

Homework 1

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Problem 1

The fuel consumption of cars is specified in Europe in terms of liters per 100 km. Convert 30 miles per gallon to this unit. Note that 1 gallon (U.S.) = 3.79 L.

Solution

We are given 30 mi/gal to convert to L/100km. The desired units in some sense are the reciprocal of the given units—volume unit of fuel per distance unit traveled compared to distance unit traveled per unit of fuel.

$$30 \frac{\text{mi}}{\text{gal}} = \frac{1}{30} \frac{\text{gal}}{\text{mi}}$$

Then we use chain multiplication to convert miles to kilometers and gallons to liters;

$$\begin{aligned} \frac{1}{30} \frac{\cancel{\text{gal}}}{\text{mi}} \cdot \frac{3.79 \text{ L}}{1 \cancel{\text{gal}}} &= \frac{3.79 \text{ L}}{30 \text{ mi}} \\ \frac{3.79 \text{ L}}{30 \cancel{\text{mi}}} \cdot \frac{1 \cancel{\text{mi}}}{1.609 \text{ km}} &= \frac{3.79 \text{ L}}{48.27 \text{ km}} \\ \frac{3.79 \text{ L}}{48.27 \text{ km}} &= 0.0785 \frac{\text{L}}{\text{km}} = 7.85 \frac{\text{L}}{100 \text{ km}} \end{aligned}$$

Problem 2

Check the following for dimensional consistency where t is time (s), ν is speed (m/s), a is acceleration (m/s²), and x is position (m):

1. $x = \frac{\nu^2}{2a}$
2. $x = \frac{1}{2}at$
3. $t = \sqrt{\frac{2x}{a}}$

Solution

Checking the dimensional consistency effectively means checking the arithmetic of the units used for each quantity. Meaning that for example the first equation;

$$x = \frac{\nu^2}{2a}$$

the units of ν^2 and $2a$ must cancel such that we are left with just meters, m, and we find that they do;

$$\frac{(\text{m/s})^2}{\text{m/s}^2} = \frac{\text{m}^{\cancel{2}^1}}{\cancel{\text{s}}^2} \cdot \frac{\cancel{\text{s}}^2}{\cancel{\text{m}}} = \text{m}$$

For equation two, we are again looking for position, m, and find that it is not dimensionally consistent;

$$\frac{\text{m}}{\text{s}^2} \cdot \text{s} \neq \text{m}$$

For equation three, we are looking for time in seconds, s and find that it is dimensionally consistent;

$$\sqrt{\frac{\text{m}}{\text{m/s}^2}} = \text{m}^{1/2} \cdot \frac{\text{s}^{2 \cdot 1/2}}{\text{m}^{1/2}} = \text{s}$$

Problem 3

A can of paint that covers 20.0 m^2 costs \$24.60. The walls of a room $13.0 \text{ ft} \times 18.0 \text{ ft}$ are 8.00 ft high. What is the cost of paint for the walls?

Solution

If the room is $13 \text{ ft} \times 18 \text{ ft}$ then it is

$$13\cancel{\text{ft}} \cdot \frac{0.3048 \text{ m}}{1 \cancel{\text{ft}}} = 3.9624\text{m}$$

by 5.4864m with 2.4383m high walls and the total surface area to cover is the sum of the surface area of each wall.

$$\text{area} = 2 (3.9624\text{m} \cdot 2.4383\text{m}) + 2 (5.4864\text{m} \cdot 2.4383\text{m}) = 23.039\text{m}^2$$

and so we will need to buy 2 cans of paint to cover the walls costing us \$49.20. Suppose we are lucky and the store agrees to refund the remaining paint in the partially used can, meters painted per dollar spent;

$$\frac{24.60}{20} \frac{\$}{\text{m}^2} = 1.23 \frac{\$}{\text{m}^2}$$

and with 23.039m^2 to paint we have a cost of

$$23.039\text{m}^2 \cdot 1.23 \frac{\$}{\text{m}^2} = \$28.34$$

Problem 4