CALIFORNIA INSITUTE OF TECHNOLOGY

Ph20 Assignment 4

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1 Part One

On the figure 1.1 it can be seen that using the initial conditions of position to be at x =1 and initial velocity 0, the evolution is a sinusoidal wave which increases in amplitude with time. Although the explicit Euler method is successful producing the overall shape, the fact that the amplitude changes is not the case for the analytic solution.

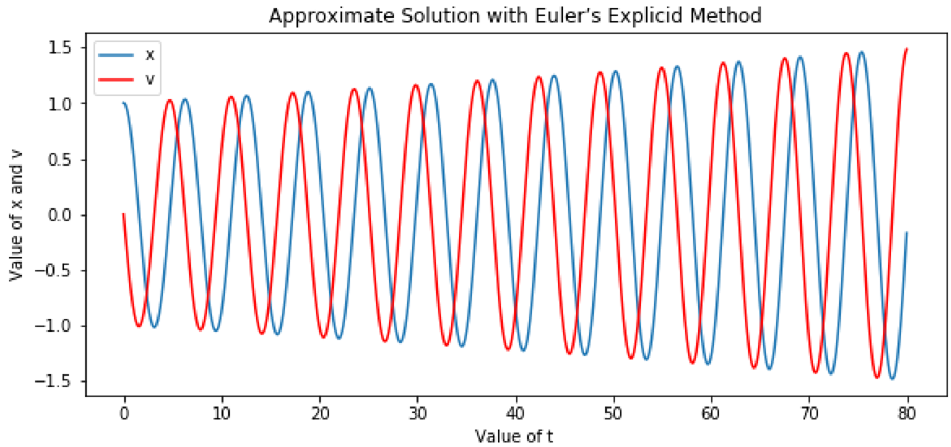


Figure 1.1: Explicit Euler's Method

Calculating the analytic solution with the same initial conditions and subtracting that from the numerical solution, the global error obtained. It can be seen in Figure 1.2 that the error (amplitude) increases as time goes by.

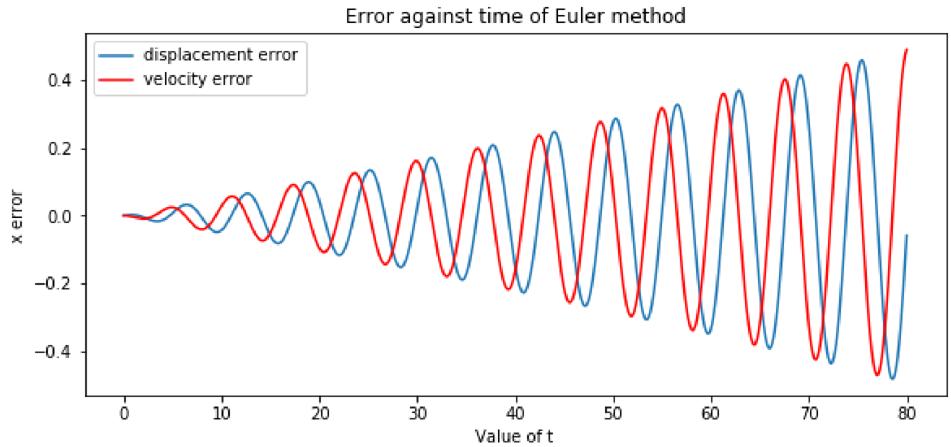


Figure 1.2: Global error of Euler Explicit method

Selecting specific values for the time step, specifically h, $\frac{h}{2}$, $\frac{h}{4}$, $\frac{h}{8}$, $\frac{h}{16}$, $\frac{h}{32}$, the truncation error was plotted to show a steady increase as the step size increased.

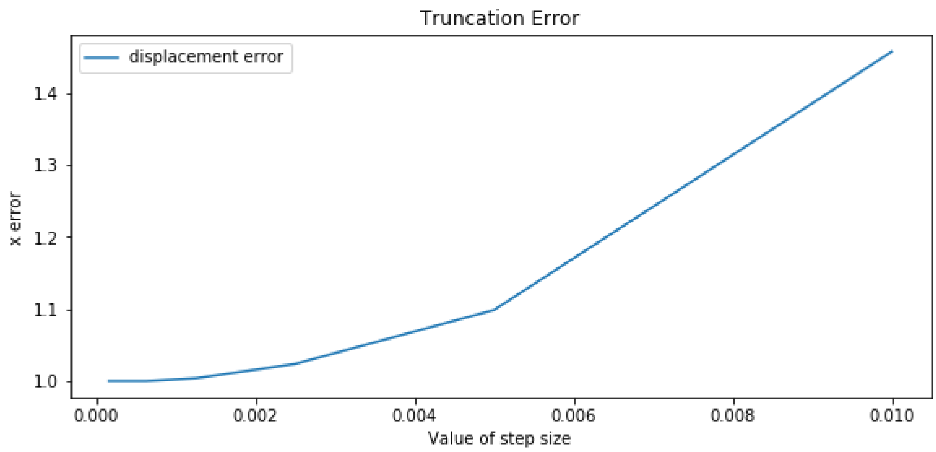


Figure 1.3: Truncation error for different step sizes

Plotting the energy using the numerical values of position and velocity, a steady increase can be observed as it goes through time. This does not make physical sense since the energy of the system should be conserved. The fact that the energy does not begin at zero makes sense, because the initial velocity was non-zero for this plot. Comparing it to the global error it can be seen that it deviates exponentially.

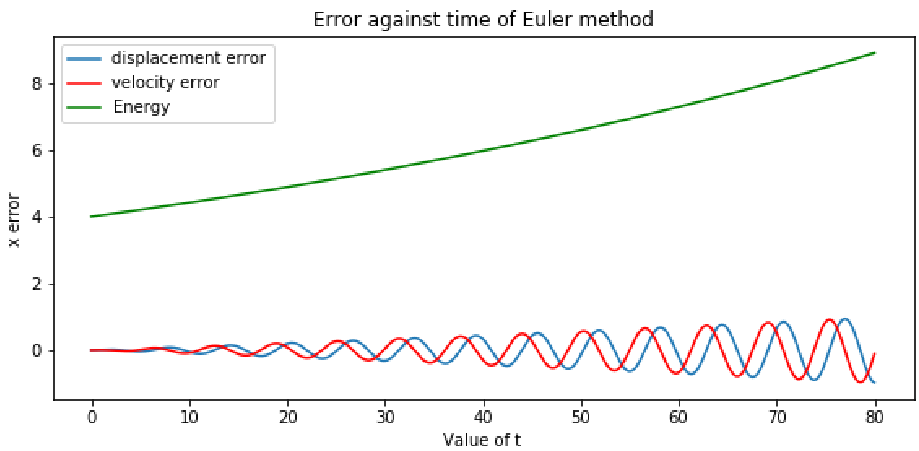


Figure 1.4: Energy with global errors using Explicit Euler Method

Using now the implicit Euler method, it can be seen that instead the Energy decreases with time. Note that the analytical solution would produce an energy that neither increases or decreases, since energy is conserved.

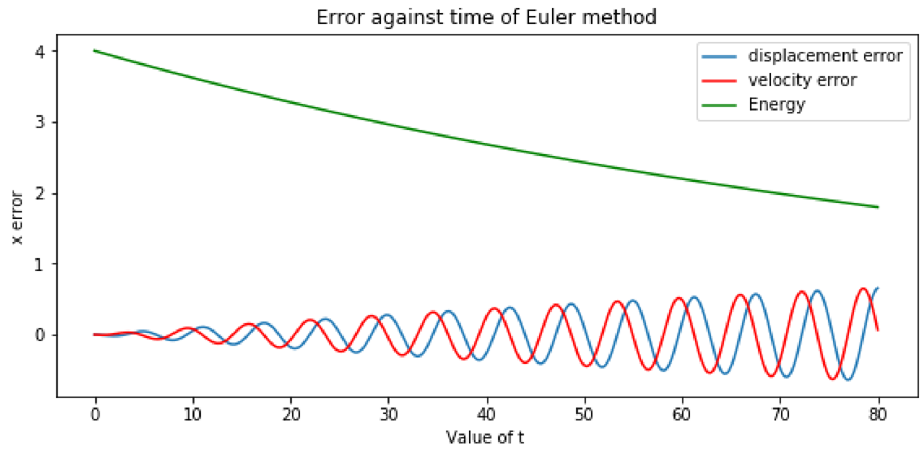


Figure 1.5: Energy with global errors using Implicit Euler Method

2 Part two

The phase space is plotted below to show the effect of the two Euler methods. While the analytic solution is a perfect circle (as it should be), the Explicit solution seems to spiral outwards and the Implicit solution seems to spiral inwards.

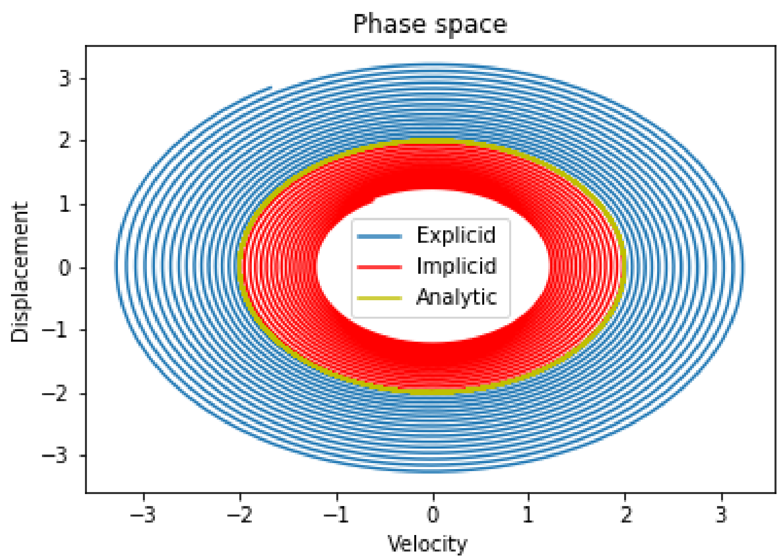
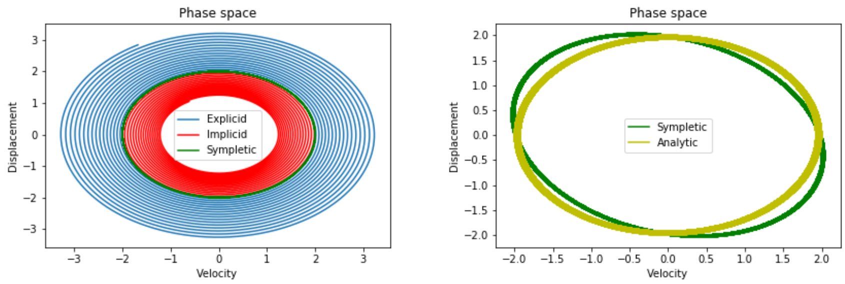


Figure 2.1: Phase space of numerical methods and Analytic solution

Using the same h, and implementing the Symplectic Euler method, it can be seen that it produces a perfect circle perfectly agreeing with the analytic solution.



(a) Phase space of all methods

(b) Phase space Sympletic & Analytic

Comparing the Symplectic solution of the phase space with the Analytic solution, it can be observed in the above figure that there is a shift from the analytical solution. It is neither a spiral in or out, but instead it deviates from the initial circle-shape.

Comparing the symplectic with the analytic solution it seems that they are identical for small intervals, as seen from figure 2.3. Increasing the interval however it can be seen that the deviations start to be apparent (figure 2.4).

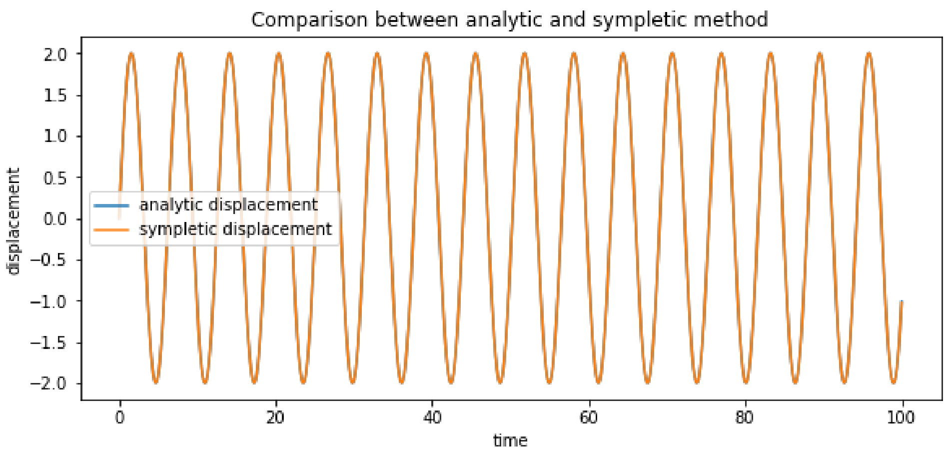


Figure 2.3: Comparison of Analytic and Symplectic method

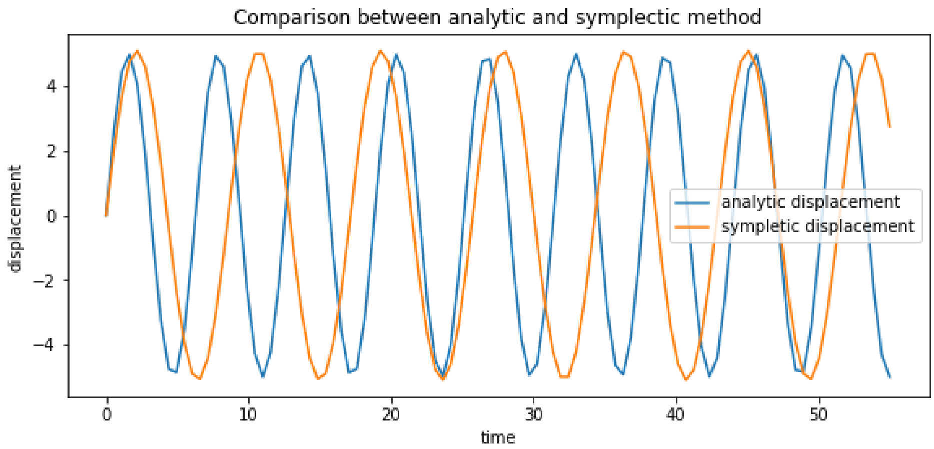


Figure 2.4: Comparison of Analytic and Symplectic method (large h)

Observing now how the error of the Symplectic method propagates for different initial conditions, it can be seen from the plots below that the Energy fluctuates (but it is conserved), and there is a phase shift between them corresponding to different initial conditions.

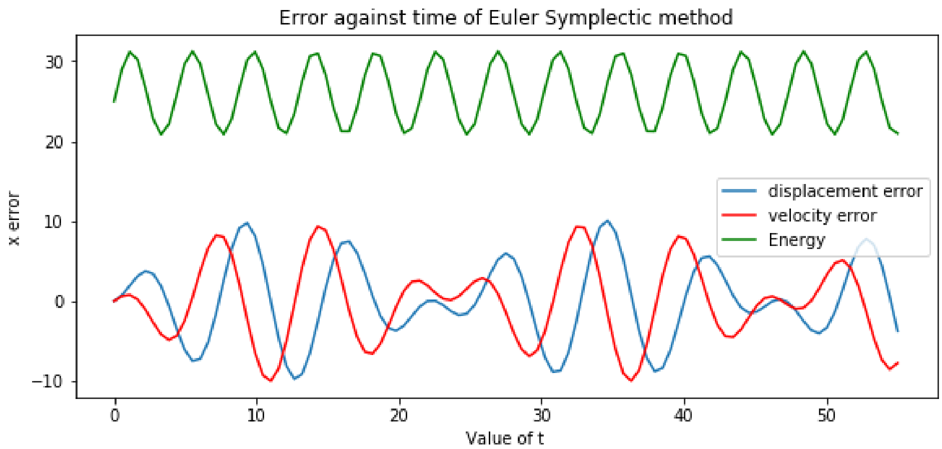


Figure 2.5: Error of Symplectic Method together with energy. Initial conditions: x = 5 v = 0

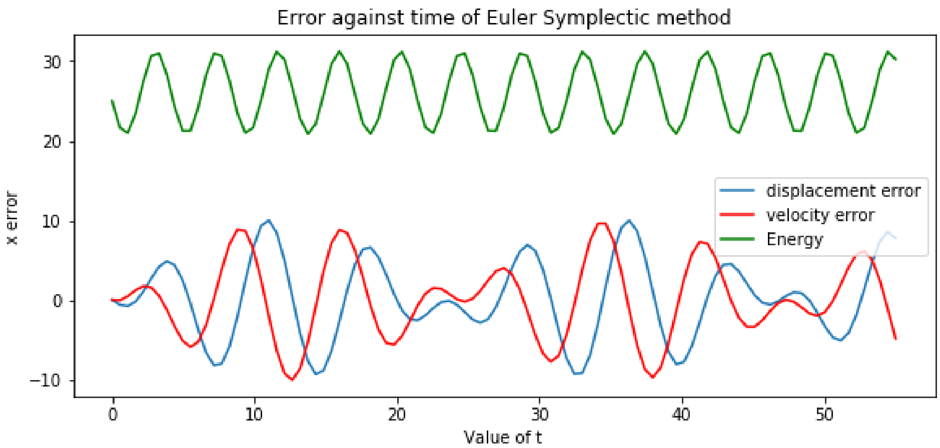


Figure 2.6: Error of Symplectic Method together with energy. Initial conditions: x = 0 v = 5