= \frac{Q}{C} \\
&= \frac{200 \times 10^{-12}}{3.5 \times 10^{-11}} \\
&= \boxed{5.71\ \mathrm{V}}
\end{aligned}
$$