# Distance and Similarity

Imagine three sentences (documents):

1. I love you.
2. I love you, you.
3. Stocks dropped.

} Intuitively, we can tell that sentence 1 and 2 are much more "similar" than 3.

**Step 1**: Vectorize the documents

So far, we're only learned about count vectorization (word count) but we'll discuss TF-IDF and embeddings soon.

| | I | love | you | stocks | dropped |
|---|---|---|---|---|---|
| 1. I love you. → | 1 | 1 | 1 | 0 | 0 |
| 2. I love you, you → | 1 | 1 | 2 | 0 | 0 |
| 3. Stocks dropped. → | 0 | 0 | 0 | 1 | 1 |

How do we quantify the degree of similarity between these documents?

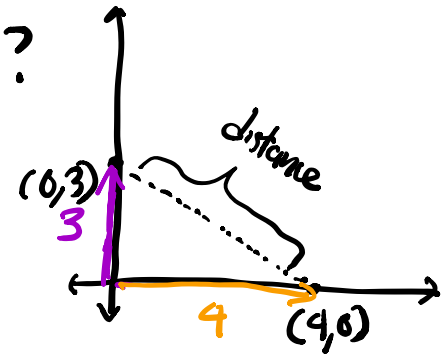
**Option #1**: Euclidean Distance

Remeber Pythagorean Theorem?

$$a^2 + b^2 = c^2$$

$$3^2 + 4^2 = c^2$$

$$c = \sqrt{3^2 + 4^2} \Rightarrow c = \sqrt{25} = 5$$



Pythagorean theorem works in 2 dimensions $(x, y)$, but we can extend it to any # of dimensions:

$$\text{distance}(a,b) = \sqrt{(a_1-b_1)^2 + (a_2-b_2)^2 + \dots (a_n-b_n)^2}$$

$$= \sqrt{\sum_i^n (a_i-b_i)^2}$$

Going back to our three sentences:

| | I | love | you | stocks | dropped |
|---|---|---|---|---|---|
| A. I love you. → | 1 | 1 | 1 | 0 | 0 |
| B. I love you, you → | 1 | 1 | 2 | 0 | 0 |
| C. Stocks dropped. → | 0 | 0 | 0 | 1 | 1 |

distance of sentences

$$\text{distance}(A,B) = \sqrt{(A_1-B_1)^2 + (A_2-B_2)^2 + (A_3-B_3)^2 + (A_4-B_4)^2 + (A_5-B_5)^2}$$

$$= \sqrt{(1-1)^2 + (1-1)^2 + (1-2)^2 + (0-0)^2 + (0-0)^2}$$

$$= \sqrt{0+0+(-1)^2+0+0}$$

$$= \boxed{1}$$

$$\text{distance}(A,C) = \sqrt{(1-0)^2 + (1-0)^2 + (1-0)^2 + (0-1)^2 + (0-1)^2}$$

$$= \sqrt{1+1+1+(-1)^2+(-1)^2}$$

$$= \sqrt{5}$$

$$\approx \boxed{2.24}$$

A is closer to B than C

But what is the issue with Euclidean distance?

1. It is sensitive to the norms (magnitudes) of the sentence vectors. For instance:

A. I love I love $\Rightarrow$ [2, 2] (I love)

B. I love $\Rightarrow$ [1, 1]

} These are basically the same! One is just a longer sentence

$$\text{distance}(A,B) = \sqrt{(2-1)^2 + (2-1)^2}$$

$$= \sqrt{2}$$ ← conceptually this should be $\approx 0$, since A and B are extremely similar.

**Norm:** the norm of a vector x is

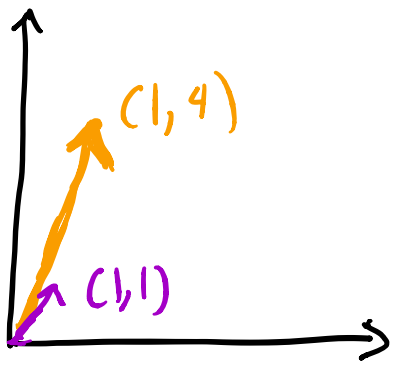
$$\|x\|_2 = \sqrt{x_1^2 + x_2^2 + x_3^2 + \ldots + x_n^2}$$

It represents the "strength" of a vector. In natural language processing, when you count vectorize a document, the longer documents will have higher norms.

A. I love love love love $\Rightarrow$ [1, 4] (I love)

B. I love $\Rightarrow$ [1, 1]

A has a higher norm than B



2. Euclidean Distance performs poorly in high-dimensional feature spaces: in high dimensions, everything will seem far apart; the ratio between a point's farthest and closest neighbors approaches 1, which essentially means all points are "relatively" uniformly distant from one another.

## An alternative? Cosine Similarity

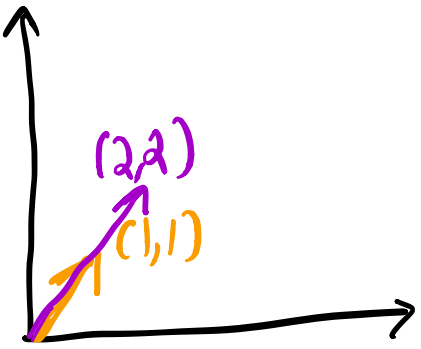
$$\cos(a,b) = \frac{a \cdot b}{\|a\|_2 \times \|b\|_2}$$

→ the numerator is a dot product, and indicates the extent to which the a and b vectors agree in their direction.

↓ the denominator is a "normalizing" factor that helps to account for the different magnitudes of a and b.

A. I love ⇒ [1,1]

B. I I love love ⇒ [2,2]




basically the same document!

Cosine similarity measures the "cosine" of the angle created by the two vectors. In this case, it is

0°. $\cos(0) = 1$, which means maximum similarity.

$$\cos(A,B) = \frac{A \cdot B}{\|A\| \times \|B\|} = \frac{1\times2 + 1\times2}{\sqrt{1^2+1^2} \times \sqrt{2^2+2^2}} = \frac{4}{\sqrt{2}\times\sqrt{8}}$$

$$= \frac{4}{\sqrt{16}} = 1$$

Remember how Euclidean distance was sensitive to the magnitudes of the vectors?

A. I love ⇒ [1,1]

B. III love love love ⇒ [3,3]

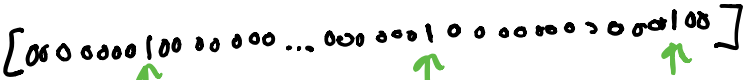
new counts

$\cos(A,B)$ is still 1!

$$\cos(A,B) = \frac{A \cdot B}{\|A\| \times \|B\|} = \frac{1\times3 + 1\times3}{\sqrt{1^2+1^2} \times \sqrt{3^2+3^2}} = \frac{6}{\sqrt{2}\times\sqrt{18}} = \frac{6}{\sqrt{36}} = 1$$

Cosine similarity is more frequently used in NLP because all of our text data is typically sparse matrices. In my examples, I used only a dimension of 2, but in practice, we'll often have dimensions > 20,000 (typically one dimension for each unique word in the English language). This means when you count vectorize your document, MOST of your elements will be 0s: ~ 20,000 elements

A. I love cats ⇒ 


B. I hate dogs ⇒ 

Cosine similarity's dot product takes into account only the elements where the documents are non-zeros. Thus, it typically performs better on high-dimensional data than Euclidean distance.

* Make sure ... *

given documents, you can

1. convert to count vectors
2. find Euclidean distance
3. find cosine similarity
4. understand the relationship between cosine distance and similarity:

cos distance = 1 − similarity
cos similarity = 1 − distance.