

# Homework 1

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## Problem 1

The fuel consumption of cars is specified in Europe in terms of liters per 100 km. Convert 30 miles per gallon to this unit. Note that 1 gallon (U.S.) = 3.79 L.

### Solution

We are given 30 mi/gal and need to convert it to L/100km. The desired units are the reciprocal of the given units—volume of fuel per distance traveled compared to distance traveled per unit of fuel.

$$30 \frac{\text{mi}}{\text{gal}} = \frac{1}{30} \frac{\text{gal}}{\text{mi}}$$

Next, we use chain multiplication to convert miles to kilometers and gallons to liters:

$$\begin{aligned} \frac{1}{30} \frac{\cancel{\text{gal}}}{\text{mi}} \cdot \frac{3.79 \text{ L}}{1 \cancel{\text{gal}}} &= \frac{3.79}{30} \frac{\text{L}}{\text{mi}} \\ \frac{3.79}{30} \frac{\text{L}}{\cancel{\text{mi}}} \cdot \frac{1 \cancel{\text{mi}}}{1.609 \text{ km}} &= \frac{3.79}{48.27} \frac{\text{L}}{\text{km}} \\ \frac{3.79}{48.27} \frac{\text{L}}{\text{km}} &= 0.0785 \frac{\text{L}}{\text{km}} = 7.85 \frac{\text{L}}{100 \text{ km}} \end{aligned}$$

## Problem 2

Check the following equations for dimensional consistency where  $t$  is time (s),  $\nu$  is speed ( $\text{m s}^{-1}$ ),  $a$  is acceleration ( $\text{m/s}^2$ ), and  $x$  is position (m):

1.  $x = \frac{\nu^2}{2a}$
2.  $x = \frac{1}{2}at$
3.  $t = \sqrt{\frac{2x}{a}}$

### Solution

Checking the dimensional consistency involves verifying that the units balance out correctly on both sides of the equations.

**Equation 1:**  $x = \frac{\nu^2}{2a}$  The units of  $\nu^2$  and  $2a$  must cancel such that we are left with just meters (m):

$$\frac{(\text{m/s})^2}{\text{m/s}^2} = \frac{\text{m}^2}{\text{s}^2} \cdot \frac{\text{s}^2}{\text{m}} = \text{m}$$

Therefore, Equation 1 is dimensionally consistent.

**Equation 2:**  $x = \frac{1}{2}at$  We are again looking for position in meters (m):

$$\frac{\text{m}}{\text{s}^2} \cdot \text{s} \neq \text{m}$$

Equation 2 is not dimensionally consistent.

**Equation 3:**  $t = \sqrt{\frac{2x}{a}}$  Here, we are looking for time in seconds (s):

$$\sqrt{\frac{\text{m}}{\text{m/s}^2}} = \sqrt{\text{s}^2} = \text{s}$$

Equation 3 is dimensionally consistent.

## Problem 3

A can of paint that covers  $20.0 \text{ m}^2$  costs \$24.60. The walls of a room  $13.0 \text{ ft} \times 18.0 \text{ ft}$  are  $8.00 \text{ ft}$  high. What is the cost of paint for the walls?

### Solution

We need to start by deriving an equation that yields the number of cans of paint required to cover the surface area,  $A$ , as a function of the given dimensions.

$$A = 2h(x_1 + x_2) \text{ ft}^2$$

where  $x_1$  and  $x_2$  are the lengths of the respective edges and  $h$  is the height of the walls. If each can is capable of covering  $y \text{ m}^2$  then then it will cover  $3.28084y \text{ ft}^2$  and so our equation for  $n$  cans becomes;

$$n = \left\lceil \frac{2h(x_1 + x_2)}{3.28084y} \right\rceil = \left\lceil \frac{2 \cdot 8.00(13.0 + 18.0)}{3.28084 \cdot 20} \right\rceil = 3$$

3 cans at \$24.60 per can gives us a total cost of \$73.80 to paint the walls.

## Problem 4

Consider a race car on a  $5.00 \text{ km}$  track. Car A finishes the race in  $4.00 \text{ h}$  and is 1.50 laps ahead of B at this time. What is B's time for the race?

### Solution

We determine Car B's race time by finding how far it has traveled in the time it took Car A to finish the race and then determine the time it would have taken Car B to complete the full distance. Since Car B was  $7.5 \text{ km}$  behind Car A, Car B traveled  $292.5 \text{ km}$  in 4 hours. We can then solve for the time  $t_B$  it took Car B to finish the full  $300 \text{ km}$ :

$$\begin{aligned} \frac{292.5 \text{ km}}{4 \text{ h}} &= \frac{300 \text{ km}}{t_B \text{ h}} \\ t_B &= \frac{300 \text{ km} \cdot 4 \text{ h}}{292.5 \text{ km}} \approx 4.10 \text{ h} \end{aligned}$$

## Problem 5

On its maiden voyage in July 1952, the liner *United States* won the coveted Blue Ribbon for the fastest crossing of the Atlantic from New York to Cornwall, U.K. The trip took 3 days 10 hours 40 min at an average speed of 34.5 knots ( $65.5 \text{ km/h}$ ). This was 10 hours 2 minutes less than the 14-year-old record held by the *Queen Mary*. What was the average speed of the *Queen Mary*?

### Solution

The distance across the Atlantic is the same for both ships, so we have:

$$x = v_1 \cdot t_1 = v_2 \cdot t_2$$

Solving for  $v_2$ :

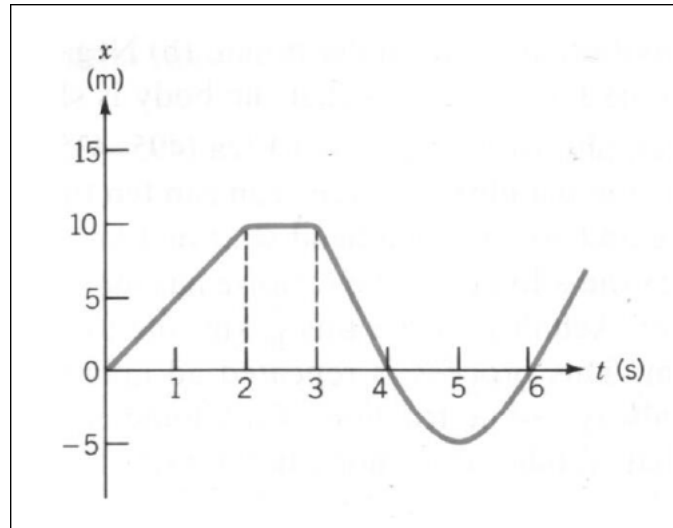
$$v_2 = \frac{v_1 \cdot t_1}{t_2}$$

Substituting the given values:

$$v_2 = \frac{65.5 \text{ km h}^{-1} \cdot 82.67 \text{ h}}{92.70 \text{ h}} \approx 58.41 \text{ km h}^{-1}$$

## Problem 6

Based on the graph below, estimate the instantaneous velocity at the following times: (a) 1.0 s; (b) 2.5 s; (c) 3.5 s; (d) 4.5 s; (e) 5.0 s;



## Solution

- (a)  $5 \text{ m s}^{-1}$
- (b)  $0 \text{ m s}^{-1}$
- (c)  $-10 \text{ m s}^{-1}$
- (d)  $-3 \text{ m s}^{-1}$
- (e)  $0 \text{ m s}^{-1}$

## Problem 7

At  $t = 2.25 \text{ s}$ , a particle is at  $x = 7.00 \text{ m}$  and has velocity  $v = 3.50 \text{ m s}^{-1}$ . At  $t = 7.00 \text{ s}$ , it is at  $x = -5.10 \text{ m}$  and has velocity  $v = 6.00 \text{ m s}^{-1}$ . Find: (a) its average velocity; (b) its average acceleration.

## Solution

The average velocity is defined as:

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

Substituting the values:

$$v_{\text{avg}} = \frac{-5.10 \text{ m} - 7.00 \text{ m}}{7.00 \text{ s} - 2.25 \text{ s}} = \frac{-12.10 \text{ m}}{4.75 \text{ s}} \approx -2.55 \text{ m s}^{-1}$$

The average acceleration is defined as:

$$a_{\text{avg}} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$$

Substituting the values:

$$a_{\text{avg}} = \frac{6.00 \text{ m s}^{-1} - 3.50 \text{ m s}^{-1}}{7.00 \text{ s} - 2.25 \text{ s}} = \frac{2.50 \text{ m s}^{-1}}{4.75 \text{ s}} \approx 0.53 \text{ m/s}^2$$

## Problem 8

The position of a particle is given by  $x = 4.50e^{-0.30t}$ , where  $x$  has units of meters. What is (a) the average velocity between 2.00 and 3.00 s; (b) the instantaneous velocity at 2.00 s; (c) the instantaneous acceleration at 2.00 s.

### Solution

#### Part a

The average velocity is the change in position over the change in time. First, find the positions at  $t = 2.00 \text{ s}$  and  $t = 3.00 \text{ s}$ :

$$x(3) = 4.50e^{-0.9}, \quad x(2) = 4.50e^{-0.6}$$

The average velocity is then:

$$v_{\text{avg}} = \frac{4.50e^{-0.9} - 4.50e^{-0.6}}{3 - 2} \approx -0.64 \text{ m s}^{-1}$$

#### Part b

The instantaneous velocity is the derivative of the position function:

$$\begin{aligned} v(t) &= \frac{d}{dt} (4.50e^{-0.30t}) \\ &= -0.30 \cdot 4.50e^{-0.30t} \\ &= -1.35e^{-0.30t} \end{aligned}$$

Evaluating at  $t = 2.00 \text{ s}$ :

$$v(2) = -1.35e^{-0.6} \approx -0.74 \text{ m s}^{-1}$$

#### Part c

The instantaneous acceleration is the derivative of the velocity function:

$$a(t) = \frac{d}{dt} (-1.35e^{-0.30t}) = 0.405e^{-0.30t}$$

Evaluating at  $t = 2.00 \text{ s}$ :

$$a(2) = 0.405e^{-0.6} \approx 0.25 \text{ m/s}^2$$