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Development of Expertise in Mathematical Problem Solving

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SUMMARY

Previous studies have found that the strategies used by expert and novice problem solvers differ. Novices tend to use means–ends analysis, which involves working backward from the goal, whereas experts prefer to work forward from the givens of a problem. Experiment 1 was designed to study the course of development of expertise using a subset of kinematics problems. After solving many problems, subjects demonstrated the switch from a means–ends to a forward-chaining strategy. This was associated with the conventional concomitants of expertise such as a decrease in the number of moves required for solution. In addition, the speed at which expertise developed varied for different categories of problems. Subjects appeared to categorize problems according to the order in which equations would be required, with these categories being discovered at nonuniform rates. This was assumed to be due to the differential rate of acquisition of schemas associated with different categories of problems.

Experiments 2 and 3, again using kinematics problems, tested the hypothesis that the means–ends strategies used by novices retarded the acquisition of appropriate schemas. It was suggested that under a means–ends strategy, moves are controlled by the problem goal, which reduces the information obtained by problem solvers concerning problem structure. The use of nonspecific rather than specific goals was found to enhance the acquisition of expertise as measured by the use of a forward-oriented strategy, the number of moves required for solution, and the number of equations written without substitutions.

Experiments 4 and 5, using geometry problems, duplicated the enhanced rate of strategy alteration found with reduced goal specificity. The results of Experiments 6 and 7, again using geometry problems, indicated that reduced goal specificity also enhanced the rate at which problem solvers induced appropriate problem categories.

It was concluded that in circumstances in which the primary reason for presenting problems is to assist problem solvers in acquiring knowledge concerning problem structure, the use of conventional problems solved by means–ends analysis may not be maximally efficient.

The distinctive strategies used by novice and expert problem solvers has generated interest in recent years. Larkin, McDermott, Simon, and Simon (1980a, 1980b) and Simon and Simon (1978) in analyzing physics problem solutions suggested that novices solved their problems using means–ends analysis. Solutions were generated by initially choosing an equation containing the goal and then working backward toward the problem givens by choosing new equations that might solve for unknowns in preceding equations. Once equations allowing solution were obtained, the process was reversed with work directed forward toward the goal.

As an example, assume a body uniformly accelerates from rest. Assume that the time (t) that it accelerates and the distance (s) that it travels are given. The goal is to find its final

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velocity (V). This can be accomplished by calculating average velocity (v) using the equation $s = vt$ followed by final velocity using $v = .5V$. A subject using means–ends analysis would note that the equation $v = .5V$ includes the desired goal. Because v is unknown and is needed to find V, finding the value of v becomes a subgoal. The next step, working backwards from the goal, is to note that v can be calculated from $s = vt$, because s and t are known. The solution to the problem can now be obtained by working forward from the givens, using $s = vt$ to find v followed by $v = .5V$ to reach the goal, which is V.

In contrast to novices, experts appeared to eliminate the initial backward-working mode and began by generating equations forward from the givens toward the goal. An equation was chosen only if it contained a single unknown that could thus be solved for. Using the previous example, experts might begin by choosing $s = vt$, calculating v, and then using $v = .5V$ to attain the goal. The initial means–ends mode—noting an equation ($v = .5V$) containing the goal and then working backward to an equation ($s = vt$) to calculate a desired unknown from the givens—was not used.

Simon and Simon (1978), in explaining the distinction between novice and expert physics problem solvers, suggested that experts solve problems with the assistance of physical intuition. This allows a physical representation with concrete referents to be constructed in memory by translation from the problem statements. Appropriate equations are then generated from this schema. Because the schema is based on the problem givens, a working-forward strategy is used. Novices, having insufficient knowledge to construct an appropriate physical representation, cannot work forward from the givens and must use means–ends analysis to guide them to solution.

Chi, Feltovich, and Glaser (1981) and Chi, Glaser, and Rees (1982) indicated that a differential ability to categorize problems may also be an important factor distinguishing novice from expert problem solvers. Using physics problems, they found that whereas experts categorized problems on the basis of physics principles, novices categorized according to features mentioned in the problem statement. For example, experts may place several problems in the same category because they can be solved by the principle of conservation of energy, whereas novices may place in a single category all problems dealing with an inclined plane. The categorization of problems according to physical principles results, according to these authors, in the activation of knowledge structures or schemas associated with the principle. The schemas in turn determine the particular equations to be used for solution. Because the schema does not depend on the problem goal, this leads to a forward working strategy.

Using more general terms, Greeno (1980a, 1980b) has suggested that appropriate, domain-specific knowledge is an essential component of problem-solving skill. The work cited above provides an indication of possible characteristics of that knowledge.

In summary, whereas a novice uses conventional means–ends analysis to solve problems, an expert categorizes problems according to solution principles and applies those principles in a forward-working manner to the givens of the problem. The expert's knowledge-based strategy is dominated by previous experience. He or she knows into which category the problem should be placed and knows which moves are most appropriate, given that particular type of problem.

Although recent work has isolated the distinctive strategies used by novices and experts, the switch from one strategy to the other has not been observed, nor have factors that might facilitate the acquisition of expertise been examined. Acquiring detailed expertise in physics normally requires lengthy preparation, making it difficult to carry out the required experimental work. Simply giving a novice a limited number of problems to solve in a given area is not likely to result in discernible strategy alterations (see Simon & Simon, 1978). An enormous number of problems spread over a long period of time might be needed. If, in order to observe the acquisition of expertise, it is essential to duplicate the time and effort normally expended by people before becoming experts, then the number and type of studies possible becomes severely limited. An alternative is required.

One way of resolving the issue is to use a very small subset of problems from the problem domain. A small subset of problems may require relatively limited knowledge. For example, physics problems all soluble by the same two or three equations could be presented. Expertise would require the ability to distinguish between only two or three categories based on solution mode, resulting in the immediate use of particular equations that allow the solution of an unknown. Problem solvers may become expert in solving such problems relatively rapidly, thus more readily allowing studies that try to monitor and manipulate the development of expertise.

What factors might facilitate the development of expertise? Several recent studies using puzzle problems provide us with hypotheses. Puzzles may be considered "knowledge reduced" problems. Unlike the semantically rich, mathematically based problems faced by physics or geometry problem solvers, these puzzles require very little knowledge. Learning many equations or theorems is unnecessary. Nevertheless, knowledge is not eliminated, and the acquisition of suitable knowledge such as an appropriate rule will obviously facilitate problem solution. In this context a novice may be considered someone who has not acquired appropriate solution rules and thus uses means–ends analysis to solve the puzzle whereas an expert uses a previously learned, rule-based approach. Sweller, Mawer, and Howe (1982), using sample arithmetic problems soluble by the alternation of two operators, found that whereas problem solvers had little difficulty solving the problems by a means–ends strategy, the use of this strategy seemed to prevent acquisition of the alternation rule. Rule induction only occurred if subjects were provided with additional information either during or after problem solution. Mawer and Sweller (1982), using similar problems, found that frequent, appropriately placed subgoals assisted rule acquisition. Sweller and Levine (1982) found that by eliminating a goal expressed as a specific problem state and thus preventing the use of means–ends analysis, rule induction and problem solution were rapid. On the maze problems used, the presence of the goal could, under some circumstances, completely prevent rule induction and solution.

Several conclusions may be derived from these studies. Although means–ends and knowledge-based strategies may be alternative problem-solving techniques, a means–ends approach may be a significant impediment to knowledge acquisition. It may be difficult to learn important aspects of the problem structure using means–ends analysis. This may be due to the manner in which problem-solving decisions are controlled under a means–ends strategy (see Sweller, 1983). Under means–ends analysis, control of moves is vested heavily in the goal. Moves are designed to reduce differences between the current problem state and the goal. Problem solvers' attention may as a consequence be primarily directed to this factor rather than to essential aspects of the problem structure, thus reducing learning. Under these circumstances, where knowledge or schema acquisition is an aim of problem solving, it may be useful to reduce the influence of the goal by altering its characteristics in a manner designed to reduce its function as a control mechanism. This may modify or eliminate the use of means–ends analysis and enhance learning by shifting control of decisions from the goal to factors that may more readily allow knowledge or schema acquisition.

Experiment 1 was designed to enable detailed analysis of the switch from a means–ends to a knowledge-based strategy using kinematics problems. Using similar problems, Experiments 2 and 3 studied the effects of reduced goal specificity on this switch and on the development of expertise in general. Experiments 4 and 5 tested the effects of goal specificity on strategy development using geometry problems, whereas Experiments 6 and 7, again using geometry problems, used Einstellung as a measure of schema acquisition.

Experiment 1

Experiment 1 was designed to study the development of a knowledge-based problem-solving strategy using kinematics problems. Expertise in solving these problems normally requires extensive experience. Studies comparing expert and novice problem solving have used experts with such experience. These studies have found no discernible alteration in novices' strategies after the solution of about 20 problems (e.g., see Simon & Simon, 1978). Under these circumstances, even a relatively limited problem-solving domain such as kinematics is likely to be too extensive to allow ready observation of the transition from a novice to expert problem-solving strategy.

The difficulty may be diminished by reducing the size of the domain. This was accomplished in the present experiment by acquainting problem solvers with only three equations and presenting problems soluble by two of these. The same two equations could be used for each problem, reducing the problem domain even further. The equations used were $V = at$, $s = vt$, and $v = .5V$, where $V =$ final velocity, $v =$ average velocity, $a =$ acceleration, $t =$ time, and $s =$ distance. All problems involved bodies uniformly accelerating from rest and could be solved using the equations $s = vt$ and $v = .5V$. No other equations were required although two of the problems could also be solved using $V = at$ alone. These were included in order to test for Einstellung. After solving many problems using a combination of $s = vt$ and $v = .5V$, problem solvers may fail to see that problems can be solved by a single, previously unused equation. Furthermore, previous research has indicated that Einstellung tends to occur when problem solvers are using a forward-working strategy but does not occur when means–ends analysis is being used (see Mawer & Sweller, 1982; Sweller, 1983; Sweller et al., 1982).

It might be noted that the three equations used are basic in the sense that they are sufficient to allow solution of all kinematics problems involving uniform acceleration from rest. Other equations normally taught to physics students such as $s = .5at^2$, can all be derived from these three. The additional equations will facilitate solution in many instances by obviating the need for simultaneous equations or by reducing problem solution lengths. The full list of equations normally taught is, nevertheless, not essential.

Using problems of the above type, we may predict that expertise will develop in the following manner. We can expect novice problem solvers to begin by using means–ends analysis. With moves controlled by the goal, the equation containing the goal term should be considered before it can be used in a calculation. With increased expertise, control should shift to previously learned schemas. Problem solvers may learn to distinguish between categories of problems based on solution methods. There are essentially only two categories of problems. The first category consists of problems in which one of the givens is V. This allows immediate use of the equation $v = .5V$ followed by $s = vt$, which contains the goal as an unknown. The other category consists of problems containing the givens s and t. This allows immediate use of $s = vt$ followed by $v = .5V$ thus solving for V, which is the goal. In addition to, or as a substitute for, distinguishing between these two categories, some problem solvers may learn that solution can be attained by calculating v as the first step on each problem. Once problem solvers have learned to categorize the problems or learned that they all can be solved by calculating v, the use of means–ends analysis should decrease to be replaced by a forward-working strategy. In the hope of facilitating this effect, many of the problems were presented to problem solvers several times in identical form.

Method

Subjects. The subjects were 14 mathematics graduates taking teacher education courses.

Apparatus and procedure. All problems were presented on a computer-controlled visual display screen. Problem-solving steps were keyed in by subjects and recorded on the screen and in computer memory for later access. Pencil and paper were not used, because the screen provided a complete substitute. Arithmetic calculations were unnecessary, because these were carried out by the computer. The program accepted correct statements of any of the three equations used (e.g., $s = vt$ or $v = s/t$), correct assignment of values to givens (e.g., $s = 10$), and correct substitutions (e.g., $s = 10 \cdot t$). Where substitutions allowed immediate arithmetic calculations (e.g., $s = 10 \cdot 20$), these were carried out automatically by the computer and the result was screened (e.g., $s = 200$). Any incorrect statements were followed by a general message indicating that an error had been made and an alternative was needed. When the correct value was found for the goal, a message indicating that the problem had been solved appeared and the next problem was screened.

A total of 25 different problems was used. Thirteen of them were presented five consecutive times in identical form (including numerical values), resulting in subjects being given a total of 77 problems including the 12 that were given only once. The problems were presented in the following order. The first 6 were given singly, followed by the 13-given five times each. These in turn were followed by 6 problems presented once only. The same problems in the same order were presented to all subjects except that the order of presentation of the two groups of 6 singly presented problems was counterbalanced, with half of the subjects solving the problems of one group first and the other half solving the problems of the other group first. Whichever group of problems remained provided the last 6 problems. The first problem of both of the groups of 6 singly presented problems could be solved solely by using the equation $V = at$ or by using both $s = vt$ and $v = .5V$. These two problems could be used as a test for Einstellung. Because the problems that intervened between the two Einstellung problems all required both $s = vt$ and $v = .5V$ for solution, we might expect that although the first problem would be solved using $V = at$, the second would tend to be solved using $s = vt$ and $v = .5V$.

In general there were two categories of problems, excluding the Einstellung problems. The givens of V-category problems contained a value for final velocity. Working forward, the first equation that needed to be used was $v = .5V$ followed by $s = vt$, which contained the goal term. The following is an example of this type of problem:

> A pile driver takes 3.732 sec to fall onto a pile. It hits the pile at 30.46 m/sec. How high was the pile driver raised?

The *st* category contained problems with s and t in the givens. These required the initial use of $s = vt$ followed by $v = .5V$ for solution. The following is an example:

> In 18 sec a racing car can start from rest and travel 305.1 m. What speed will it reach?

An example of an Einstellung problem is:

> A train takes 16 sec and accelerates at 2.31 m/sec/sec until it is 296 m from where it started. What is the train's final velocity?

Table 1 summarizes the categories of problems used in Experiment 1 and their characteristics.

Excluding the two Einstellung problems, of the 23 different problems used, 8 required the use of $s = vt$ first, with V as the goal, and 15 required $v = .5V$. Eight of the latter 15 problems had s as the goal, and the remaining 7 had t. There was thus an approximately equal number of problems requiring the calculation of s, t, or V as the final goal step. The goal of both Einstellung problems required the calculation of V.

It should also be noted that, again excluding the two Einstellung problems, four of the problems contained acceleration among the givens. Because these problems also contained V and t, knowing the acceleration provided no useful information. Acceleration could be calculated in all other problems, but, with the exception of the Einstellung problems, knowing its value did not facilitate solution.

All subjects were acquainted with the three equations and were informed that these were the only equations that could be used to solve the problems. They could request that the equations be shown to them at any time during the experiment. The computer kept a record of all problem-solving steps taken, and, in addition, all subjects were encouraged to generate verbal protocols, which were tape recorded.

Results

Table 2 indicates the strategy used (backward or forward) by each subject on each problem, excluding repeated problems. (Repeated problems will be discussed later.) The following criteria were used to determine strategy. If an equation containing the goal term was written or verbalized before the value of a variable was calculated from the givens, a backward (means–ends) strategy was assumed. A forward strategy was assumed if this order was reversed, with the value of a variable being calculated from the givens before the equation containing the goal term was written or verbalized.

Excluding the first problem, which was a control for Einstellung, the first five problems may be used as indicators of the problem-solving strategies used initially by the 14 subjects. Two of the subjects (Subjects 1 and 2) worked forward consistently. The first equation typed in was invariably one not containing the goal but allowing immediate calculation of the value of a variable. For these two subjects, no evidence could be obtained from verbal protocols of a means–ends strategy. As can be seen from Table 2, the remaining 12 subjects used means–ends analysis on at least three of the first five problems. We may conclude that a means–ends strategy was preferred over-

Table 1
Categories of Problems Used in Experiment 1

Problem category	Variables				Order of equations using a forward strategy
	V	s	t	a	
st	?	×	×		$s = vt$; $v = .5V$
V(a)	×	?	×		$v = .5V$; $s = vt$
V(b)	×	×	?		$v = .5V$; $s = vt$
Einstellung	?	×	×	×	$V = at$ or $s = vt$; $v = .5V$

Note. V = final velocity; s = distance; t = time; a = acceleration; v = average velocity; ? = unknown; × = given. Average velocity is never a given nor an unknown. Problem category *st* has s and t as givens and requires a different equation order than category V, which has V and either t, as in the case of category V(a), or s, in the case of V(b), as givens.

whelmingly by most problem solvers on the initial five problems.

The last five problems may be used to indicate whether practice on the intervening problems has resulted in alterations in problem-solving strategies. We have no grounds for assuming that the two subjects who worked forward on the initial problems would change their strategy, and, indeed, both continued to work in this manner. Of the 12 subjects who tended to work backward on the initial problems, most demonstrated an increased tendency to work forward on the last five problems. The mean number of problems on which subjects used means–ends analysis during the first five and last five problems was 3.9 and 2.2, respectively. The difference between these means is significant using a t test for related samples, $t(13) = 3.07$. (The .05 level of significance is used throughout this article.)

These results indicate that extensive practice can induce a strategy alteration. It might also be noted that evidence for the development of expertise should be obtainable from the number of times subjects wrote down algebraic equations without variable substitution, that is, the number of times $s = vt$ and $v = .5V$ were written. We would expect the equations to be written less frequently during the last five problems compared with the first five problems. The mean number of times equations with no substitutions were written during the first five and last five problems was 11.5 and 5.0, respectively. The difference between these means using a t test for related samples is significant, $t(13) = 5.75$.

Similarly, we might expect increases in expertise to result in a decreased number of moves required for solution. The mean number of moves required to solve all of the first

Table 2
Strategies Used by Each Subject on Unique (Nonrepeated) Problems in Experiment 1

		Subjects													
Problem	Category	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	e	d	d	d	d	d	d	d	d	d	d	d	d	d	i
2	v	f	f	b	b	b	b	b	b	b	b	b	b	b	b
3	v	f	f	b	b	b	b	b	b	b	b	b	b	b	b
4	st	f	f	b	b	f	b	b	b	b	b	b	f	f	f
5	v	f	f	b	b	b	b	b	b	b	b	b	b	b	f
6	v	f	f	b	b	b	b	b	b	b	b	b	b	b	b
7	v	f	f	b	b	b	b	b	b	b	b	b	b	b	f
8	st	f	f	b	b	f	b	b	b	f	f	f	b	f	b
9	v	f	f	b	b	b	b	b	b	b	b	b	b	b	b
10	v	f	f	b	b	b	b	b	b	b	b	b	f	b	b
11	st	f	f	b	b	f	b	b	b	b	b	f	b	f	b
12	v	f	f	b	b	f	b	b	b	b	b	f	b	b	b
13	st	f	f	b	b	f	f	f	b	f	f	b	b	f	f
14	v	f	f	b	b	f	b	b	b	b	b	b	b	b	b
15	v	f	f	b	b	b	b	b	b	b	b	b	f	f	b
16	st	f	f	b	b	f	f	b	f	f	f	b	b	b	f
17	st	f	f	b	b	f	b	f	f	f	f	f	b	f	f
18	v	f	f	b	b	f	b	b	b	b	b	f	b	b	b
19	st	f	f	b	b	b	f	f	f	f	b	b	b	f	b
20	e	d	d	d	i	i	i	d	i	i	d	i	i	i	i
21	v	f	f	b	b	f	b	b	f	b	f	f	b	b	b
22	v	f	f	b	b	f	b	b	f	b	f	f	f	b	b
23	st	f	f	b	b	f	f	f	f	f	f	f	f	f	f
24	v	f	f	b	b	f	b	b	f	f	f	b	f	b	b
25	v	f	f	b	b	f	b	b	f	b	f	f	f	b	b

Note. Problems 7–19 were repeated five times each. The first and last six problems presented to Subjects 1–7 were presented as the last and first six problems, respectively, to Subjects 8–14. e = Einstellung problems; v = problems with final velocity among the givens; st = problems with distance and time as the givens; d = direct solution on an Einstellung problem; i = indirect solution on an Einstellung problem; b = backward strategy; f = forward strategy.

and last five problems was 35.9 and 26.8, respectively. This difference is also significant, $t(13) = 5.64$.

The processes by which strategy alteration occurs is of interest. As indicated previously, the problems can be placed into two categories according to solution mode. The first category consists of problems including distance and time in the givens and final velocity as the goal. Working forward, these problems require the use of $s = vt$, followed by $v = .5V$. These are called *st* problems because distance and time always appear in the givens. The second category includes final velocity and either distance or time in the givens, with the goal consisting of whichever one of distance or time that did not occur as a given. These problems require the use of $v = .5V$, followed by $s = vt$. These are called *V* problems because a value for final velocity always appears in the givens. The results may be analyzed to detect any evidence that subjects have distinguished between these two categories.

Evidence for categorization may be obtained if subjects treat problems falling into the two categories differentially. If at any stage, subjects work forward on problems falling into one category but use means–ends analysis on problems falling into the other category, this may indicate familiarity with the working-forward category. (Again, we will only consider the first presentation of those problems that were presented more than once.) There were 10 subjects who worked forward on some problems but backward on others, thus providing data that could be searched for evidence of categorization. The two subjects who consistently worked forward (Subjects 1 and 2) and two who consistently worked backward (Subjects 3 and 4) could not provide evidence of this type of categorization.

There was one subject who appeared to categorize the problems according to solution mode immediately. Subject 5 worked forward on all except one *st* problem but may have learned the characteristics of the *V* category during the course of the experiment. This subject began to work forward on the eighth *V* problem and from then on continued to work forward on all but one of the remaining problems. His protocols indicated that on the latter problems he had learned always to calculate average velocity first.

Four additional subjects (6, 7, 8, and 9) provided evidence of inducing problem categories during the course of the experiment. Initially, all of these solved the problems using means–ends analysis. However, they all shifted to a forward-working strategy on later *st* problems. The occurrence of this shift varied from the second to the fifth *st* problem. Once they shifted from a means–ends to a forward strategy, three of these subjects (6, 7, and 9) continued to work forward on all except, at the most, one subsequent *st* problem and backward on all except, at the most, one *V* problem. Subject 8 differed from the preceding three subjects in that she subsequently abandoned the use of means–ends analysis on *V* problems as well. Means–ends analysis was only used on the first *V* problem immediately after it had been abandoned on *st* problems and not used thereafter. This subject's protocols indicated that on the latter problems she used the rule "always calculate average velocity first."

The remaining 5 subjects (10, 11, 12, 13, and 14) of the 10 who mixed working backward and forward provided less clear evidence that they had induced the two categories. Nevertheless, it needs to be noted that in all five cases an *st* problem was the first on which a forward-working strategy was used despite there being approximately twice as many *V* problems as *st* problems. Even these subjects may thus provide some evidence for categorization.

It might be noted that in some cases it was difficult to determine whether a subject had induced the *st* category followed by the *V* category or had induced the *st* category followed by induction of the rule that average velocity should always by calculated first. Protocols indicated that some subjects may have followed the latter pattern. Only one subject may have learned the rule without first providing evidence of categorization. Other subjects who appeared to learn the rule provided evidence of categorization by first using different solution strategies (in terms of working backward or forward) on the two types of problems.

From the preceding data it is clear that subjects found the *st* category considerably more obvious than the *V* category. This may be due to the uniformity of the *st* problem type. All *st* problems had distance and time as the givens. The V category effectively consisted of two subcategories. Although both subcategories included final velocity as a given, they also contained either distance or time as a given (see Table 1). This increase in complexity may account for the difficulty subjects had in discovering this problem type.

The data on categorization may be summarized as follows. Evidence for categorization may be obtained from the differential rates at which many subjects began to work forward. The *st* category was induced before the V category despite there being approximately twice as many problems in the latter. A few subjects may have come to the experiment with this category already known. If subjects induced the V category, this invariably occurred after discovering the *st* problem type.

It should be noted that the evidence, which we have suggested indicates categorization, could in fact be generated by an alternative mechanism. It could be argued that subjects had a response set manifested by writing the equation $s = vt$ on first being presented with each problem. If it was possible to calculate an unknown, the unknown was calculated immediately. This possibility invariably occurs on *st* problems, and so on these subjects worked forward from the givens. On V category problems, nothing can be calculated immediately using $s = vt$. To proceed the subject must use the other equation, $v = .5V$. This results in working backward. Hence, with this response set a subject will work forward on the *st*-category problems and backward on the V-category problems.

Evidence that subjects are not operating under the influence of such a response set may be found by examining strategy changes on the repeated presentation problems. All of the subjects discussed previously who may have provided evidence of learning the *st* category during the course of the experiment, as indicated by their forward strategy on *st* problems, also switched to a forward strategy during the solution of the repeated presentations of the V-category problems. For example, Subject 6 began each of the repeat presentation V-category problems by working backward but by the last repeat presentation of each problem was working forward. This shift in strategy across repeat problems was quite general. Because there were 13 problems that were repeated five times, each subject could be given scores out of 13, indicating the number of times on each of the five repeats that a forward-oriented strategy was used. For repeat problem numbers 1–5, the mean numbers of times a forward-oriented strategy was used per subject were 4.9, 8.8, 9.6, 10.2, and 10.4, respectively. Although subjects tended to work backward on the first presentation of a problem, by the fifth presentation they overwhelmingly preferred to work forward. There is a significant decrease in number of problems on which means–ends analysis was used across the five repeats, $F(4, 52) = 20.07$.

These results provide evidence against the occurrence of response sets. We might expect a response set to operate on all problems whether repeated or not. It should occur on repeated V-category problems as well as initial problems of this type. This did not happen. It is of course possible that subjects manifested a response set on the initial presentation of a problem but used a learned schema to generate solutions of repeat problems. Nevertheless, we believe it more parsimonious to assume that a working-forward strategy indicated an acquired schema whether it occurred on initial or repeat problems.

The Einstellung problems may be used to test whether Einstellung is more likely to be associated with a forward- or a backward-oriented strategy. Both Einstellung problems are essentially *st*-category problems in that they can be solved by using the equation $s = vt$ followed by $v = .5V$. Their special character resides in the fact that they can also be solved by simply using the equation $V = at$. One of these problems was presented to all subjects as the initial problem. Most subjects, using means–ends analysis, should solve the problem using the equation $V = at$. When given a similar problem (the alternative Einstellung problem) toward the end of the experimental session, most subjects should demonstrate Einstellung by solving the problem using the equations $s = vt$ followed by $v = .5V$. Furthermore, if as suggested previously, Einstellung is more likely to be associated with a forward-oriented strategy, then we can predict that subjects who demonstrate Einstellung are more likely to be using a forward-oriented strategy on a similar problem. Excluding the Einstellung problems, the initial five and final five problems each contain an *st* problem that can be used to check the strategy used by subjects near the beginning and end of the experiment. The *st* problems are appropriate because they resemble the Einstellung problems in structure. As pointed out above, the only difference is that *st* problems do not contain a given value for acceleration.

Table 3 indicates the interaction between the strategy used on the first and last *st* problems and the equations used on the Einstellung problems. As can be seen, by using $V = at$ to solve the first Einstellung problem, subjects were using a direct, non-Einstellung solution. This was associated with working backward on the first *st* problem. On the last Einstellung problem the less efficient combination of $s = vt$ and $v = .5V$ tended to be used, thus demonstrating Einstellung. This was associated with working forward on the last *st* problem. We consequently have evidence of an association between strategy used and Einstellung.

Discussion

Experiment 1 has allowed us to observe the development of characteristics previously shown to be associated with expertise. We have found that sufficient practice on simple kinematics problems resulted in many subjects switching from a means–ends to a forward-working strategy. The forward-working strategy developed first on a category of problems requiring a particular solution mode. This can be interpreted as indicating that with practice, subjects were able to classify at least some of the problems according to solution mode, thus leading to a forward orientation. Means–ends analysis may have continued to be used on those problems that subjects could not classify according to solution mode. In addition we have found that Einstellung tends to be associated with a forward rather than a means–ends strategy.

Chi et al. (1982) indicated that experts classified problems according to broad principles such as "conservation of energy." The schemas associated with these principles direct the solution process, resulting in a forward-oriented strategy. The present results may indicate that expert schemas can also be based on substantially narrower concepts than general physical principles. Our subjects seem to have learned that problems containing distance and time with final velocity as the goal can always be solved by $s = vt$ followed by $v = .5V$. This category of problems is presumably represented by a schema based not on a physical principle but rather on a highly specific combination of givens and goal. Detection of this combination may result in the activation of a schema leading to the use of a particular sequence of equations determined by the schema.

We may hypothesize a hierarchy beginning with schemas based on broad principles and successively narrowed to schemas based on specific solution procedures. There are many more schemas based on narrow solution procedures than on broad principles, and so we might expect that with increased expertise, problem solvers may be more and more likely to use a schema based on a particular solution procedure. Schemas based on general principles may be the first of the solution mode schemas to be developed, with particular solution procedures following. This of course does not prevent schemas based on narrow solution procedures from developing early, if, as occurred in Experiment 1, conditions are specifically arranged to encourage the development of these schemas rather than those based on broad principles.

The results also have implications for the development of expertise through the use of a means–ends strategy. Although evidence for the acquisition of knowledge-based schemas was obtained, this knowledge appears to have developed extremely slowly. In the case of a few subjects, it may not have developed at all because there was no discernible alteration in problem-solving strategy over problems. This occurred despite the solution of an enormous

Table 3
Number of Subjects in Each Combination of the Einstellung Occurrence and the Strategy Used on the Initial and Final, Critical st *(Distance/Time) Problems in Experiment 1*

Problems	B × E	B × NE	F × E	F × NE
First five	0	8	1	5
Last five	1	1	8	4

Note. B = backward strategy; F = forward strategy; E = Einstellung; NE = failure to demonstrate Einstellung.

number of problems (77, including repeated problems). We need to question why learning under the applicable conditions appeared inordinately slow.

We suggested previously that means–ends analysis, although an efficient problem-solving strategy, may be inefficient as a means of acquiring knowledge of problem structure. The aim of the strategy is to eliminate differences between each current problem state and the goal state. The goal, as a consequence is a major factor controlling moves and may be rapidly attained, resulting in problem solution. The conditions that allow the goal to be attained are not necessarily those that will facilitate learning. When learning, it may be more important to consider how a problem state has been attained rather than the difference between the current problem state and the goal. This may involve consideration of previously made steps rather than possible future steps. Consideration of previously made steps may be facilitated if moves are controlled by aspects of the problem structure other than the goal. We might expect problem solvers to learn more of the general problem structure if their moves, rather than being controlled by the goal, are controlled by factors based on the problem structure, such as relations between possible moves or relations between moves and knowns or unknowns. Consequently, a heavy reliance on means–ends analysis may act as an impediment to the development of knowledge-based schemas, thus perpetuating use of a means–ends strategy (see Sweller, 1983; Sweller & Levine, 1982; Sweller et al., 1982). Experiment 2 was designed to test this possibility.

Experiment 2

If using means–ends analysis retards the acquisition of schemas that may benefit problem solution, then solution by alternative techniques may be beneficial under some circumstances. Means–ends analysis appears to be the natural problem-solving strategy used by most problem solvers presented with novel problems. We may presume that this is so because of its efficacy in allowing problem solvers to attain the problem goal.

Because means–ends analysis involves substantial control of moves by the goal, the strategy is likely to be enhanced under conditions in which the goal is presented to the problem solver as a specific problem state. The less specific the goal, the more difficult it becomes clearly to reduce differences between it and a given problem state. A nonspecific goal is less likely to control moves, and a problem-solving strategy other than means–ends analysis may need to be found. The acquisition of schemas appropriate to the problem may thus be enhanced. Sweller and Levine (1982) provided evidence supporting this suggestion. Using maze problems, they found that under some conditions, the presence of a specific goal in the form of a visible exit point could retard acquisition of the problem structure to the point where problem solution became unattainable.

The kinematics problems of Experiment 1 each had a specific goal, which was to find the value of a particular unknown. Essentially identical problems could be presented with goals not specified as a problem state, thus eliminating both control by the goal and the use of means–ends analysis. Take the previously given example:

In 18 sec a racing car can start from rest and travel 305.1 m. What speed will it reach?

The last sentence provides a specific goal. It might be replaced by the nonspecific goal statement:

Calculate the value of as many variables as you can.

If problem solvers are only given the equations $s = vt$ and $v = .5V$, successful solution of both problems results in the values of identical variables being calculated. Possible contaminating effects are thus eliminated. The only difference is that the nonspecific goal cannot control specific moves. Means–ends analysis is effectively prevented due to the difficulty of reducing differences between a given problem state and the goal. If the acquisition of knowledge concerning problem structure is inhibited by the use of means–ends analysis, manipulation of goal specificity in the above manner may provide some evidence of the effect.

Method

Subjects. The 20 subjects were 10-, 11-, and 12-year students from a Sydney metropolitan high school.

Apparatus and procedure. The general apparatus and procedure were identical to Experiment 1. Twenty problems were used; half were soluble by using the equation $s = vt$ followed by $v = .5V$, and the other half soluble by using the reverse sequence. The *goal* group was presented the 20 problems, once each, with the first and last 4 problems counterbalanced. Half of the subjects solved one set of 4 problems initially, while the other half was presented the alternate set initially. The remaining set was then presented as the final 4 problems.

The first and last 4 problems presented to the *no-goal* group were identical to those presented to the goal group, with counterbalancing again used. The 12 intermediate problems differed from the goal group problems in that the final goal sentence (e.g., "Find the distance traveled") was in each case altered to "Calculate the value of as many variables as you can." In all other respects the problems were identical.

The 10-, 11-, and 12-year students were evenly distributed between the two groups.

Results and Discussion

Our first concern is whether the strategy alteration displayed by some subjects in Experiment 1 has been duplicated in Experiment 2 and whether any change has differentially affected the two groups. Table 4 provides the mean number of problems on which subjects worked forward for the first and last four problems. Analysis of variance (ANOVA) indicated a significant effect due to goal specificity, $F(1, 18) = 7.2$; a significant effect due to practice, $F(1, 18) = 18.2$; and a significant Goal Specificity × Practice interaction, $F(1, 18) = 13.2$.

The interaction is of primary interest. Almost all subjects from both groups began solving their problems by using means–ends analysis. Given the slow rate of strategy change exhibited in Experiment 1, we would expect most goal subjects to still be using this strategy on the last four problems of the present experiment. This has occurred with only a slight increase in the number of problems on which a forward-oriented strategy has been used. None of the goal group subjects worked forward on all of their last four problems. On these problems, most (7 out of 10) no-goal subjects used this strategy. Subjects given problems to solve with a nonspecific goal have switched strategies far more readily than those presented the same problems in conventional form.

Although a nonspecific goal has assisted subjects in switching to a strategy known to be used by more expert problem solvers, we need to know whether the groups differ with respect to other variables associated with expertise, such as number of times subjects wrote algebraic equations without variable substitution and number of moves required to solve the problems. Table 5 contains the mean number of times subjects wrote an equation with no variable substitution. Each subject was given two scores consisting of the total number of times such an equation was written on the first and last four problems. An ANOVA indicated a significant goal specificity effect, $F(1, 18) = 9.5$; a significant practice effect, $F(1, 18) = 11.9$; and a significant Goal Specificity × Practice interaction, $F(1, 18) = 10.8$.

These significant effects again are caused almost entirely by a large alteration in score on the last 4 problems of the no-goal group. In detail, whereas only 2 of the 10 no-goal subjects wrote both equations without substituting any values on all 4 problems, 7 of the goal group subjects did similarly. While solving their 20 problems, the no-goal subjects appear to have gained sufficient familiarity with both the equations and the problem structure so that they no longer required a visual representation of the equations. This allows immediate substitution of values into the appropriate equations. The goal group still appeared to require the assistance of a visual representation in order to incorporate the equations into the problem structure.

Given the decreased use by the no-goal group of algebraic equations without substi-

Table 4
Mean Number of Problems on Which Subjects Worked Forward on the Initial and Final Four Problems of Experiment 2

Group	Initial	Final
Goal	.5	.7
No-goal	.5	3.0

Table 5
Mean Number of Equations Written Without Variable Substitution on the Initial and Final Four Problems in Experiment 2

Group	Initial	Final
Goal	7.2	7.1
No-goal	7.1	2.8

tutions on the final problems, we would expect the total number of moves for this group on these problems to be decreased. Table 6 provides the relevant means. These are based on the total number of equations, irrespective of form or content, written by each subject on the first and last four problems. Analysis of these results indicated a nonsignificant goal specificity effect ($F < 1$); a significant practice effect, $F(1, 18) = 17.5$; and a nonsignificant Goal Specificity × Practice interaction, $F(1, 18) = 2.5$.

As indicated above, because no-goal subjects wrote substantially fewer equations without substituted values on their final problems than did goal subjects, we would expect the no-goal group to require fewer moves to solve the final problems than the goal group. This should yield a significant Goal Specificity × Practice interaction, which was not obtained. Presumably, the reduction in the number of moves by no-goal subjects due to reduced use of equations without substituted values is at least partly compensated by another factor tending to increase the number of moves. Inspection of the data indicates that on the last four problems, although no-goal subjects tended to write equations and substitute given values in a single move, the equations were normally written in the form in which they were most commonly learned rather than in a form that made the unknown the subject of the equation. Thus, if the problem givens indicated that $s = 10$ and $t = 5$, no-goal subjects tended to write $10 = v \cdot 5$, because they had learned the equation as $s = vt$. This equation then needed to be transposed to make v the subject. Two moves ($10 = v \cdot 5$ and $v = 10/5$) are required to reach this point. In contrast, goal subjects tended first to write both equations in fully algebraic form (in the case of the previously used example, $s = v \cdot t$) and then in a single step to rearrange the equation to make v the subject and substitute numerical values for s and t. This also requires two steps ($s = v \cdot t$ and $v = 10/5$). The no-goal group thus derives no advantage from substituting values into equations immediately.

Table 6
Mean Number of Moves Used to Solve the Initial and Final Four Problems in Experiment 2

Group	Initial	Final
Goal	7.7	6.8
No-goal	7.9	5.9

Table 7
Mean Number of Moves Required to Solve the Initial and Final Critical Problem Not Requiring Algebraic Manipulation in Experiment 2

Group	Initial	Final
Goal	6.9	6.4
No-goal	7.0	5.3

This analysis suggests that the failure to obtain a difference between groups in number of moves to solution is due to a lack of sophistication in algebraic manipulation by the school children used. No-goal subjects, although more readily learning which equation had to be used first in order to solve the problem, were unable to manipulate mentally the equation without visual assistance and thus lost the advantage they had gained over the goal group. Reducing goal specificity may assist problem solvers to acquire information concerning problem structures in a simple problem domain, but it is unlikely to improve algebraic skills if only a limited number of problems are used. Because most goal group subjects wrote the equations before substituting values, they were able to use the written form to assist them in simultaneously manipulating and substituting.

If the above suggestion is valid, we should be able to increase differences between the two groups using problems that can be solved without reorganizing the equations from the conventionally learned forms. Problems of this type were in fact used in the current experiment. Any problem in which s was the goal had v and t as givens and could be solved by initially calculating v using $v = .5V$ and then calculating s using $s = vt$. Neither equation requires reorganization on this type of problem. Both the first and last four problems contained one example from this category. Table 7 provides the mean number of moves required to solve these two problems. Analysis of these scores indicated a nonsignificant effect due to goal specificity, $F(1, 18) = 1.1$; a significant practice effect, $F(1, 18) = 14.2$; and a nonsignificant interaction, $F(1, 18) = 4.2$. Although this interaction is not significant, a possible effect may in fact have occurred. It might be noted that on the final, relevant, critical problem, 5 of the 10 no-goal subjects solved the problem in the minimum number of moves possible. None of the goal subjects did similarly. A Fisher exact probability test indicated that this difference is significant ($p = .006$). (All Fisher exact probability tests reported in this article used Overall's, 1980, correction.)

Experiment 3

Although the results of Experiment 2 were clear-cut with respect to strategy used and number of equations written without substituted values, there was ambiguity with respect to the number of moves to solution. No-goal subjects may have solved problems not requiring equation reorganization more rapidly (as measured by number of moves) than goal subjects. Because expertise normally is associated with more rapid solution, this factor needs to be clarified. Experiment 3 investigated the effects of reduced goal specificity by using only problems that do not require equation reorganization.

Method

Subjects. The 20 subjects were 9-, 10-, and 11-year students from a Sydney metropolitan high school.

Apparatus and procedure. The general procedure was identical to that of Experiment 2. Each subject was required to solve 10 problems. The last 2 were critical problems, and as in the previous experiments these were interchanged with the first 2 problems for half of the subjects in the two groups used. For the no-goal group, the intermediate 6 problems were presented with a nonspecific goal. All 10 problems presented to the goal group were conventional. In all other respects the problems presented to the two groups were identical. Each problem had a similar format in that final velocity and time were given and a value for distance was the goal on those problems containing a specific goal. As pointed out previously, this type of problem can be solved without reorganizing the two equations, $v = .5V$ and $s = vt$.

Table 8
Mean Number of Equations Written Without Variable Substitution on the Initial and Final Two Problems of Experiment 2

Group	Initial	Final
Goal	3.8	4.0
No-goal	3.8	1.0

Table 9
Mean Number of Moves Required to Solve the First and Last Two Problems of Experiment 3

Group	Initial	Final
Goal	14.9	13.2
No-goal	13.8	9.9

Results and Discussion

Unlike Experiments 1 and 2, there are too few critical problems to realistically assign each subject a score representing the number of problems on which a particular strategy was used. As a substitute, each subject can be classified according to strategy. Nine of the 10 no-goal group subjects worked forward on the last two critical problems. The equivalent figure for the goal group was 1 out of 10. A Fisher exact probability test indicated a significant difference between the two groups on this measure ($p = .0002$). On the first two problems, all subjects in both groups used means–ends analysis.

Table 8 contains the mean number of times equations were written without numerical substitution on the first and last two problems. Analysis of these data indicated a significant goal specificity effect, $F(1, 18) = 27.7$; a significant practice effect, $F(1, 18) = 16.4$; and a significant Goal Specificity × Practice interaction, $F(1, 18) = 21.8$. Inspection of Table 8 indicates that these effects have all been caused by a large reduction in the number of equations written without numerical substitution by the no-goal group on the last two problems.

The mean number of steps required to solve the first and last two problems can be found in Table 9. An ANOVA indicated a significant goal specificity effect, $F(1, 18) = 5.5$; a significant practice effect, $F(1, 18) = 49.5$; and a significant Goal Specificity × Practice interaction, $F(1, 18) = 7.6$. Inspection of Table 9 indicates that these effects are largely caused by a decrease in the number of moves required by no-goal subjects to solve the final two problems. Six of the 10 no-goal subjects solved the final two critical problems in the minimum number of moves, whereas all of the goal subjects required more than the minimum number of moves on these two problems.

The results of Experiment 3 are unambiguous. Within 10 problems most of the no-goal subjects were solving this particular category of problems in a manner similar to experts. On the other hand, there was little discernible change by subjects presented the same problems with a conventional goal.

Experiment 4

The previous experiments and the bulk of previously reported work on novice–expert distinctions in problem solving have used physics problems. We might expect our findings to generalize to other transformation problems. Geometry problems have been chosen for this purpose. Greeno and his co-workers (Greeno, 1976, 1978; Greeno, Magone, & Chaiklin, 1979) have illuminated some of the processes involved in problem solving using geometry problems, but there appears to be no work on novice–expert distinctions.

There are many distinctions between geometry and physics problems, some of which are relevant to present considerations. Although physics problems can involve diagrammatic representations, they are not essential. Subjects in the previous experiments were not presented with diagrams, nor were they permitted to draw their own. The problems could be solved algebraically with no reference to their spatial characteristics. Geometry problems on the other hand invariably require the presentation of a diagram that is central to the problem. The diagram provides a convenient, readily visualized representation of the problem. In the case of physics problems, a similar representation either does not occur at all or must be constructed. Knowledge acquired by experts may thus be quite different for the two types of problems. We might expect geometry experts to have learned a variety of geometric configurations to which they can readily apply appropriate theorems. Rather than becoming familiar with equations that can be applied to givens in a physics problem, experts may become familiar with particular theorems and particular diagrams to which the theorems apply.

Experiment 4 tested whether the substitution of a nonspecific for a specific goal in a conventional geometry problem resulted in similar strategy alterations to those obtained using physics problems. All problems could be solved using two theorems: (a) When two lines intersect, the vertically opposite angles so formed are equal. (b) Each exterior angle of a triangle is equal to the sum of the two interior opposite angles. These two theorems replaced the equations used in the physics problems, whereas geometric diagrams replaced the physics problem statements.

Method

Subjects. The 24 subjects were 9- and 10-year students from a Sydney metropolitan high school.

Apparatus and procedure. Information transmission was computer-controlled. A geometric diagram was presented on a visual display screen. (Figure 1 provides an example.) Most angles of less than 180° were alphabetically labeled, and some also had numerical values provided. A few angles were not labeled in order to ensure that only one solution path was available. Subjects were not able to use unlabeled angles to attain solution.

Subjects could type in the value of any initially unknown angle. The theorem used to derive the value had to be indicated by typing in a V for the theorem "Vertically opposite angles are equal" or an E for the theorem "Each exterior angle of a triangle is equal to the sum of the two interior opposite angles." If the equation was correct, the newly found value appeared on the diagram. If it was incorrect, a message to this effect was screened. As an example, with reference to Figure 1, subjects could type in H = 50V. This would result in "50" appearing on the diagram near Angle H. In addition, subjects could indicate

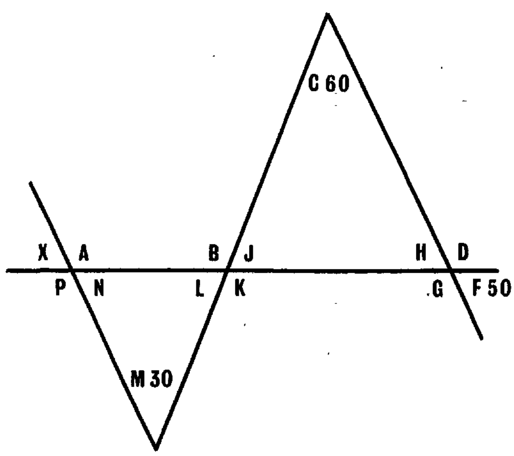


Figure 1. Example of a problem used in Experiment 4. (Solution working forward: **a:** H = F = 50 [V]; **b:** B = H + C = 100 [E]; **c:** N = B − M = 80 [E]; **d:** X = N = 80 [V]. V = vertically opposite angles theorem; E = exterior angles theorem. Angle X is the unknown. Only labeled angles can be calculated, and numbered angles are given.)

relations between angles without finding the value of a new angle. This would not result in the diagram altering. For example, again with reference to Figure 1, subjects could type X = NV, which would provide them with a note that Angle X was vertically opposite to Angle N. This would not result in the diagram altering. In both cases the equations written would remain on the screen until the problem was solved. In addition to the above procedure, all subjects were asked to provide verbal protocols of their problem-solving techniques. These were tape recorded.

There were two groups of subjects. Twelve goal group subjects were required to solve 20 problems each. These required four moves for solution, using the two theorems. One of the angles was labeled X, and the goal of the problems was to find the value of X. The first and last 4 problems were counterbalanced within the group in a manner similar to the previous experiments. Twelve no-goal group subjects were given problems identical to those of the goal group except that the middle 12 problems had a nonspecific goal. Rather than being told, "Calculate the value of X," they were told, "Calculate the value of as many angles as possible." The angles that could be calculated were in fact identical to those that the goal group subjects had to calculate in order to attain the goal. The angle labeled X for the goal group was randomly allocated a letter other than X in the case of the no-goal group.

All subjects were familiarized with the two theorems and shown a diagram containing a triangle with some of its sides extended to produce a variety of vertically opposite and exterior angles. All vertically opposite and exterior angle relations were pointed out. In addition, subjects were told that although the problems to be presented could be solved in a variety of ways, only the vertically opposite and exterior angle theorems could be used. The computer program would not accept relations based on alternative theorems.

Results and Discussion

The problem-solving strategy used on the initial and final four critical problems is again of primary interest. Although this strategy could be readily discerned from the verbal protocols, unlike the physics experiments, the order in which equations were written was virtually identical for each subject on every problem. Equations were written in a forward-oriented mode even when the verbal protocols indicated that means–ends analysis was being used. Subjects were unwilling to say and write their problem solving steps simultaneously when working backward. As a consequence, when using means–ends analysis, they would work from the goal back to a given, stating each step but writing nothing. When working forward toward the goal, they would write each step thus leaving a written, forward-working record only, despite the use of means–ends analysis.

This distinction between physics and geometry problem solving may be of interest. We suggest that the presence of a geometric diagram provides problem solvers with a clear simultaneous representation of most of the problem and its problem states. This allows them to locate readily their current problem state and its relation to other states by reference to the diagram. There may consequently be a reduced need for problem solvers to provide themselves with a statement of each problem state they encounter. In algebra word problems, of which the currently used physics problems are an example, a similar representation of the problem is not normally available. The lists of connected problem states written down by subjects working backward may be an attempt to provide a cognitive equivalent to the geometric diagram.

Because no useful data were obtained from the written record, all analyses are based on verbal protocols. Table 10 provides the mean number of problems on which subjects worked forward for both the initial and final four problems. A subject was assumed to be working forward if he or she calculated the value of all required angles before mentioning the goal. Analysis of these results indicated a significant goal specificity effect, $F(1, 22) = 5.6$; a significant practice effect, $F(1, 22) = 12.1$; but a nonsignificant Goal × Practice interaction, $F(1, 22) = 2.2$. The interaction is of primary interest. We have predicted that the scores for the no-goal group should increase more rapidly due to practice than the scores of the goal group. More of the final four problems should be solved using a forward-oriented strategy by the no-goal group compared with the goal group. This has not occurred to a significant extent, thus failing to replicate the findings using the physics experiments.

Inspection of the means of Table 10 provides a possible reason for the lack of a significant

Table 10
Mean Number of Problems on Which Subjects Worked Forward on the Initial and Final Four Problems of Experiment 4

Group	Initial	Final
Goal	.4	1.1
No-goal	1.3	2.9

interaction. Although there is a substantial difference between means on the last four problems in the predicted direction, the means of the first four problems suggest that the groups may not have been analogous. The no-goal group had fewer problems solved in a backward manner, thus decreasing the extent to which a reduction in the use of this strategy could be observed. In fact, 4 of the 12 no-goal group subjects solved three or more of their initial problems in a forward manner. Only 1 goal group subject did likewise. There is obviously no opportunity for these subjects to switch from a backward-oriented to a forward-oriented strategy. The best we could expect is for these subjects to continue working forward. This in fact they did; all of them worked forward on all of the final four problems. The unequal proportions of these subjects in the two groups may have prevented a significant interaction by preventing a sufficiently substantial reduction in the scores of the no-goal group.

Because the primary purpose of the experiment was to test the prediction that a switch in strategy from a backward to a forward orientation would be facilitated by the use of a nonspecific rather than a specific goal, it may be appropriate to eliminate from the analysis those subjects who began by solving problems using a forward strategy. If we eliminate subjects who worked forward on three or more of the initial four problems, we are left with 8 no-goal subjects and 11 goal subjects. Table 11 provides the mean number of problems solved using a forward strategy on the first and last four problems for the two reduced groups. An ANOVA indicated no significant effect due to goal specificity, $F(1, 17) = 2.7$; a significant effect due to practice, $F(1, 17) = 15.4$; and a significant interaction, $F(1, 17) = 5.1$. Of the subjects who began by using a backward strategy, more in the no-goal than the goal group switched to a forward strategy.

Table 11
Mean Number of Problems on Which Subjects Worked Forward on the Initial and Final Four Problems of Experiment 4

Group	Initial	Final
Goal	.2	.8
No-goal	0	2.4

Note. These data exclude subjects who worked forward consistently.

Experiment 5

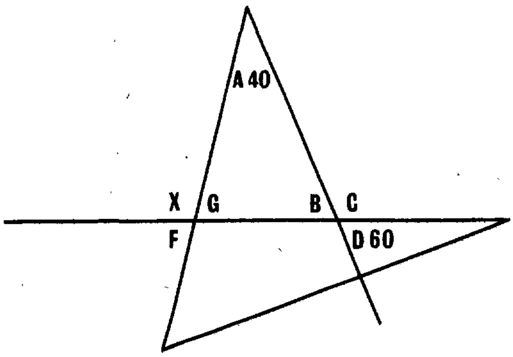
Experiment 4 has provided some evidence that reduced goal specificity on geometry problems facilitates a switch from the use of a means–ends to a forward-oriented strategy. Because the effect may have been partially obscured by an unequal assignment to groups of subjects initially using the two strategies of interest, Experiment 5 is in part designed to replicate Experiment 4. In addition, Experiment 5 used simpler problems requiring fewer moves for solution. The relation between the particular moves required for solution and the geometric configuration presented to problem solvers was also simplified by having only two basic configurations with each configuration requiring a common set of moves for solution. The many different configurations used in Experiment 4 may have prevented problem solvers from developing an effective representation of each configuration and from associating appropriate moves with it. If problem solvers learn to distinguish between different configurations and their associated moves, this may facilitate a strategy shift. If, in turn, reduced goal specificity facilitates such knowledge acquisition, this may result in substantial strategy-shift differences between goal and no-goal groups.

Because the order in which problem solvers entered equations did not yield usable data in Experiment 4, Experiment 5 relied on verbal protocols alone. Subjects were presented with a penciled diagram and had to verbalize a solution. This duplicated the part of Experiment 4 that produced analyzable results. The two theorems used in Experiment 4 were used again in Experiment 5.

Method

Subjects. The 32 subjects were 9- and 10-year students from a Sydney metropolitan high school.

Materials and procedure. Figure 2 provides examples of the two types of configurations used. All problems belonging to the category represented by Figure 2a had a given value for the angle vertically opposite to the base angle on the right-hand side of the triangle (in the example, Angle D = 60°). The exterior angle on the right-hand side of the triangle was unknown. The only other given angle was the apex of the triangle (Angle A = 40°). The goal



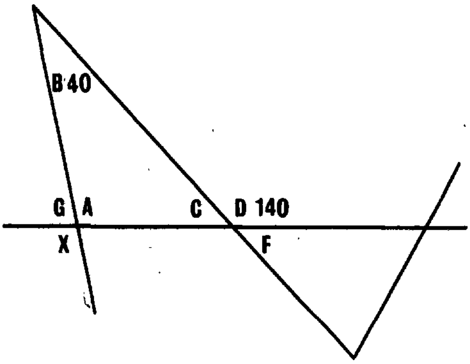


Figure 2. Examples of the two categories of problems used in Experiment 5. (Solution working forward: a: B = D = 60 [V]; X = A + B = 100 [E]. b: A = D − B = 100 [E]; X = A = 100 [V]. V = vertically opposite angles theorem; E = exterior angles theorem. Angle X is the unknown. Only labeled angles can be calculated, and numbered angles are given.)

was the exterior angle on the left-hand side of the triangle (Angle X). To solve the problem working forward, the vertically opposite angles theorem must be used first (B = D = 60) followed by the exterior angles theorem (X = A + B = 100). All problems in this category included the configuration bound by the labeled angles of Figure 2a, with the same angles labeled, although the particular labels altered from problem to problem. The unlabeled sections of the diagram also varied between problems.

Problems belonging to the category represented by Figure 2b were similar to those of the alternative category (as represented by Figure 2a) except that the unknowns and the goal were placed in different locations. The angle vertically opposite to the base angle on the right-hand side of the triangle was unknown (Angle F). The exterior angle on the right-hand side of the triangle was known (Angle D). The vertically opposite angle to the base angle on the left-hand side of the triangle provided the goal (Angle X). To solve these problems working forward, the exterior angles theorem (A = D − B = 100) followed by the vertically opposite angles theorem (X = A = 100) had to be used.

Twelve problems were used, with the first 2 and last 2 problems counterbalanced as in previous experiments. There were equal numbers of problems conforming to the two configurations with one of each configuration represented in the first 2 and last 2 problems. Sixteen subjects were assigned to both a goal group and a no-goal group. In other respects Experiment 5 was identical to Experiment 4.

Results and Discussion

As was the case with Experiment 3, there are too few critical problems to assign realistically each subject a score representing the number of problems on which a particular strategy was used. As a substitute, we classified each subject according to strategy. Twelve of the 16 no-goal group subjects worked forward on both of the final two problems. Four of the 16 goal group subjects worked forward on these problems. A Fisher exact probability test indicated a significant difference between the two groups on this measure (p = .003). On the first two problems, 1 no-goal group subject and none of the goal group subjects worked forward on both problems. This difference is not significant (p = .24).

Far more no-goal than goal subjects have altered their strategy. The use of a no-goal procedure has clearly been effective in inducing subjects to abandon the use of means–ends analysis and to use a forward-oriented strategy instead.

Experiment 6

Experiments 4 and 5 have indicated that reduced goal specificity hastens the development of a forward-oriented strategy. The two experiments were not designed to demonstrate that the use of this strategy was associated with improved knowledge acquisition. Thus, although no-goal group subjects may have been using the problem-solving strategy previously shown to be associated with expertise, we have no independent evidence that they were, in fact, more expert. Furthermore, although one of the measures of expertise used in Experiment 1 was the ability to categorize problems according to solution modes, this was not tested in any of the preceding experiments dealing with goal specificity. If a nonspecific goal enhances the development of expertise, we might expect that no-goal subjects are better able to categorize problems according to solution mode. The measure used in Experiment 1—differential rates of strategy alteration for different categories of problems—was not usable in the other experiments because by the time no-goal subjects were solving their critical problems, they tended to work forward on all problems irrespective of category. An alternative measure is required. Experiment 6 was designed to test whether reduced goal specificity enhanced learning of geometry problem structures using such a measure.

Einstellung can be used as a vehicle to test whether specific knowledge structures have developed. This may appear paradoxical, because Einstellung is normally associated with ineffective problem-solving techniques. Although this is true, the effect is nevertheless partly due to problem solvers' acquiring schemas that are inappropriately applied to new problems. These schemas can equally facilitate subsequent problem solving if applied to appropriate problems. For example, Sweller (1980) and Sweller and Gee (1978), working within a hypothesis theory framework (see Levine, 1975), found that an identical set of initial problems could either massively facilitate or retard subsequent problem solving, with retardation providing an example of Einstellung. There were critical factors determining whether facilitation or retardation occurred: (a) Problem solvers had to perceive initial and critical problems as being closely related; and (b) if facilitation was to occur, this perception had to be essentially correct, whereas if retardation was to occur, the perception had to be incorrect. In all other respects the conditions determining facilitation or retardation appeared identical. It is important to note that the same schema acquired in the initial problems could cause subsequent facilitation or retardation depending on whether it was appropriate or inappropriate to solution of the critical problem. Furthermore, Mawer and Sweller (1982) and Sweller et al. (1982) provided evidence that if problems were solved without the acquisition of an appropriate schema, facilitation and/or retardation of subsequent problem solving tended not to occur. We are thus in a position to use the occurrence of Einstellung as evidence of schema acquisition.

The geometry problems used in Experiment 6 were structured to readily allow manifestations of Einstellung. Each problem conformed to one of two problem types with each type requiring a distinct mode of solution. Critical Einstellung problems were devised, which appeared to belong to one of the two categories but in fact belonged to the other. These problems could be used to gauge the extent to which schemas based on the initial solution categories had been acquired. This in turn could be related to the problem-solving strategy used and the goal specificity of the preliminary problems.

Method

Subjects. The 32 subjects were 9- and 10-year students from a Sydney metropolitan high school.

Materials and procedure. Figure 3 provides examples of the two categories of problems used. Problems in the category exemplified by Figure 3a had the triangle oriented as shown in the diagram with the unknown (X) on the left-hand side. Working forward, they could be solved by first applying the vertically opposite angles theorem ($C = A = 80$) followed by the exterior angles theorem ($X = C + G = 140$). Problems in the category exemplified by Figure 3b had the triangle oriented as shown in the diagram with the unknown on the right-hand side and could be solved by first applying the exterior angles theorem ($C = B - A = 60$) followed by the vertically opposite angles theorem ($X = C = 60$).

Figures 3c and 3d demonstrate the Einstellung problems. The problem depicted in Figure 3c appears similar to those exemplified by Figure 3a. In fact, if the same procedure is used by beginning at Angle G and using the vertically opposite angles theorem ($D = G = 50$) followed by the exterior angles theorem ($A = B + D = 110$), Angle A is calculated instead of Angle X. To solve the problem, subjects must start from Angle C and, by using the exterior angles theorem ($F = C - B = 70$) followed by the vertically opposites angles theorem ($X = F = 70$), arrive at a value for Angle X by way of Angle F. The order in which theorems must be used is identical to the problem of Figure 3b rather than that of Figure 3a. A similar relation holds between the problems exemplified in Figures 3b and 3d.

The general procedure was identical to that used in Experiment 5 except that there were 14 rather than 12 problems. The 2 additional problems were the Einstellung problems of Figures 3c and 3d. Thus the 16 goal group subjects were presented 12 conventional problems followed by the Einstellung problems, whereas the 16 no-goal group subjects were presented identical problems except that Problems 3–10 did not have a specific goal. As was the case in Experiment 5, Problems 11 and 12 were again used to determine the strategy favored after solving the initial problems.

Results and Discussion

The shift from a means–ends to a forward-oriented strategy was substantial for the no-

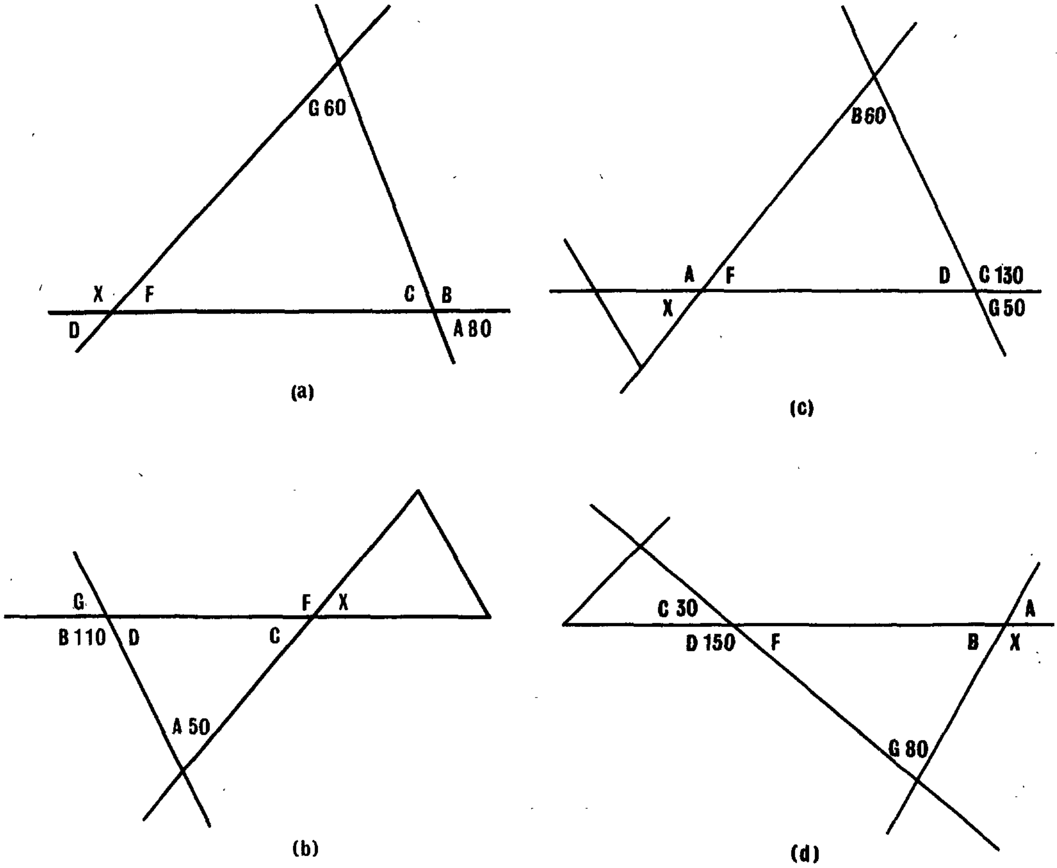


Figure 3. Examples of the two categories of problems used in Experiments 6 and 7 (a and b) and the two Einstellung problems (c and d). (Solution working forward: **a:** C = A = 80 [V]; X = C + G = 140 [E]. **b:** C = B − A = 60 [E]; X = C = 60 [V]. **c:** F = C − B = 70 [E]; X = F = 70 [V]. **d:** F = C = 30 [V]; X = F + G = 110 [E]. V = vertically opposite angles theorem; E = exterior angles theorem. Angle X is the unknown. Only labeled angles can be calculated, and numbered angles are given.)

goal group but slight in the case of the goal group. Ten of the 16 no-goal subjects worked forward on both Problem 11 and 12, whereas only 3 of the goal subjects worked forward on both of these problems. Using a Fisher exact test this difference is significant ($p = .007$). Using this measure on the first two problems, the difference between groups was not significant ($p = .3$). One no-goal and 2 goal subjects worked forward on both of these problems. (All 3 of these subjects continued to work forward on Problems 11 and 12.)

The primary purpose of this experiment is to gauge the extent to which the above difference in solution strategy reflects differences in the degree to which problem-solving schemas have been acquired during the initial problems. We have argued that Einstellung can be used as an indicator of the use (albeit inappropriate) of these schemas. If problem solvers who are working forward are using schemas that also predispose them to Einstellung, we would expect a positive correlation between these two variables. In addition, because no-goal subjects worked forward in most cases whereas goal subjects did not, we would expect a difference between the two groups in the extent to which Einstellung was demonstrated.

With respect to Problems 11 and 12, subjects were given a score of 0, 1, or 2, representing the number of problems on which they worked forward. They were similarly given scores of 0, 1, or 2 depending on the number of problems on which they demonstrated Einstellung. A subject was assumed to be under the influence of mental set if he or she calculated first the value of an angle identical to the one that led to solution on the earlier problems. Angle D of Figure 3c is an example of such an angle. In the problem of Figure 3c this angle cannot assist in calculating Angle X. A Spearman rank-order correlation using the 32 subjects from both groups indicated a significant correlation of .41 between strategy used and Einstellung. Subjects who worked forward tended to demonstrate Einstellung. Given this correlation and given the difference between groups in strategy used, we might expect a difference between groups in the extent to which Einstellung was demonstrated. In fact, both groups had 5 subjects who exhibited Einstellung on both problems. There were more subjects in the no-goal group (14 out of 16) than the goal group (11 out of 16) who demonstrated Einstellung on at least one of the two problems used to test for mental set. This difference, nevertheless, was not significant using a Fisher exact probability test ($p = .11$).

The failure to obtain a significant difference between groups on the Einstellung problems may in part have been due to ceiling effects. There are essentially two factors, relevant to present considerations, that determine whether Einstellung will occur (see Sweller & Gee, 1978). These are, first, the strength of the schemas or the degree to which the schemas have been assimilated by subjects. These schemas give rise to Einstellung. The strength of the schemas, and hence Einstellung, can be determined by, for example, the number of preliminary problems. The second factor of current concern determining whether the effect will occur is the extent to which problem solvers perceive the critical problems as being related to the initial problems. In the present experiment, the strength of the schemas may have been sufficiently high for the mental set effect to be close to the asymptote for both groups. This would result in any difference between them not being detectable by a Fisher exact test. The fact that all subjects did not demonstrate Einstellung on both problems despite the strength of the effect can be attributed to the lack of essential perceptual identity between the preliminary and critical problems. Although perceptual identity can be obtained on some types of problems, it is not attainable on geometry problems. Minor diagrammatic differences as occurred in the present experiment must invariably intrude, although they must be minimized as they were in the current study.

This analysis assumes that a schema may be sufficiently strong to generate Einstellung maximally but not strong enough to induce problem solvers maximally to work forward. The fact that goal group subjects tended to work backward but also tended to demonstrate Einstellung supports this suggestion. Inspection of results generated by individual subjects provides further evidence. Six subjects, one in the no-goal group and five in the goal group, worked backward on the two problems preceding the Einstellung problems but nevertheless exhibited mental set on at least one problem. These subjects were classified as working backward because they first mentioned the goal. They followed a similar strategy on the Einstellung problems. The goal was first mentioned followed by the angles, which would have been apropriate in solving the preceding problems but were no longer appropriate on the Einstellung problems. The schema used by these subjects to generate a solution was presumably sufficiently well assimilated to cause Einstellung but not sufficiently strong to eliminate any mention of the goal.

Given this analysis, a difference between groups in terms of Einstellung should be obtainable by reducing the strength of the schema used to generate the phenomenon. This might be accomplished by reducing the number of preliminary problems. Fewer problems may have less effect on the no-goal group than the goal group, because the former group may, in the present experiment, have developed appropriate schemas to an extent far greater than that needed to induce both a strategy change and Einstellung. The goal group, on the other hand, with fewer preliminary problems, may not demonstrate Einstellung to the same extent as in the present experiment. Experiment 7 was designed to test this suggestion.

Experiment 7

In order to eliminate the ceiling effects that appear to have affected Einstellung in Experiment 6, this experiment attempted to reduce the extent to which schemas generating the effect could develop. This was done by reducing the number of preliminary problems from 12 to 6.

Method

Subjects. The 34 subjects were 9- and 10-year students from a Sydney metropolitan high school.

Materials and procedure. The general procedure was identical to that used in Experiment 6. The no-goal group (17 subjects) was presented four problems with a nonspecific goal. Two of these used a configuration similar to Figure 3a; the other two were similar to Figure 3b. These problems were followed by two conventional problems. For half of the group the latter two problems were identical to Problems 1 and 2 of Experiment 6, whereas for the remaining subjects Problems 11 and 12 were used. These problems were used to ascertain the strategy used on conventional problems. The use of two sets of problems duplicated the procedure used in Experiment 6. In that experiment and the previous experiments, the early problems were counterbalanced with the critical problems resulting in half of each group being presented with problems differing from the other half. In order to reduce the number of initial problems, the two conventional problems first presented to subjects in the previous experiment were eliminated. The two problems used to provide information concerning the strategy used were followed by two Einstellung problems identical to those used in Experiment 6 (see Figures 3c and 3d). The goal group (17 subjects) was identical to the no-goal group except that the first four problems incorporated a conventional goal.

Results and Discussion

Eleven of the 17 no-goal subjects used a forward-oriented strategy on both Problems 5 and 6. Three goal subjects used this strategy on these problems. A Fisher exact probability test indicated a significant difference between the two groups ($p = .003$). A significant Spearman rank-order correlation of .67 was obtained between the number of problems on which a forward-oriented strategy was used on these two problems and the number of problems on which Einstellung was demonstrated. All 17 no-goal subjects exhibited Einstellung on at least one of the critical problems. The equivalent score for the goal group was 10 out of the 17 subjects. This difference is significant by a Fisher exact probability test ($p = .001$).

The results of this experiment are quite unambiguous. Problems with nonspecific goals have yielded both a strategy alteration and Einstellung to a greater extent than conventional problems with specific goals. The two effects have been highly correlated. The simplest explanation for this conjunction of effects is that the acquisition of schemas appropriate for the initial problems has been enhanced where nonspecific goals have been used. Problem solvers have more rapidly learned to identify the two categories of problems presented to them and the moves appropriate to those categories. As a consequence, they have been able to work forward rather than use means–ends analysis, which in turn has resulted in Einstellung on problems that appeared similar but in fact were different.

General Discussion

Several broad conclusions can be derived from these experiments. The results indicate that the switch from a means–ends strategy frequently used by novice problem solvers to a forward-chaining strategy frequently used by experts can be observed readily using the same subjects. This may be contrasted with the discrete groups of novices and experts used in previous studies. Other normal accompaniments of expertise, such as reduced moves to solution and reduced use of equations with unsubstituted variables, were associated with the switch in strategy. Evidence for acquired problem categorization was indicated both by differential rates of strategy alteration for different problem categories and by the occurrence of mental set associated with problem solutions relevant to particular categories. Our major finding concerned the rate at which expertise, as defined by these several factors, developed. The use of conventional problems presented in a conventional manner was relatively ineffective in altering the problem-solving modes characteristically used by novices. Expertise was acquired more rapidly if problems were presented with nonspecific rather than specific goals.

We may account for these findings by assuming that the information processing required to used means–ends analysis retards the acquisition of schemas characteristic of expert problem solvers. A problem solver who is attempting to reduce differences between his or her current problem state and the goal may not be processing those aspects of the problem structure required for expert problem solving. He or she may not be attending to those features that allow categorization into convenient problem types and to which particular solution steps can be applied. Means–ends analysis can operate perfectly without the acquisition of these schemas. Similarly, the development of an adequate representation of essential components of problem solving, such as, in the present experiments, equations or theorems, may be retarded if a large segment of processing capacity is devoted to reducing differences between current problem states and the goal. We thus make the paradoxical suggestion that the strategy conventionally used by novices to solve problems is unlikely to allow the rapid development of expertise.

The experiments reported in this article varied goal specificity in order to reduce the use of means–ends analysis. This in turn facilitated the acquisition of appropriate schemas characteristic of expert problem solvers. There are theoretical grounds for assuming that alterations in goal specificity constitute one example of a class of manipulations, all of which might have similar effects. We suggest that the particular schemas acquired during problem solving are heavily dependent on the method by which problem solving moves are controlled. The use of a means–ends strategy results in the problem goal being the primary mechanism controlling moves. Reductions in goal specificity shift control of moves from the goal to the givens in the first instance. Subsequently, control is vested in a combination of the givens and in previously made moves. In the present experiments, previously made moves consist of calculated values of kinematics variables or angles. Both the givens and previously made moves are essential in categorizing problems according to solution schemas. Acquiring a schema requires learning which moves should be made when presented with a particular problem type. Under many circumstances, a problem type is largely identified by its givens (see Hinsley, Hayes, & Simon, 1977). We might expect that subjects would more readily learn to distinguish problem types and their associated appropriate moves if moves are controlled, indeed generated solely, by problem givens and the previous moves needed to attain the current problem state. This is more likely to occur when goal specificity is low. It may also occur under other circumstances, which we outline.

Any mechanism that provides problem solvers with feedback concerning the appropriateness of their moves could be expected to decrease control of those moves by the goal. Feedback from external sources or from the structure of the problem can act as an alternative control mechanism to the goal. Unlike the goal, which through means–ends analysis emphasizes future possible moves (see Egan & Greeno, 1974), feedback emphasizes immediately preceding moves. As suggested above, any control mechanism that has this emphasis may facilitate the acquisition of appropriate schemas. The use of subgoals may provide feedback in this manner (see Mawer & Sweller, 1982; Sweller, 1983). The attainment of a subgoal known to lead to the goal indicates to problem solvers that the preceding moves were appropriate. Subgoals, by deemphasizing the goal, may have an effect similar to the use of nonspecific goals.

The schemas acquired by problem solvers in the present experiments were inevitably simpler and more basic than those studied by other researchers using experts of many years standing in the subject domains of interest. Our subjects learned that a particular problem category could be solved by a particular sequence of equations or theorems. Surface structures such as the shape of a geometric diagram played an important role in these schemas. This contrasts with, for example, Chi et al.'s (1982) experts who categorized problems according to broad physical principles that could then be used for solution. We believe, nevertheless, that with respect to the factors reported in the present experiments, there is no evidence for qualitative differences between simple and complex or between surface and deep structure schemas. It requires much more time to acquire a complex schema, but many of its important properties appear indistinguishable from a simpler schema. The use of schemas to solve problems seems to result in problem categorization, strategy alterations, and more efficient solution under appropriate circumstances, irrespective of the complexity of the problems and their related schemas. We thus might expect the present findings concerning the effects of goal specificity to generalize to more complex problems.

One other comment concerning the nature of the particular problems used is needed. With the exception of Experiment 1, all problems were structured in such a manner as to prevent the possibility of calculating values for irrelevant variables or angles. This was essential in order to prevent no-goal groups from calculating variables or angles not calculated by goal groups. Valid comparisons consequently could be made between the two groups. Under more realistic conditions allowing the calculation of irrelevant values, we might expect the information gained by a no-goal procedure to be even greater than in the present experiments. A no-goal procedure should result in a greater exploration and thus greater knowledge of the available problem space than a conventional procedure. Problem solvers who calculate all that can be calculated from a set of givens can be expected to learn more than problem solvers who merely calculate values needed for solution. This, nevertheless, was not the focus of the present experiments and would have been a confounding variable had it not been controlled for.

In conclusion, we suggest that the real goal of problem solution needs to be analyzed. If the goal is simply to solve a particular problem, then a conventional problem format is satisfactory. Alternatively, a reduced goal specificity procedure might be preferred if, as normally occurs in educational contexts, the real goal is to gain additional insights into the problem structure and the structure of the subject matter that gave rise to the problem.

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