Lecture Notes

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1 Angular Kinematics Review

Angular position

$$\theta(t) = \frac{S}{r}$$

Angular velocity

$$\omega(t) = \frac{d\theta(t)}{dt}$$

units are just s^{-1} and angular acceleration is;

$$\alpha(t) = \frac{d\omega(t)}{dt} = \frac{d^2\theta(t)}{dt^2}$$

then tangential velocity is

$$\boxed{v_t = r\omega}$$

and centripetal/radial acceleration is

$$a_r = r\alpha \quad \text{or} \quad a_r = \frac{v^2}{r} = r\omega^2$$

2 Rotational Interia

Imagine a rod rotating counter clockwise; ω then would be positive, in reality $\vec{\omega}$, and the direction would be out of the page. Because we want to model with constant velocity we need a unique direction, therefore we define the direction of angular velocity as a cross product. Some notation

- Out of the page $\rightarrow \odot$
- Into the page $\rightarrow \otimes$

Rotational Kinetic Energy

In linear motion

$$K = \frac{1}{2}mv^2$$

For some infinitesimal section of the rod, the velocity would be

$$v_i = r_i\omega$$

so then the infinitesimal kinetic energy is;

$$K_i = \frac{1}{2}m_i v_i^2 = \frac{1}{2}m_i r_i^2 \omega_i^2$$

or

$$\sum_i K_i = \frac{1}{2}\omega^2 \sum_i m_i r_i^2$$

so we define rotational intertia;

$$I \coloneqq \sum_i m_i r_i^2 \tag{1}$$

Therefore rotational kinetic energy is given by

$$K = \frac{1}{2} I \omega^2$$

rotational inertia is how hard it is to get something rotating and if it is rotating how hard it is to get it to stop. Time to integrate;

$$\begin{aligned} I &= \sum_i m_i r) i^2 \\ &= \int r(m)^2 \, dm \end{aligned}$$

Some variables

$$\begin{aligned} \rho &= \frac{m}{V} \\ \sigma &= \frac{m}{A} \\ \lambda &= \frac{m}{L} \end{aligned}$$

so we have

$$\frac{M}{L} = \frac{dm}{dx} \implies dm = \frac{M}{L} dx$$

then we use that to construct

$$I = \frac{M}{L} \int_{-L/2}^{L/2} x^2 dx$$

basically use density to substitute *dm* in the integral and then integrate.