Homework 8

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Problem 1

A 0.315-kg particle moves from an initial position $\vec{\mathbf{r}}_1 = 2.00\,\hat{\mathbf{i}} - 1.00\,\hat{\mathbf{j}} + 3.00\,\hat{\mathbf{k}}$ m to a final position $\vec{\mathbf{r}}_2 = 4.00\,\hat{\mathbf{i}} - 3.00\,\hat{\mathbf{j}} - 1.00\,\hat{\mathbf{k}}$ m while a force $\vec{\mathbf{F}} = 2.00\,\hat{\mathbf{i}} - 3.00\,\hat{\mathbf{j}} + 1.00\,\hat{\mathbf{k}}$ N acts on it. What is the work done by the force on the particle?

Solution

The distance traveled by the particle, $\vec{\mathbf{d}}$, is equal to the difference in final and initial position

$$\begin{aligned}\vec{\mathbf{d}} &= \vec{\mathbf{r}}_2 - \vec{\mathbf{r}}_1 \\ &= (4.00 - 2.00)\ \hat{\mathbf{i}} + (-3.00 + 1.00)\ \hat{\mathbf{j}} + (-1.00 - 3.00)\ \hat{\mathbf{k}} \\ &= 2.00\,\hat{\mathbf{i}} - 2.00\,\hat{\mathbf{j}} - 4.00\,\hat{\mathbf{k}}\end{aligned}$$

Then work is the dot product of $\vec{\mathbf{F}}$ and $\vec{\mathbf{d}}$

$$W = \vec{\mathbf{F}} \cdot \vec{\mathbf{d}} = (2)(2) + (-2)(-3) + (-4)(1) = \boxed{6\text{ J}}$$

Problem 2

Compute the kinetic energy for each of the cases below. Through what distance would a 800-N force have to act to stop each object?
(a) A 150-g baseball moving at 40 m/s;
(b) a 13-g bullet from a rifle moving at 635 m/s;
(c) a 1500-kg Corvette moving at 250 km/h;
(d) a 1.8×10^5-kg Concorde airliner moving at 2240 km/h.

Solution

The kinetic energy is given by

$$k = \frac{1}{2}mv^2 \tag{1}$$

Then

$$k = Fd \implies d = \frac{k}{F}$$

Using these equations to solve for each part;

Part a:

$$k = \frac{1}{2}(0.150)(40)^2 = \boxed{120\text{ J}}$$

and

$$d = \frac{120}{800} = \boxed{0.15\text{ m}}$$

Part b:

$$k = \frac{1}{2}(0.013)(635)^2 = \boxed{2620.96 \text{ J}}$$

and

$$d = \frac{2620.96}{800} = \boxed{3.276 \text{ m}}$$

Part c:

$$k = \frac{1}{2}(1500)(69.44444)^2 = \boxed{3.616 \times 10^6 \text{ J}}$$

and

$$d = \frac{3\,616\,897}{800} = \boxed{4.521 \text{ km}}$$

Part d:

$$k = \frac{1}{2}(1.8 \times 10^5)(622.2222)^2 = \boxed{3.484 \times 10^{10} \text{ J}}$$

and

$$d = \frac{3.484 \times 10^{10}}{800} = \boxed{43.555 \times 10^3 \text{ km}}$$

Problem 3

Compute the kinetic energies for each of the following. What force would be required to stop each object in 1.00 km?
(a) The 8.00×10^7-kg carrier Nimitz moving at 55 km/h;
(b) a 3.4×10^5-kg Boeing 747 moving at 1000 km/h;
(c) the 270-kg Pioneer 10 spacecraft moving at 51,800 km/h.

Solution

Part a:

Kinetic energy is

$$k = \frac{1}{2}mv^2 = \frac{1}{2}(8.00 \times 10^7)(15.27778)^2 = \boxed{9.336 \times 10^9 \text{ J}}$$

and the force required to stop it is

$$F = \frac{9.336 \times 10^9 \text{ J}}{1000 \text{ m}} = \boxed{9.336 \times 10^6 \text{ N}}$$

Part b:

Kinetic energy is

$$k = \frac{1}{2}(3.4 \times 10^5)(277.7778)^2 = \boxed{1.312 \times 10^{10} \text{ J}}$$

and the force required to stop it is

$$F = \frac{1.312 \times 10^{10} \text{ J}}{1000 \text{ m}} = \boxed{13.117 \times 10^6 \text{ N}}$$

Part c:

Kinetic energy is

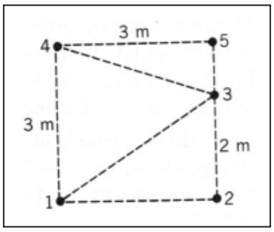
$$k = \frac{1}{2}(270)(14388.89)^2 = \boxed{2.795 \times 10^{10}\ \text{J}}$$

and the force required to stop it is

$$F = \frac{2.795 \times 10^{10}\ \text{J}}{1000\ \text{m}} = \boxed{27.950 \times 10^{6}\ \text{N}}$$

Problem 4

A 1.50-kg block is moved at constant speed in a vertical plane from position 1 to position 3 via several routes shown in the figure. Compute the work done by gravity on the block for each segment indicated, where W_{ab} means work done from a to b.
(a) $W_{1\ 3}$
(b) $W_{1\ 2} + W_{2\ 3}$
(c) $W_{1\ 4} + W_{4\ 3}$
(d) $W_{1\ 4} + W_{4\ 5} + W_{5\ 3}$



Solution

Part a:

Let $\vec{\mathbf{d}}$ be the path from position 1 to position 3, the angle between $\vec{\mathbf{F}}_g$ and $\vec{\mathbf{d}}$ is given by

$$\phi_g = 90 + \arctan\left(\frac{d_y}{d_x}\right)$$

then

$$W_{1\ 3} = (mg)(d)\cos\phi_g$$

substituting the given values values

$$W_{1\ 3} = (1.50)(9.81)(\sqrt{13})\cos\left(90 + \arctan\left(\frac{2}{3}\right)\right) = \boxed{-29.43\ \text{J}}$$

Part b:

The distance vector $\vec{\mathbf{d}}$ is orthogonal to $\vec{\mathbf{F}}_g$ for $W_{1\ 2}$, therefore that component is zero. For $W_{2\ 3}$, the angle between the $\vec{\mathbf{F}}_\mathbf{g}$ and $\vec{\mathbf{d}}$ is 180° so we have

$$W_{1\ 2} + W_{2\ 3} = 0 + (mg)(d)(-1) = (1.50)(9.81)(2)(-1) = \boxed{-29.43\ \text{J}}$$

Part c:

Similar to part a, we have;

$$\phi_g = 90 - \arctan\left(\frac{d_y}{d_x}\right)$$

for $W_{4\ 3}$, giving us

$$\begin{aligned} W_{1\ 4} + W_{4\ 3} &= (mg)(d_1)(-1) + (mg)(d_2)\left(cos\left(90 - \arctan\left(\frac{d_y}{d_x}\right)\right)\right) \\ &= (1.50 \cdot 9.81)(3)(-1) + (1.50 \cdot 9.81)(\sqrt{10})\left(cos\left(90 - \arctan\left(\frac{1}{3}\right)\right)\right) \\ &= \boxed{-29.43 \text{ J}} \end{aligned}$$

Part d:

This is similar to part b, except, for the third component of work, $\phi_g = 0$.

$$W_{1\ 4} + W_{4\ 5} + W_{5\ 3} = (mg)(d)(-1) + 0 + (mg)(d)(1) = (1.5)(9.81)(3)(-1) + (1.5)(9.81)(1)(1) = \boxed{-29.43 \text{ J}}$$

Problem 5

What is the work needed to lift 14.7 kg of water from a well 11.0 m deep. Assume the water has a constant upward acceleration of 0.700 m/s^2.

Solution

Let $\vec{\mathbf{F}}$ be the upward force acting on the bucket

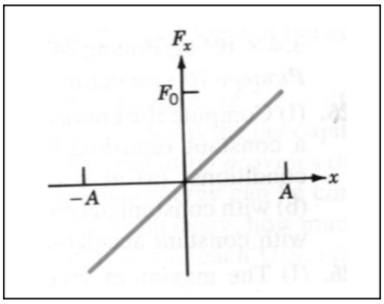
$$\vec{\mathbf{F}} = m\vec{\mathbf{a}}$$

then work W is given by

$$W = Fd\cos(0) = (14.7)(0.7)(11) = \boxed{113.19 \text{ J}}$$

Problem 6

The variation of a force with position is shown in the figure below. Find the work from (a) $x = 0$ to $x = -A$
(b) $x = +A$ to $x = 0$



Solution

Let the force function be $F(x) = x$ then the work is

$$W = \int x\,dx = \frac{x^2}{2} + C$$

so the work for $x = 0$ to $x = -A$ is given by

$$W = \left[\frac{x^2}{2}\right]_0^{-A} = \left[\frac{(-A)^2}{2} - \frac{0^2}{2}\right] = \boxed{\frac{(-A)^2}{2}}$$

Similarly for $x = +A$ to $x = 0$;

$$W = \left[\frac{x^2}{2}\right]_A^{0} = \left[\frac{0^2}{2} - \frac{A^2}{2}\right] = \boxed{-\frac{A^2}{2}}$$

Problem 7

Consider a particle on which several forces act, one of which is known to be constant in time: $\vec{\mathbf{F}}_1 = 3.00\,\hat{\mathbf{i}} + 4.00\,\hat{\mathbf{j}}$ N. As a result, the particle moves along a straight path from a Cartesian coordinate of (0.00 m, 0.00 m) to (5.00 m, 6.00 m). What is the work done by $\vec{\mathbf{F}}_1$?

Solution

Let $\vec{\mathbf{d}}$ be the distance traveled by the particle

$$\vec{\mathbf{d}} = 5.00\,\hat{\mathbf{i}} + 6.00\,\hat{\mathbf{j}}$$

then the work is the dot product of the distnace vector and the force vector;

$$W = \vec{\mathbf{F}} \cdot \vec{\mathbf{d}} = (3.00 \cdot 5.00) + (4.00 \cdot 6.00) = \boxed{39 \text{ J}}$$

Problem 8

A bungee cord exerts a nonlinear elastic force of magnitude $F(x) = k_1 x + k_2 x^3$, where x is the distance the cord is stretched, $k_1 = 204$ N/m and $k_2 = -0.233$ N/m^3. How much work must be done on the cord to stretch it 16.7 m?

Solution

We can calculate work as

$$\begin{aligned}
W &= \int_{x_1}^{x_2} k_1 x + k_2 x^3 \\
&= k_1 \int_{x_1}^{x_2} x\,dx \quad + \quad k_2 \int_{x_1}^{x_2} x^3\,dx \\
&= k_1 \left[\frac{x^2}{2}\right]_{x_1}^{x_2} \quad + \quad k_1 \left[\frac{x^4}{4}\right]_{x_1}^{x_2} \\
&= k_1 \left[\frac{x_2^2}{2} - \frac{x_1^2}{2}\right] + k_2 \left[\frac{x_2^4}{4} - \frac{x_1^4}{4}\right]
\end{aligned}$$

Substituting the given values we find;

$$W = 204\left[\frac{16.7^2}{2} - \frac{0^2}{2}\right] + -0.233\left[\frac{16.7^4}{4} - \frac{0^4}{4}\right] = \boxed{23\,916.116 \text{ J}}$$