Parallel Programming: Speedups and Amdahl's law

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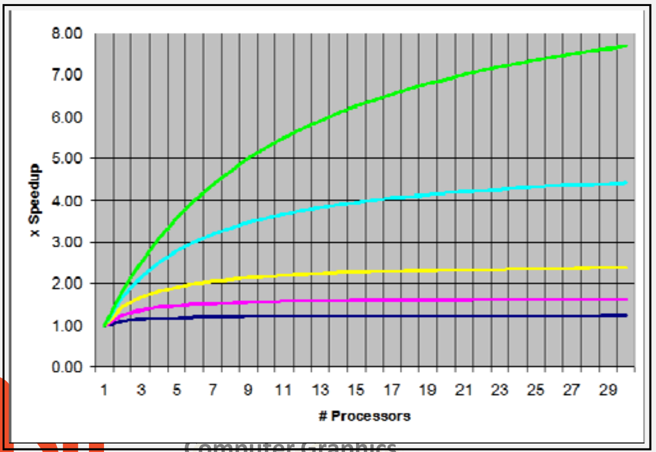
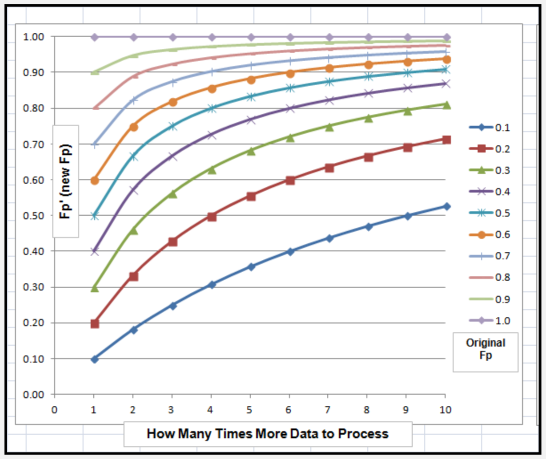




Definition of Speedup

If you are using **n** processors, your ***Speedup*$_n$** is:

$$Speedup_n = \frac{T_1}{T_n}$$

where T_1 is the execution time on one core and T_n is the execution time on n cores. Note that $Speedup_n$ should be > 1.

And your ***Speedup Efficiency*$_n$** is:

$$Efficiency_n = \frac{Speedup_n}{n}$$

which could be as high as 1., but probably never will be.

However, Multicore is not a Free Lunch: Amdahl's Law

If you put in *n* processors, you should get *n* times Speedup
(and 100% Speedup Efficiency), right? Wrong!

There are always some fraction of the total operation that is inherently sequential and cannot be parallelized no matter what you do. This includes reading data, setting up calculations, control logic, storing results, etc.

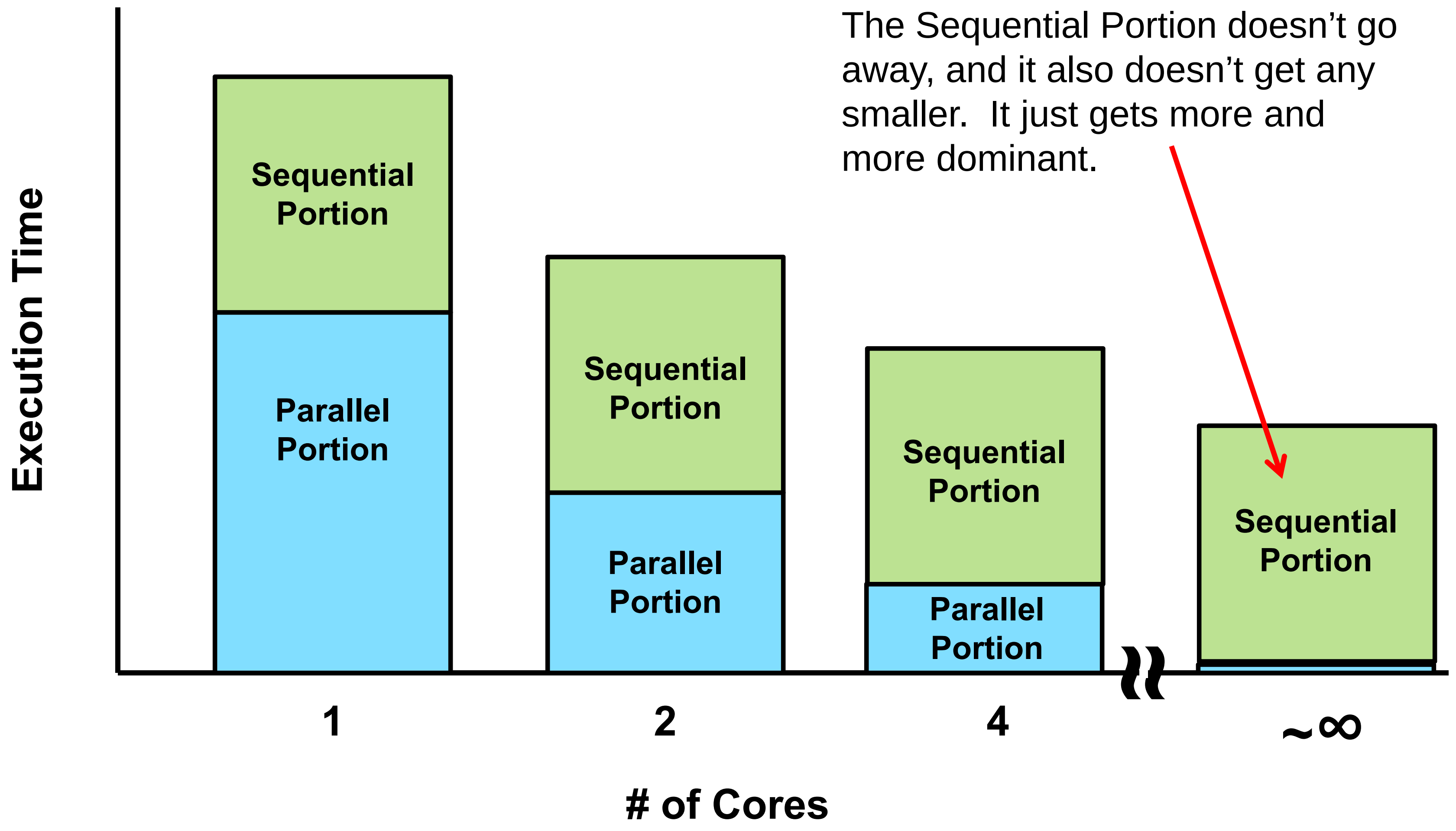
If you think of all the operations that a program needs to do as being divided between a fraction that is parallelizable and a fraction that isn't (i.e., is stuck at being sequential), then **Amdahl's Law** says:

$$Speedup_n = \frac{T_1}{T_n} = \frac{1}{\frac{F_{parallel}}{n} + F_{sequential}} = \frac{1}{\frac{F_{parallel}}{n} + (1 - F_{parallel})}$$

This fraction can be reduced by deploying multiple processors.

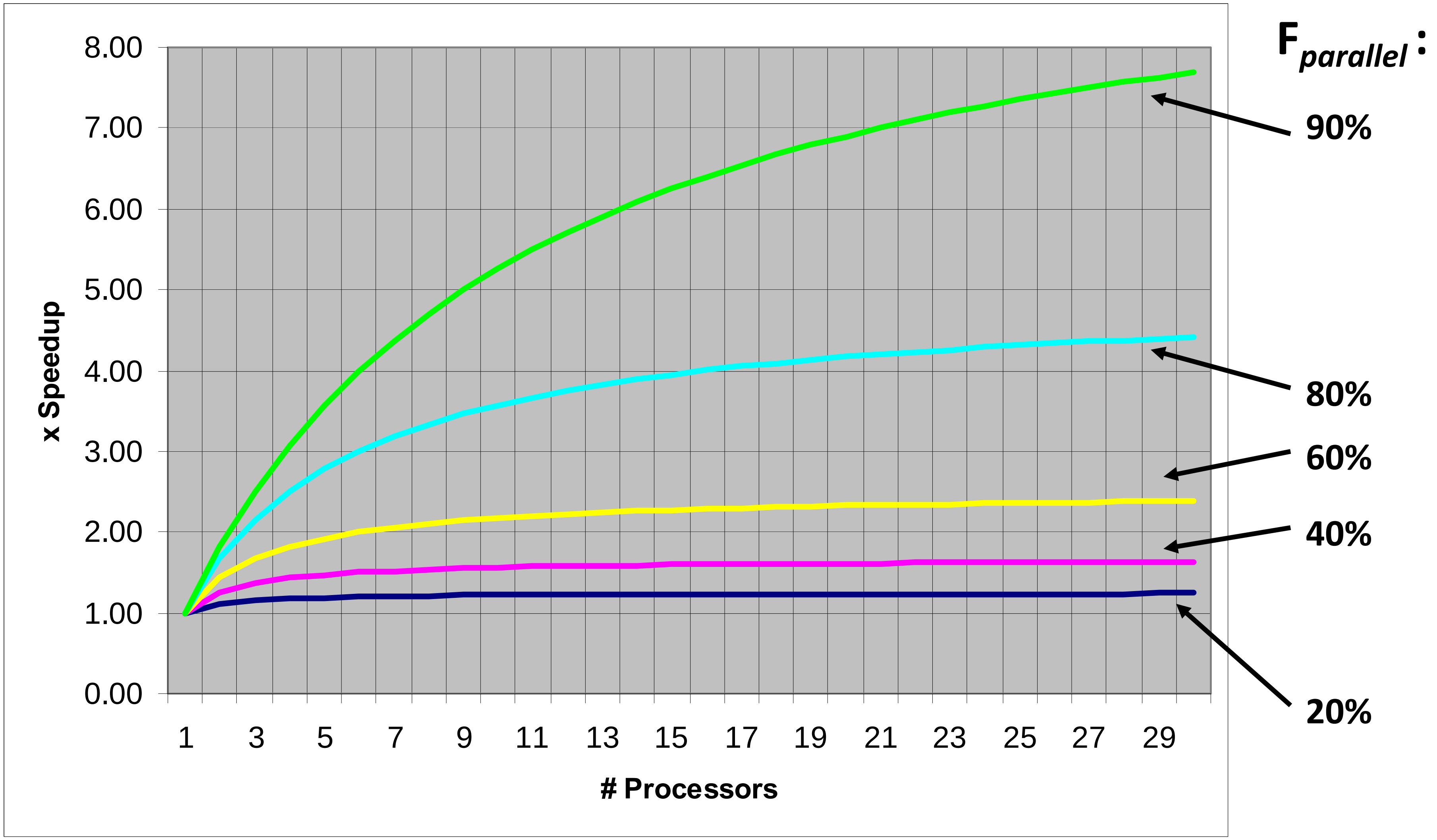
This fraction can't.

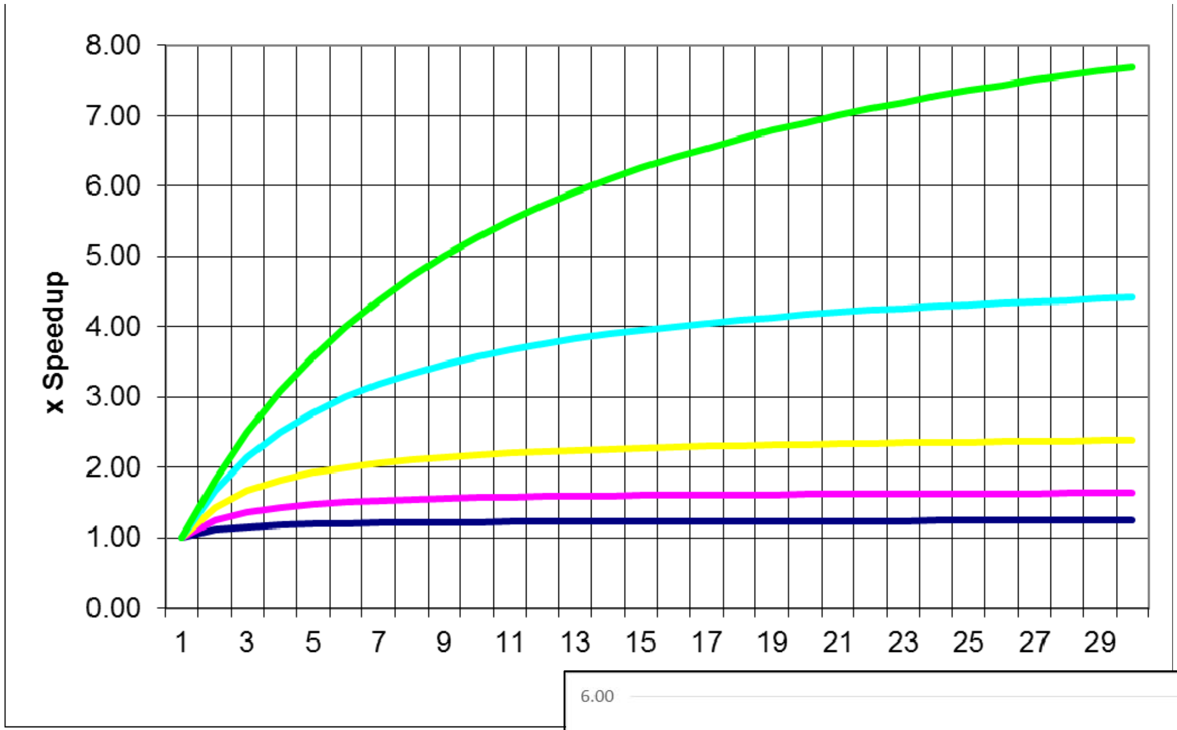
A Visual Explanation of Amdahl's Law

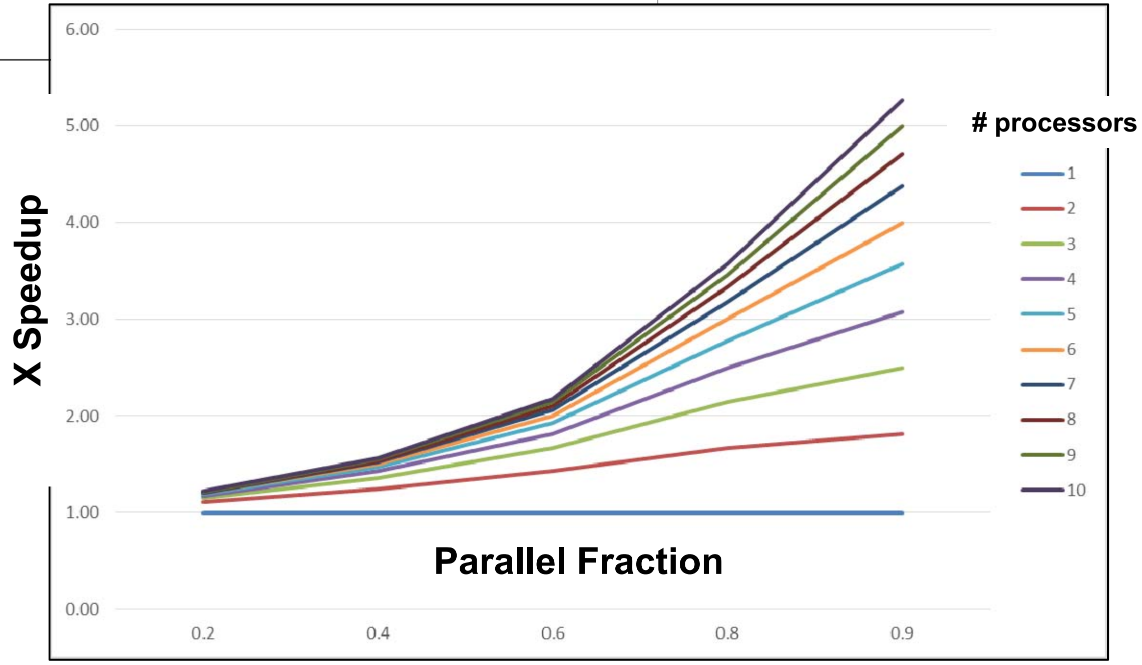


The Sequential Portion doesn't go away, and it also doesn't get any smaller. It just gets more and more dominant.

Amdahl's Law as a Function of Number of Processors and $F_{parallel}$




x Speedup
8.00
7.00
6.00
5.00
4.00
3.00
2.00
1.00
0.00
1
3
5
7
9
11
13
15
17
19
21
23
25
27
29


X Speedup
6.00
5.00
4.00
3.00
2.00
1.00
0.00
0.2
0.4
0.6
0.8
0.9
Parallel Fraction
# processors
1
2
3
4
5
6
7
8
9
10

Amdahl's Law

Note that these fractions put an upper bound on how much benefit you will get from adding more processors:

$$\max Speedup = \lim_{n\to\infty} Speedup = \frac{1}{F_{sequential}} = \frac{1}{1 - F_{parallel}}$$

Fparallel	maxSpeedup
0.00	1.00
0.10	1.11
0.20	1.25
0.30	1.43
0.40	1.67
0.50	2.00
0.60	2.50
0.70	3.33
0.80	5.00
0.90	10.00
0.95	20.00
0.99	100.00

You can also solve for $F_{parallel}$ using Amdahl's Law if you know your speedup and the number of processors

Amdahl's law says:

$$S = \frac{T_1}{T_n} = \frac{1}{\frac{F}{n} + (1-F)} \Rightarrow \frac{1}{S} = \frac{F}{n} + (1-F) = 1 + \frac{F - nF}{n} \Rightarrow \frac{1}{S} - 1 = F\frac{(1-n)}{n}$$

Solving for F:

$$F = \frac{\frac{1}{S} - 1}{\frac{1-n}{n}} = \frac{\frac{T_n}{T_1} - 1}{\frac{1-n}{n}} = \frac{\frac{T_n - T_1}{T_1}}{\frac{1-n}{n}} = \frac{\frac{T_1 - T_n}{T_1}}{\frac{n-1}{n}} = \frac{n(T_1 - T_n)}{T_1(n-1)} = \boxed{\frac{n}{(n-1)}\frac{T_1 - T_n}{T_1}} = \boxed{\frac{n}{(n-1)}\left(1 - \frac{1}{Speedup}\right)}$$

Use this if you know the timing

Use this if you know the speedup

If you've got several (n,S) values, you can take the average (which is actually a least squares fit):

$$F_i = \frac{n_i}{(n_i - 1)}\frac{T_1 - T_{n_i}}{T_1}, i = 2..N$$

$$\bar{F} = \frac{\sum_{i=2}^{N} F_i}{N-1}$$

note that when i=1, $T_{ni} = T_1$

A More Optimistic Take on Amdahl's Law: Gustafson's Observation

Gustafson observed that as you increase the number of processors, you have a tendency to attack larger and larger versions of the problem. He also observed that when you use the same parallel program on larger datasets, the parallel fraction, *F_p*, increases.

Let P be the amount of time spent on the parallel portion of an original task and S spent on the serial portion. Then

$$F_p = \frac{P}{\underset{\uparrow\ \text{Parallel Time}}{P} + \underset{\nwarrow\ \text{Serial Time}}{S}}$$

or

$$S = \frac{P - PF_p}{F_p}$$

Without loss of generality, we can set *P*=1 so that, really, *S* is now a fraction of *P*. We now have:

$$S = \frac{1 - F_p}{F_p}$$

A More Optimistic Take on Amdahl's Law: Gustafson's Observation

We know that if we multiply the amount of data to process by *N*, then the amount of parallel work becomes *NP*. Surely the serial work must increase too, but we don't know how much. Let's say it doesn't increase at all, so that we know we are getting an upper bound answer.

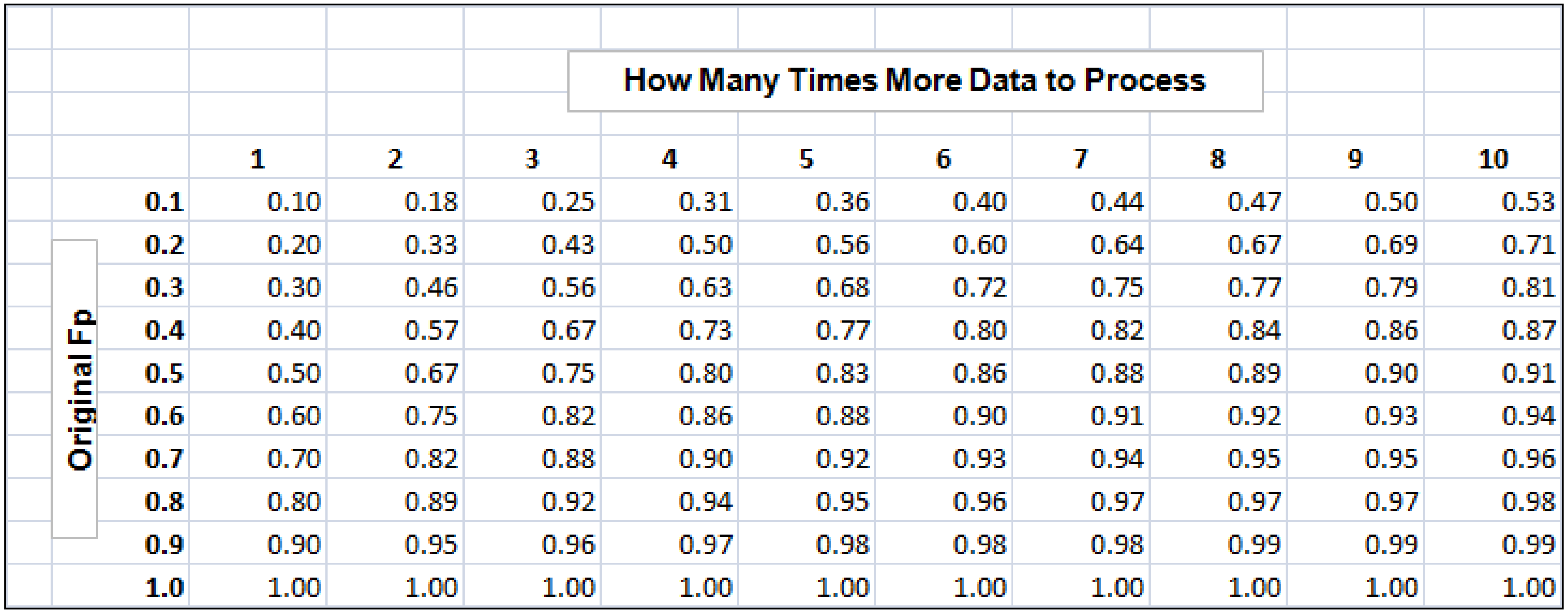
In that case, the new parallel fraction is: $F_p^{'} = \frac{P'}{P'+S} = \frac{NP}{NP+S}$

And substituting for *P* (=1) and for *S*, we have:

$$F_p^{'} = \frac{N}{N+S} = \frac{N}{N + \frac{1-F_p}{F_p}}$$

A More Optimistic Take on Amdahl's Law: Gustafson's Observation

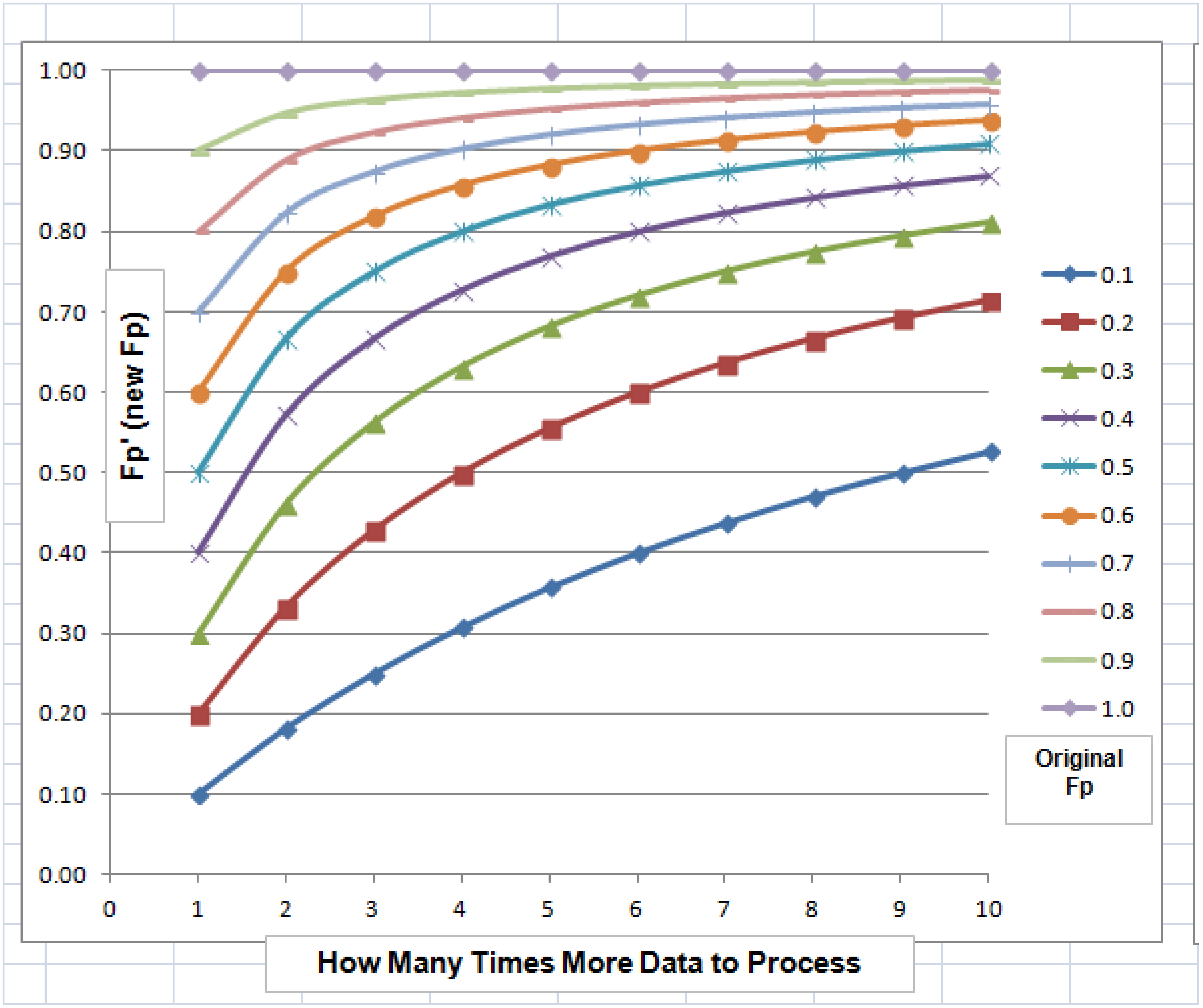
If we tabulate this, we get a table of F_p' values:



		How Many Times More Data to Process									
		1	2	3	4	5	6	7	8	9	10
Original Fp	0.1	0.10	0.18	0.25	0.31	0.36	0.40	0.44	0.47	0.50	0.53
	0.2	0.20	0.33	0.43	0.50	0.56	0.60	0.64	0.67	0.69	0.71
	0.3	0.30	0.46	0.56	0.63	0.68	0.72	0.75	0.77	0.79	0.81
	0.4	0.40	0.57	0.67	0.73	0.77	0.80	0.82	0.84	0.86	0.87
	0.5	0.50	0.67	0.75	0.80	0.83	0.86	0.88	0.89	0.90	0.91
	0.6	0.60	0.75	0.82	0.86	0.88	0.90	0.91	0.92	0.93	0.94
	0.7	0.70	0.82	0.88	0.90	0.92	0.93	0.94	0.95	0.95	0.96
	0.8	0.80	0.89	0.92	0.94	0.95	0.96	0.97	0.97	0.97	0.98
	0.9	0.90	0.95	0.96	0.97	0.98	0.98	0.98	0.99	0.99	0.99
	1.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

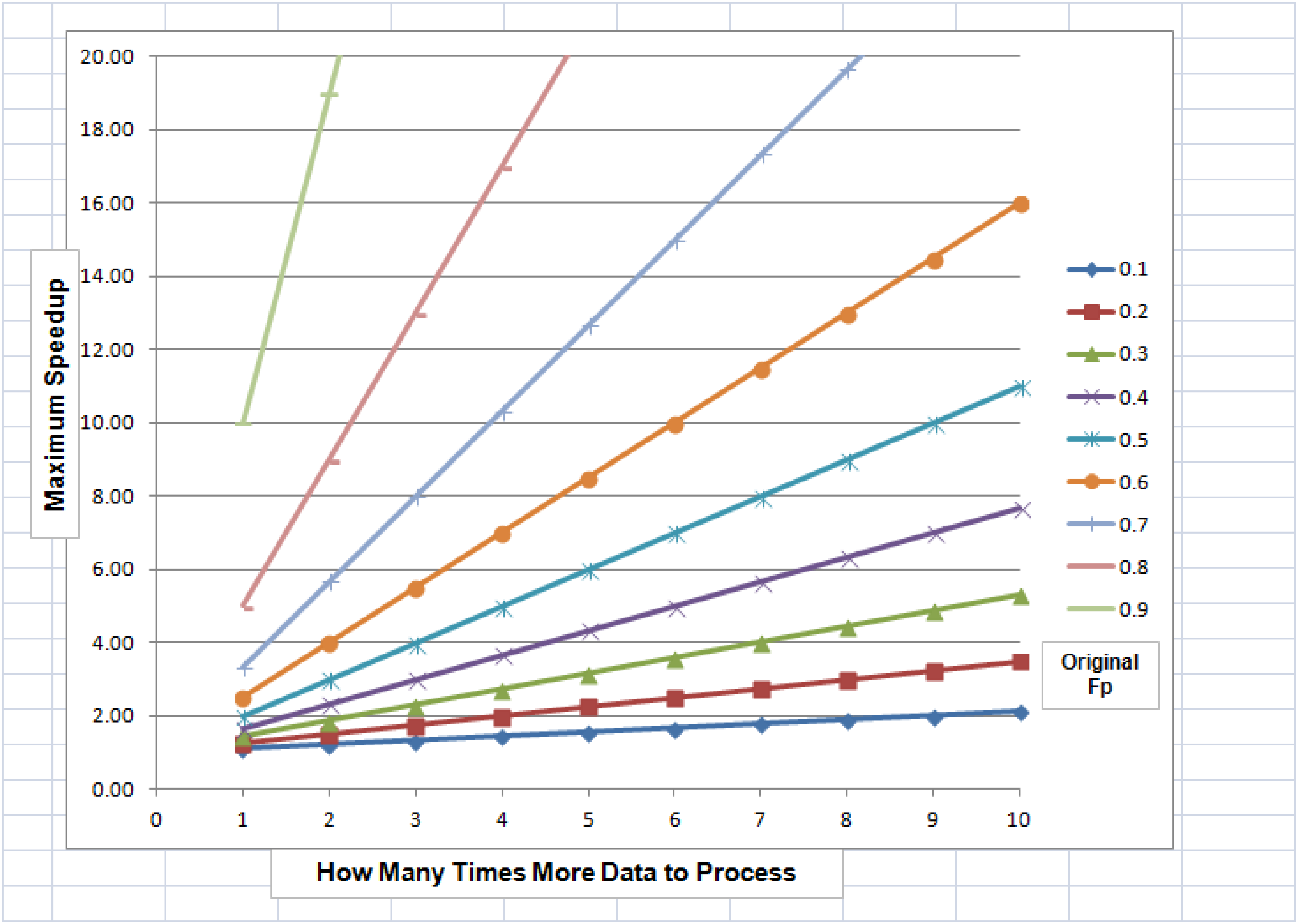
A More Optimistic Take on Amdahl's Law: Gustafson's Observation

Or, graphing it:



A More Optimistic Take on Amdahl's Law: Gustafson's Observation

We can also turn F_p' into a Maximum Speedup:


Maximum Speedup
How Many Times More Data to Process
Original Fp