Chapter 2 - Motion Along a Straight Line

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2.1 Position, Displacement, and Average Velocity

key ideas

- The position x of a particle on an x axis locates the particle with respect to the origin
- The sign of the position indicates the direction the particle is located with respect to the origin
- Displacement Δx is the change in position of the particle

$$\Delta x = x_2 - x_1$$

- Average velocity is the displacement of a particle over the time interval

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

- The sign of v_{avg} indicates the direction of the motion with respect to the origin. Average velocity is not a function of distance traveled it is a function of initial and final position.
- On a graph of x and t, average velocity is the slope of the line connecting two points on the graph.
- Average speed, s_{avg} is the total distance travel over the time interval

$$s_{avg} = \frac{\text{total distance}}{\Delta t}$$

2.1.1 Motion

Kinematics is the classification and comparison of motions. Initially we restrict motion in a couple of ways to examine the most basic cases and get comfortable with the fundamental concepts of kinematics.

1. We are only considering motion in a straight line–Newton didn't have a conception of curved space time so we don't need one either.
2. The forces that cause motion are not considered, simply the motion itself and changes to motion.
3. A moving object is either a point-like particle such as an electron or something rigid enough to behave like one. I.e. the whole body of the object moves at the same rate.

2.1.2 Position and Displacement

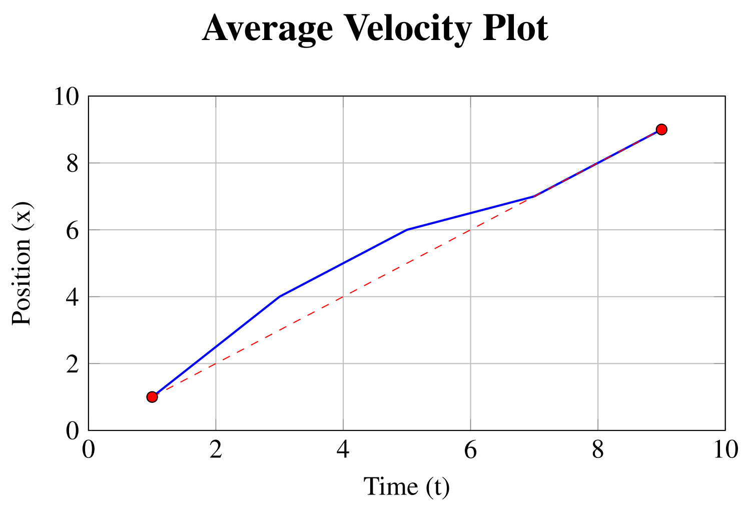
To locate an object means to find its position as a measure of length from the origin. In one dimension, movement to the right of the origin is positive and movement to the left of the origin is negative. The sign gives us a notion of direction and combined with the magnitude of the distance gives us our first vector quantity **displacement**.

Displacement, Δx, is not necessarily with respect to the origin it can be measured between any two points of an objects path of motion.

$$\Delta x = x_2 - x_1$$

2.1.3 Average Velocity and Average Speed

We can think of change in position as a graph of plotted as a function of position (x), and time (t).



Average Velocity: Average velocity, v_{avg}, is defined as the displacement (change in position) of an object over the time interval during which the displacement occurs:

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

Here, Δx is the displacement, and Δt is the time interval. The sign of v_{avg} indicates the direction of motion relative to the origin, making it a vector quantity. On a graph of position versus time, average velocity corresponds to the slope of the line connecting two points on the graph.

Average Speed: Average speed, s_{avg}, differs from average velocity as it considers the total distance traveled by the object rather than its displacement. It is defined as:

$$s_{avg} = \frac{\text{total distance}}{\Delta t}$$

Unlike average velocity, average speed is a scalar quantity and does not consider the direction of motion. It only accounts for how much ground an object has covered during the time interval.

- **Average velocity** is a vector and depends on the net displacement, considering direction.
- **Average speed** is a scalar and only considers the total distance covered, irrespective of direction.

2.2 Instantaneous Velocity and Speed

key ideas

- The instantaneous velocity of a particle, v is;

$$v = \lim_{t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

- If v_{avg} is the slope of the line between two points then it is relatively intuitive that the instantaneous velocity is the derivative of the function $x(t)$.
- Speed is the magnitude of v, $|v|$.

There really isn't much to this section beyond the key ideas beyond the importance of the fact that speed is agnostic to direction and we cannot derive velocity from speed because it is not a vector quantity, it has incomplete information.

2.3 Acceleration

key ideas

- Average acceleration is the ratio between Δv and Δt.

$$a_{avg} = \frac{\Delta v}{\Delta t}$$

- The sign indicates direction.
- Acceleration a is the derivative of velocity with respect to time.

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

The units for acceleration are generally $length/time^2$. Acceleration is responsible for sensations not velocity or speed. Sometimes these large changes in velocity are expressed in terms of g units.

$$1\ g = 9.8\ \mathrm{m/s^2}$$

Sign indicates direction of the acceleration **not** whether the particle is speeding up or slowing down. When the signs of v and a match, the object is speeding up when they do not, the object is slowing down.

2.4 Constant acceleration

There are a number of equations that we can use to solve problems with less information, if and only if acceleration is constant. Acceleration of 0 is still constant acceleration. We have a number of equations that are used to solve for a given variable;

- Velocity: $v = at + v_0$.
- Position: $x - x_0 = v_0 t + \frac{1}{2}at^2$
- Unknown time: $v^2 = 2a(x - x_0) + v_0^2$
- Unknown acceleration: $x - x_0 = \frac{v+v_0}{2}t$
- Unknown initial velocity: $x - x_0 = vt - \frac{at^2}{2}$

Refer to my derivations of constant acceleration equations for more details[1].

2.5 Free-fall Acceleration

Free-fall acceleration is one of the most impactful examples of motion in a straight line. The units g are derived from the rate of acceleration in free-fall. All objects in free-fall in a vacuum are subject to the same downward acceleration. We have mostly described motion in one dimension along an x-axis, i.e. a horizontal line. The axis for free-fall acceleration is the vertical y-axis with the negative direction being downward towards the earth's surface.

2.6 Graphical Integration in Motion Analysis

key ideas

- On a graph of a versus t, Δv is given by;

$$v_1 - v_0 = \int_{t_0}^{t_1} a\,dt$$

[1] https://github.com/kyroh/physics/blob/main/4A-Classical-Mechanics/derivations/constant-acceleration.pdf

- On a graph of v versus t, Δx is given by:

$$x_1 - x_0 = \int_{t_0}^{t_1} v dt$$

We know that velocity is the derivative of position with respect to time and that acceleration is the derivative of velocity with respect to time and so it should be relatively intuitive that by the fundamental theorem of calculus we can derive velocity from acceleration and position from velocity by integration.