Spacecraft in a non-spherical gravitational field

During Apollo missions, it was noted that the trajectory of a spacecraft circling the Moon would deviate from a theoretical ellipsoid orbit strongly enough to require unplanned course corrections. In this checkpoint you will investigate in a simple 2d model how the presence of a single mass concentration ("mascon") below the Moon's surface could cause such deviations.

What will be given in a Jupyter notebook:

- mass of the Moon and its radius,
- initial position and velocity of the spacecraft,
- equations of motion in a radially symmetric gravitational field

Tasks to do:

Task	Points
1. Plot a trajectory of the spacecraft for $t = 0 \ldots 24$h assuming radially symmetric gravity. The trajectory should be a closed curve (what kind of curve?), with multiple revolutions. Don't forget to label the axes. You will get extra points if you make an animation of the spacecraft's trajectory instead of a static plot.	25 + 5 for the animation
2. Determine the orbital period T. Your solution must be calculated numerically, i.e., not simply using the known analytical expression. The obtained value must be within +/-1 s of the correct value.	15
3. Now add a correction that makes the field non-spherical. The correction rotates with the Moon (one full rotation every $T_{Moon} = 27.3$ days). How long does it take until the spacecraft hits the Moon? The time must be accurate to +/-1 s. Assume the Moon's surface is a sphere and use the following equations of motion: $$\frac{d^2x}{dt^2} = -\frac{GMx}{(x^2+y^2)^{3/2}} - \frac{qGM\,x'}{(x'^2+y'^2)^{3/2}}$$ $$\frac{d^2y}{dt^2} = -\frac{GMy}{(x^2+y^2)^{3/2}} - \frac{qGM\,y'}{(x'^2+y'^2)^{3/2}}$$ where $q = 0.00025$, and $$x' = x + 0.8R\cos\left(\frac{2\pi t}{T_{Moon}}\right),\ y' = y + 0.8R\sin\left(\frac{2\pi t}{T_{Moon}}\right)$$ Also, make a plot of the height of the spacecraft above the Moon's surface as a function of time. Don't forget to label axes and include units.	20
4. Which coordinate (x or y) of the position of the spacecraft after one revolution (orbital period T from task 2) is more sensitive to small changes in the amplitude of the correction? To answer this, calculate the derivatives of $dx/dq, dy/dq$ at $t = T$, for $q = 0$.	10
5. Given three positions of the spacecraft at $t = 0, t = T/2$, and $t = T$ (see the Jupyter notebook), determine the amplitude of the correction q assuming this is not known. Mind that the correct value of q may now be different to that assumed in task 3. The answer must be accurate to 10%.	10
6. What is the minimum initial height of a circular orbit (orbit centre = Moon's centre of mass) such that, for the perturbation from task 3, the spacecraft does not collide with the Moon but remains gravitationally bound to it? The answer must be accurate to +/-1 km.	15

The mark for the checkpoint is the sum of points for all tasks. What will be marked: (i) numerical correctness, (ii) efficiency i.e. how long it takes to execute the code (it shouldn't take more than a few minutes), (iii) coding style.

The Jupyter notebook provides the most comprehensive set of instructions for the task. In the case of discrepancy between this document and the Jupyter notebook, please follow instructions as specified in the Jupyter notebook.