FOUNDATIONS OF OPTIMIZATION: IE6001

Examples of LP Problems

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Diet Problem

Determine most economical diet, with basic nutritional requirements for good health.

- n different foods: ith sells at price c_i/unit,
- m basic nutritional ingredients: jth ingredient's daily intake for individual is at least b_j units (healthy diet),
- one unit of food i contains a_{ji} units of jth ingredient,
- x_i: number of units of food i in diet.

Diet Problem (cont)

minimize total cost:

$$c_1 x_1 + c_2 x_2 + \ldots + c_n x_n$$

subject to:

$$\begin{array}{ccccccccc}
a_{11}x_1 & + & a_{12}x_2 & + & \ldots & + & a_{1n}x_n & \geq & b_1 \\
a_{21}x_1 & + & a_{22}x_2 & + & \ldots & + & a_{2n}x_n & \geq & b_2 \\
\vdots & & \vdots & & & & \vdots & & \vdots \\
a_{m1}x_1 & + & a_{m2}x_2 & + & \ldots & + & a_{mn}x_n & \geq & b_m
\end{array}$$

non-negativity of food quantities

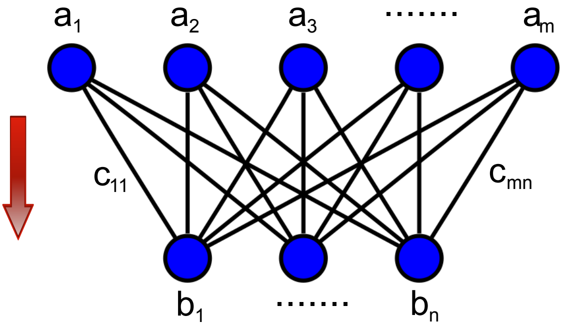
$$x_1 \geq 0, x_2 \geq 0, \ldots, x_n \geq 0$$

Transportation Problem

- Quantities $a_1, a_2, \ldots, a_m$ of a product to be shipped from m locations,
- demanded in amounts $b_1, b_2, \ldots, b_n$ at n destinations,
- c_{ij}: unit cost of transporting product from i to j,
- x_{ij}: amounts to be shipped from i to j ($i = 1, \ldots, m$; $j = 1, \ldots, n$).

Determine x_{ij} to satisfy shipping requirements and minimize total cost.

Transportation Problem (cont)



Transportation Problem (cont)

minimize total cost: $\sum_{i,j} c_{ij} x_{ij}$ subject to:

$$\sum_{j=1}^{n} x_{ij} = a_i \qquad \text{(total shipped form } i = 1, \ldots, m\text{)}$$

$$\sum_{i=1}^{m} x_{ij} = b_j \qquad \text{(total required by } j = 1, \ldots, m\text{)}$$

$$x_{ij} \geq 0 \qquad \text{consistency: } \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

Air Traffic Control

- Airplane n, $n = 1, \ldots, N$, arrives at the airport within the time interval $[a_n, b_n]$ in the order of $1, 2, \ldots, N$.
- For safety reasons, the airport wants to set the arrival time for each plane such that the time interval between two consecutive flights is as large as possible.
- Denote by t_n be the arrival time of airplane n, and let Δ be the smallest inter-arrival time.

maximize the inter-arrival time: Δ subject to:

$$t_n - t_{n-1} - \Delta \geq 0 \quad n = 2, \ldots, N$$

$$a_n \leq t_n \leq b_n \quad n = 1, \ldots, N$$

$$\Delta \geq 0$$

Newsboy Problem

- In the morning, the newsboy buys x newspapers from the publisher at a wholesale price c per piece.
- The publisher provides at most u newspapers.
- The demand (amount of newspapers that can be sold) is uncertain. It amounts to d_s with probability $p_s > 0$ for different scenarios $s = 1, \ldots, S$, where $\sum_{s=1}^{S} p_s = 1$.
- During the day, the newsboy sells y_s newspapers in scenario s at retail price $q > c$.
- In the evening, unsold newspapers become worthless.
- What is the newsboy's strategy to maximize expected profit?

Newsboy Problem (cont)

$$\text{maximize expected profit: } -c\,x + q\sum_{s=1}^{S} p_s y_s$$

subject to:

$$0 \le x \le u \qquad \text{(amount bought in the morning)}$$

$$0 \le y_s \le x \qquad \text{(sell at most } x \text{ in each scenario } s = 1, \ldots, S\text{)}$$

$$y_s \le d_s \qquad \text{(demand constraint for } s = 1, \ldots, S\text{)}$$

Data Fitting

- Given input variables $a_n \in \mathbb{R}^d$ and output variables b_n, $n = 1, \dots, N$; e.g. $a_n =$(height, age) and $b_n =$weight of person n.
- Is there a linear relationship between inputs and outputs? Put differently, is there an $x \in \mathbb{R}^d$ with the property that $a_n^T x \approx b_n$?
- The least squares problem is to find x such that

$$\sum_{n=1}^{N} (a_n^T x - b_n)^2$$

is minimized.

- To give less/more weight to outliers, one can choose different objective criteria. We can minimize

$$\sum_{n=1}^{N} |a_n^T x - b_n| \quad \text{or} \quad \max_n |a_n^T x - b_n| \,.$$

- These problems can be rewritten as linear programs.