Physics beyond the Standard Model

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Yukawa interaction

Higgs doublet $H = \begin{pmatrix} G^+ \\ v + \frac{h^0 + iG^0}{\sqrt{2}} \end{pmatrix}$ with $v = 174\,\mathrm{GeV}$.

Charge-conjugate doublet: $\widetilde{H} = \begin{pmatrix} v + \frac{h^0 - iG^0}{\sqrt{2}} \\ -G^- \end{pmatrix}$

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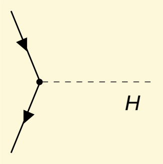

Yukawa lagrangian:

$$-L_Y = Y^d_{jk}\, \overline{Q}^j_L\, H\, d^k_R + Y^u_{jk}\, \overline{Q}^j_L\, \widetilde{H}\, u^k_R + Y^l_{jk}\, \overline{L}^j_L\, H\, e^k_R + \text{h.c.}$$

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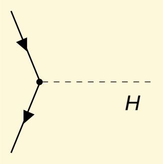

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Here neutrinos are (still) massless.
The Yukawa matrices Y^f are arbitrary complex 3×3 matrices.
The mass matrices $M^f = Y^f v$ are not diagonal!

$\Rightarrow$ $u^j_{L,R}, d^j_{L,R}$ do not describe physical quarks!
We must find a basis in which Y^f is diagonal!

Any matrix can be diagonalised by a bi-unitary transformation. Start with

$$\widehat{Y}^u = S_Q^\dagger Y^u S_u \quad \text{with } \widehat{Y}^u = \begin{pmatrix} y_u & 0 & 0 \\ 0 & y_c & 0 \\ 0 & 0 & y_t \end{pmatrix} \quad \text{and } y_{u,c,t} \geq 0$$

This can be achieved via

$$Q_L^j = S_{jk}^Q Q_L^{k\prime}, \qquad u_R^j = S_{jk}^u u_R^{k\prime}$$

with unitary 3×3 matrices S^Q, S^u.
This transformation leaves L_{gauge} invariant (“flavour-blindness of the gauge interactions”)!

Next diagonalise Y^d:

$$\widehat{Y}^d = V^\dagger S_Q^\dagger Y^d S_d \quad \text{with } \widehat{Y}^d = \begin{pmatrix} y_d & 0 & 0 \\ 0 & y_s & 0 \\ 0 & 0 & y_b \end{pmatrix} \quad \text{and } y_{d,s,b} \geq 0$$

with unitary 3×3 matrices V, S^d.
Via $d_R^j = S_{jk}^d d_R^{k\prime}$ we leave L_{gauge} unchanged, while

$$-L_Y^{\text{quark}} = \overline{Q}_L V \widehat{Y}^d H \, d_R \; + \; \overline{Q}_L \widehat{Y}^u \widetilde{H} \, u_R \; + \; \text{h.c.}$$

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This breaks up the $SU(2)$ doublet Q_L. $\Rightarrow$ L_{gauge} changes!

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The transformation $d_L^j = V_{jk} d_L^{k\prime}$ changes the W-boson couplings in L_{gauge}:

$$L_W = \frac{g_2}{\sqrt{2}} \left[\overline{u}_L V \gamma^\mu d_L W_\mu^+ + \overline{d}_L V^\dagger \gamma^\mu u_L W_\mu^- \right]$$

The Z-boson couplings stay flavour-diagonal because of $V^\dagger V = 1$.

V is the Cabibbo-Kobayashi-Maskawa (CKM) matrix.

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

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Leptons: Only one Yukawa matrix Y^l; the mass matrix $M^l = Y^l v$ of the charged leptons is diagonalised with

$$L_L^j = S_{jk}^L L_L^{k\prime}, \qquad e_R^k = S_{jk}^e e_R^{k\prime}$$

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No lepton-flavour violation!

$\Rightarrow$ Add a ν_R to the SM to mimick the quark sector or add a Majorana mass term $Y^M \frac{\bar{L}HH^T L^c}{M}$.

The lepton mixing matrix is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix.

Methodology of new physics searches

Renormalisability, power counting and decoupling:

The Standard-Model Lagrangian is renormalisable by power counting, meaning that it has no interaction terms beyond mass dimension 4.

Counting rule:

Lagrangian: $[\mathcal{L}] = 4$

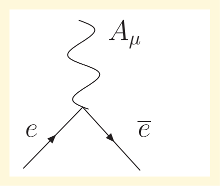
fermion field: $[\psi] = 3/2$

boson filed: $[A_\mu] = [\phi] = 1$

Example:

Photon-electron coupling $\overline{e}\gamma^\mu e A_\mu g_e$:

$$2[e] + [A_\mu] = 2\frac{3}{2} + 1 = 4 \quad \Rightarrow \quad [g_e] = 0$$



In a renormalisable theory one can systematically calculate quantum corrections (loop corrections); the results are applicable to arbitarily high energies.

If a theory has couplings with negative dimension, one can still calculate quantum corrections, but the results are only meaningful for energies well below the mass scale inferred from these couplings.

If the Standard Model is an effective theory superseded by a more complete theory at higher energies, higher-dimension terms are perfectly allowed. Such terms arise automatically, if very heavy, yet undiscovered particles are present: At low energies the interaction mediated by such a particle appears point-like, e.g.

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Famous example: Fermi theory valid for $p^2 \ll M_W^2$ and $G_F \propto 1/M_W^2$ has dimension -2.

Decoupling theorem of (Appelquist and Carazzone), applied to the Standard Model: If the Standard-Model Lagrangian is complemented by couplings to a new field with mass $M \gg v$, then all physical effects of this new field measurable at energies below M are encoded in higher-dimensional (i.e. dimension-5 and dimension-6) operators added to to the Standard-Model Lagrangian.

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The decoupling theorem does not hold, if masses are increased by increasing a coupling constant! A sequential fourth fermion generation involves masses proportional to Yukawa couplings, $m_{f'} = y_{f'} v$, and in a loop process involving $y_{f'}$ the increase of $y_{f'}$ can compensate for the $1/m_{f'}$ suppression of the loop integral!

Where to look for new physics?

Accidental features:

Properties of the Standard Model, which are not the consequences of the gauge symmetry $SU(3) \times SU(2) \times U(1)_Y$, are vulnerable to effects of new physics:

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i) Effects of the chosen particle content of the Standard Model, in particular accidental symmetries,

ii) Transition amplitudes which are suppressed, because certain Standard-Model parameters are small. The smallness of parameters is often linked to approximate global symmetries.

Examples for i):

The Standard Model possesses only one Higgs doublet.

- Adding more Higgs doublets leads to flavour-changing neutral Higgs couplings like $\bar{s}dH^{0'}$ with dramatic consequences: Flavour-changing neutral current (FCNC) processes like $\mathrm{K}-\overline{\mathrm{K}}$ mixing, which only occur at loop level in the Standard Model, can now appear at tree-level: Enhancement factor of $16\pi^2 M_W^2/M_{\mathrm{new}}^2$.

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- Adding Higgs fields in other SU(2) representations (e.g. adding a Higgs triplet) will spoil the tree-level relation
$$1 = \frac{M_W^2}{M_Z^2 \cos\theta_W} = \frac{M_W^2}{M_Z^2\sqrt{1 - g_Y^2/g_2^2}}$$
by new tree-level contributions. (Loop corrections occur already in the Standard Model.)

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  The Standard Model has failed this test, because neutrinos have been found to oscillate!

Examples for ii):

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- Only one Yukawa coupling is large, $y_t \approx 1$. especially flavour-changing elements of the Yukawa matrices are small, exhibiting approximate flavour symmetries.

  $\Rightarrow$ FCNC processes are excellent for new physics searches.