

Supplementary Material for the Paper: “Deterministic Multicast Using Temporal Graph-based Routing and Scheduling in Non-terrestrial Networks”

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In this document, we present the detailed derivations for **Lemma 1**, **Application**, **Theorem 1**, and **Theorem 2**.

APPENDIX A PROOF OF **Lemma 1**

We verify **Lemma 1** using proof by contradiction. Assume that the task data is transmitted along a loop consisting of the links (v, w) , (w, r) , (r, p) , \dots , (q, u) and (u, v) . According to Equation (10), we can obtain: $L(w) = (h_{v,w} - 1) \cdot |\tau| + l_{v,w}^{h_{v,w}}$, $L(r) = (h_{w,r} - 1) \cdot |\tau| + l_{w,r}^{h_{w,r}}$, $L(p) = (h_{r,p} - 1) \cdot |\tau| + l_{r,p}^{h_{r,p}}$, \dots , $L(u) = (h_{q,u} - 1) \cdot |\tau| + l_{q,u}^{h_{q,u}}$ and $L(v) = (h_{u,v} - 1) \cdot |\tau| + l_{u,v}^{h_{u,v}}$, where $h_{v,w}$, $h_{w,r}$, $h_{r,p}$, \dots , $h_{q,u}$ and $h_{u,v}$ represent the cycles within which task data transmitted along different links, and $l_{v,w}^{h_{v,w}}$, $l_{w,r}^{h_{w,r}}$, $l_{r,p}^{h_{r,p}}$, \dots , $l_{q,u}^{h_{q,u}}$ and $l_{u,v}^{h_{u,v}}$ denote the delay of these links. Since link delays are nonnegative, it follows that: $(h_{v,w} - 1) \cdot |\tau| < L(w)$, $(h_{w,r} - 1) \cdot |\tau| < L(r)$, $(h_{r,p} - 1) \cdot |\tau| < L(p)$, \dots , $(h_{q,u} - 1) \cdot |\tau| < L(u)$ and $(h_{u,v} - 1) \cdot |\tau| < L(v)$. According to Inequality (12), $L(v) \leq (h_{v,w} - 1) \cdot |\tau|$, $L(w) \leq (h_{w,r} - 1) \cdot |\tau|$, $L(r) \leq (h_{r,p} - 1) \cdot |\tau|$, \dots , $L(q) \leq (h_{q,u} - 1) \cdot |\tau|$, and $L(u) \leq (h_{u,v} - 1) \cdot |\tau|$ hold. Combining the above inequality groups, we derive two key results:

- On one hand, $h_{v,w} < h_{w,r}$, $h_{w,r} < h_{r,p}$, \dots , and $h_{q,u} < h_{u,v}$ hold, indicating that $h_{v,w} < h_{u,v}$.
- On the other hand, $(h_{u,v} - 1) \cdot |\tau| < L(v)$ and $L(v) \leq (h_{v,w} - 1) \cdot |\tau|$ hold, implying that $h_{u,v} < h_{v,w}$.

Since the two results are contradictory, the initial assumption does not hold. Therefore, we conclude that the task data transmission is loop-free. The proof is thus complete.

APPENDIX B APPLICATION OF ALGORITHM 1

As illustrated in Fig. 1(a), the input of **Algorithm 1** consists of the pruned and enhanced time-expanded graph (TEG) \mathcal{G} and a time-critical (TC) multicast task denoted as $m = \langle s, \{d_1, d_2\}, 33.33\text{ms}, 0.3\text{Mb}, 1\text{ms}, 11\text{ms} \rangle$. The detailed procedure of **Algorithm 1** is described as follows:

- **Initialization process.** The desired time-featured (TF) tree \mathcal{T} is initialized as \emptyset . The “pend-capacity” $C(\cdot)$ is set to ∞ for s^1 and to 0 for all other nodes. The “arrival-time” $L(\cdot)$ is initialized to 1ms for s^1 and to ∞ for all other nodes. In addition, the “pre-node” $P(\cdot)$ of each node is set to NULL, and the priority queue Q is initialized to contain the entire node set \mathcal{V} .
- **Search process.** In **Iteration 1**, node s^1 is extracted from Q due to its maximal “pending-capacity”, $C(s^1) = \infty$. Since the storage edge (SE) (s^1, s^2) is the only outgoing edge from s^1 , the parameters of s^2 are updated as $C(s^2) = \infty$, $L(s^2) = 2\text{ms}$, and $P(s^2) = s^1$ (steps 13–15). In **Iteration 2**, node s^2 is popped from Q . Considering the transmission edge (TE) (s^2, w^2) from s^2 , the arrival cycle number of the task data transmitted along it is determined as $I(w) = 3$. Consequently, the actual arrival node through cross-cycle transmission is w^3 , whose “pending-capacity” is improved from 0 to 2Mb, together with updated parameters $L(w^3) = 5\text{ms}$ and $P(w^3) = s^2$ (steps 6–11). Furthermore, by checking the SE (s^2, s^3) , we obtain $C(s^3) = \infty$, $L(s^3) = 4\text{ms}$, and $P(s^3) = s^2$. During **Iterations 3–6**, nodes s^3 , s^4 ($C(s^4) = \infty$, $L(s^4) = 6\text{ms}$, $P(s^4) = s^3$), s^5 ($C(s^5) = \infty$, $L(s^5) = 8\text{ms}$, $P(s^5) = s^4$), and s^6 ($C(s^6) = \infty$, $L(s^6) = 10\text{ms}$, $P(s^6) = s^5$) are sequentially selected. In **Iteration 9**, node w^3 is extracted from Q . Due to the violation of the *forwarding time constraint* (step 6), the two TEs, (w^3, v^3) and (w^3, d_1^3) , are not visited. However, through TE (w^3, w^4) , the parameters of w^4 are updated to $C(w^4) = 2\text{Mb}$, $L(w^4) = 6\text{ms}$, and $P(w^4) = w^3$. In **Iteration 10**, the TE (w^4, v^4) is first examined, yielding $C(v^5) = 2\text{Mb}$, $L(v^5) = 9\text{ms}$, and $P(v^5) = w^4$. Subsequently, the TE (w^4, d_1^4) is checked, obtaining $C(d_1^4) = 1\text{Mb}$, $L(d_1^4) = 7\text{ms}$, and $P(d_1^4) = w^4$. Finally, the SE (w^4, w^5) is examined, resulting in $C(w^5) = 2\text{Mb}$, $L(w^5) = 8\text{ms}$, and $P(w^5) = w^4$. In **Iteration 11**, node v^5 is selected as the relay, updating the parameters of v^6 to $C(v^6) = 2\text{Mb}$, $L(v^6) = 10\text{ms}$, and $P(v^6) = v^5$. In **Iteration 12**, node v^6 is popped, and its unique outgoing edge (v^6, d_2^6) is checked, resulting in $C(d_2^6) = 1\text{Mb}$, $L(d_2^6) = 11\text{ms}$, and $P(d_2^6) = v^6$. In **Iteration 13**, node w^5 with $C(w^5) = 2\text{Mb}$ is extracted from Q . It is determined that the “pending-capacity” of v^6 cannot be further improved through cross-cycle transmission along TE (w^5, v^5) (step 8). In contrast, the parameters of w^6 are updated to $C(w^6) = 2\text{Mb}$, $L(w^6) = 10\text{ms}$, and $P(w^6) = w^5$. Subsequently, w^6 is selected in **Iteration 14** without triggering any parameter updates for other nodes. In **Iterations 15 and 16**, nodes d_1^4 and d_2^6 are selected, producing the corresponding virtual nodes d_1' and d_2' with parameters $C(d_1') = 1\text{Mb}$, $L(d_1') = 7\text{ms}$, $P(d_1') = d_1^4$; and $C(d_2') = 1\text{Mb}$,

$L(d'_2) = 11\text{ms}$, $P(d'_2) = d_2^6$, respectively. In the **final two iterations**, nodes d'_1 and d'_2 are extracted from \mathcal{Q} , indicating that their “pending-capacities” have been maximized, and the **search process** terminates.

- **Construction process.** Since both $C(d'_1) > 0$ and $C(d'_2) > 0$ are satisfied, the final \mathcal{T} can be constructed through backtracking (steps 21-31). The resulting tree is $\mathcal{T} = (s^1, s^2), (s^2, w^2), (w^3, w^4), (w^4, d_1^4), (w^4, v^4), (v^5, v^6), (v^6, d_2^6)$, as illustrated in Fig. 1(b).

Task: $m = \langle s, \{d_1, d_2\}, 33.33\text{ms}, 0.3\text{Mb}, 1\text{ms}, 1\text{ms} \rangle$

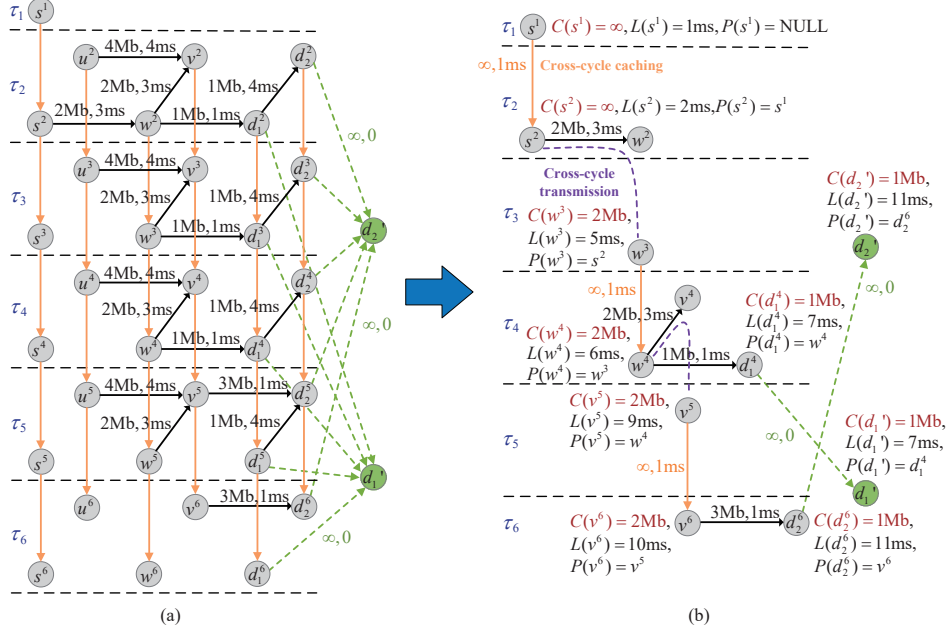


Fig. 1. (a) The input of the proposed algorithm; (b) the constructed TF tree.

APPENDIX C

PROOF OF THEOREM *Theorem 1*

For **Algorithm 1**, the key is to demonstrate that each node extracted from \mathcal{Q} has determined its maximum “pending-capacity” $C(\cdot)$. This assertion remains valid for s^h with $C(s^h) = \infty$. Moving on to the K -th extracted node, denoted as u^h (or d'_n), we identify that its “pending-capacity” cannot be further improved by relaying via any node out of \mathcal{Q} (steps 8 to 9, 13 to 14, and 16 to 17). Regarding potential relaying via a node in \mathcal{Q} , we adopt proof by contradiction for analysis, accounting for three cases:

- If reaching u^h via $w^i \in \mathcal{Q}$ along $(w^i, u^i) \in \mathcal{E}_t$ enables $C(u^h) < \min \{C(w^i), c_{w,u}^i\} \leq C(w^i)$, where $h = \lceil \frac{1}{|\mathcal{T}|} \cdot ((i-1) \cdot |\mathcal{T}| + l_{w,u}^i) \rceil$, a contradiction arises because $C(w^i) \leq C(u^h)$ is enforced by step 4;
- If reaching u^h via $u^{h-1} \in \mathcal{Q}$ along $(u^{h-1}, u^h) \in \mathcal{E}_s$ enables $C(u^h) < \min \{C(u^{h-1}), c_{u,u}^h\} = C(u^{h-1})$, a contradiction also arises due to $C(u^{h-1}) \leq C(u^h)$;
- If reaching d'_n via $d_n^i \in \mathcal{Q}$ along $(d_n^i, d'_n) \in \mathcal{E}_v$ enables $C(d'_n) < \min \{C(d_n^i), c_{d_n,d'_n}^i\} = C(d_n^i)$, it encounters a contradiction with $C(d_n^i) \leq C(d'_n)$.

Therefore, the “pending-capacity” of the K -th extracted node cannot be improved.

Intuitively, the bottleneck capacity of the output TF tree \mathcal{T} , calculated as $C(\mathcal{T}) = \min_{(u^h, v^h) \in \mathcal{T}} c_{u,v}^h = \min_{d'_n \in \mathcal{V}_v} C(d'_n)$, must reach its maximum since each $C(d'_n)$ is maximized when the algorithm terminates. Additionally, the input \mathcal{G} has ensured that \mathcal{T} does not contain edges with insufficient capacity or unsatisfied delay, and step 6 enforces the correct forwarding timing at each hop. The proof is completed.

APPENDIX D

PROOF OF THEOREM *Theorem 2*

In **Algorithm 1**, we assume that the input \mathcal{G} and the introduced \mathcal{Q} are stored in an adjacency list and a binary heap, respectively. The initialization in step 2 takes $O(|\mathcal{V}|)$ time. During each iteration from steps 3 to 19, it requires $O(1)$ time to extract the node u^h with the maximum bottleneck capacity from \mathcal{Q} and $O(\log |\mathcal{V}|)$ time to update \mathcal{Q} (in step 4). Furthermore, examining each edge (i.e., transmission edge, storage edge, or virtual edge) and updating the parameters (i.e., “pending-capacity” $C(\cdot)$, “arrival-time” $L(\cdot)$, and “pre-node” $P(\cdot)$) of the node actually reached along that edge takes $O(\log |\mathcal{V}|)$ time. Denoting the out-degree of u^h as $\deg(u^h)$, the time complexity of the process from step 5 to 18 is at most $O(\deg(u^h) \cdot \log |\mathcal{V}|)$. At worst, we must traverse all nodes in \mathcal{V} once before extracting all the virtual nodes $d'_n \in \mathcal{V}_v$ from \mathcal{Q} . Therefore, the time complexity reaches $O(\sum_{u^h \in \mathcal{V}} \deg(u^h) \cdot \log |\mathcal{V}|) = O(|\mathcal{E}| \cdot \log |\mathcal{V}|)$. Additionally, the backtracking process takes at most $O(|\mathcal{E}|)$ time. Thus, the time complexity of **Algorithm 1** can be calculated as $O(|\mathcal{V}| + |\mathcal{E}| \cdot \log |\mathcal{V}| + |\mathcal{E}|)$. Since \mathcal{G} is a connected graph, it follows that $|\mathcal{E}| + 1 \geq |\mathcal{V}| \geq 2$ holds, and the time complexity can be further simplified to $O(|\mathcal{E}| \cdot \log |\mathcal{V}|)$. The proof is completed.