Financial Engineering MTH319
Coursework assignment 1

Yu Kong ID 1718810

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Abstract

Firstly, this coursework used the Monte Carlo simulation to simulate the the option price with predetermined parameters like number of steps, number of paths, the strike price of option, maturity and volatility etc.

Secondly, performance analysis with changing the number of steps and paths will be illustrated, then estimation of the probability functions of strike price at maturity is also conducted.

Finally, delta hedging strategy and delta-gamma hedging trading strategy will be simulated, and the replication error of both strategies will be analyzed under the change of sigma.

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1 Part 1

1.1 General methodology

1. construct the 'EulerMilsteinMCStock' function for simulating the price of the Stock by formed a matrix 'S' which column denoted by number of paths (M) and row is denoted by the number of steps (N). we have two different schemes Euler and Milstein to conduct simulation then we use variable 'ST' to store the stock price when maturity.

scheme Euler:

$$\tilde{X}(t_{i+1}) = \tilde{X}(t_i) + \frac{1}{2}(\mu(t_i), \tilde{X}^*(t_{i+1})\Delta t_i + \sigma(\tilde{X}(t_i)\Delta W(t_i)))$$

scheme Milstein:

$$\tilde{X}(t_{i+1}) = \tilde{X}(t_i)+\mu(t_i, \tilde{X}(t_i))\Delta t_i+\sigma(\tilde{X}(t_i)\Delta W(t_i))+\frac{1}{2}\sigma(t_i, \tilde{X}(t_i))\sigma\prime(t_i, \tilde{X}(t_i))(\Delta W(t_i)^2-\Delta t_i)$$

2. construct the 'MCoptionprice ' function to simulate the option price by discounting the payoffs over all the simulated paths.

(1) The principal for working out the option price is:

$$C_0(K, t_k) = e^{rt_k}\{\frac{1}{n_{paths}}\sum_{j=i}^{n_{paths}}[S(t_k, \omega_j) - K]^+\}$$

(1)The option result: the price of option priced by the Monte Carlo simulation under number of paths equal to 5000 is 25.804981672894446

1.2 plot result

the stock price paths:

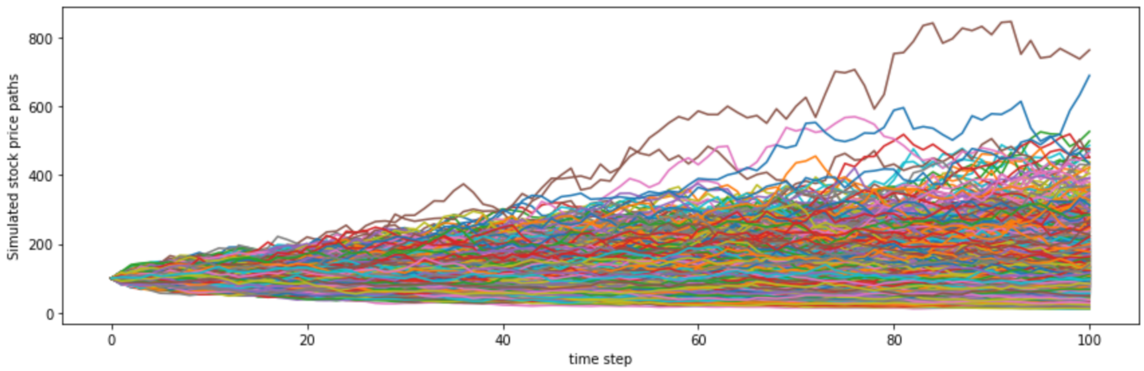


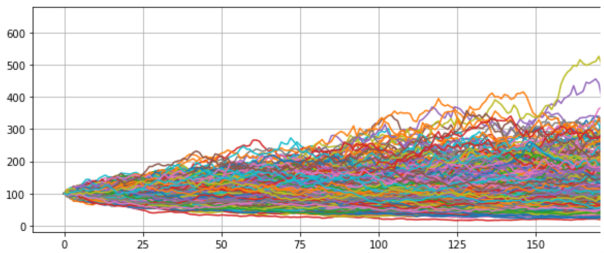
Figure 1: simulated stock price

2 Part 2

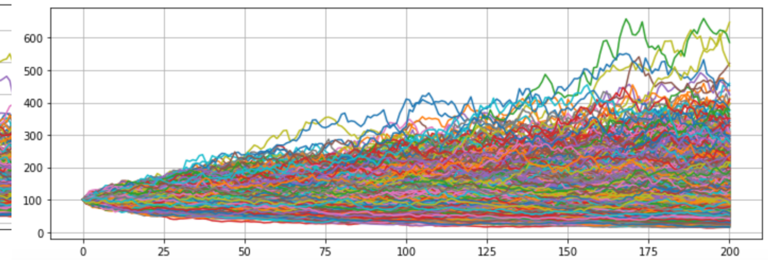
2.1 Changing the number of paths

When the number of paths increased to 1000,3000, 50000 with other setting unchanged, the price of option has benn listed as below:
when the number of paths=1000 the price of the option is:24.330631797815634
when the number of paths=3000 the price of the option is: 26.27119315740072
when the number of paths=5000 the price of the option is: 25.804981672894446
when the number of paths= 8000 the price of the option is: 26.23909224225027

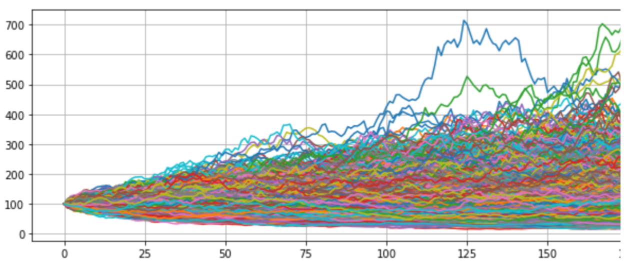
2.2 corresponding plots



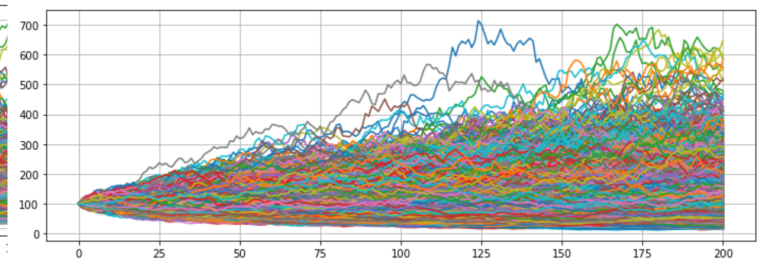
(a) M=1000.



(b) M=3000.



(c) M=5000.



(d) M=8000.

Figure 2: Changing the number of paths

2.3 Changing the number of steps

When the number of Steps increased to 500,1000, 40000 with other setting unchanged, the price of option has benn listed as below:

when the number of Steps=500 the price of the option is: 51.05317730375977

when the number of Steps= 1000 the price of the option is: 95.35600081612247

when the number of Steps= 4000 the price of the option is: 691.5863595902179

2.4 corresponding plots

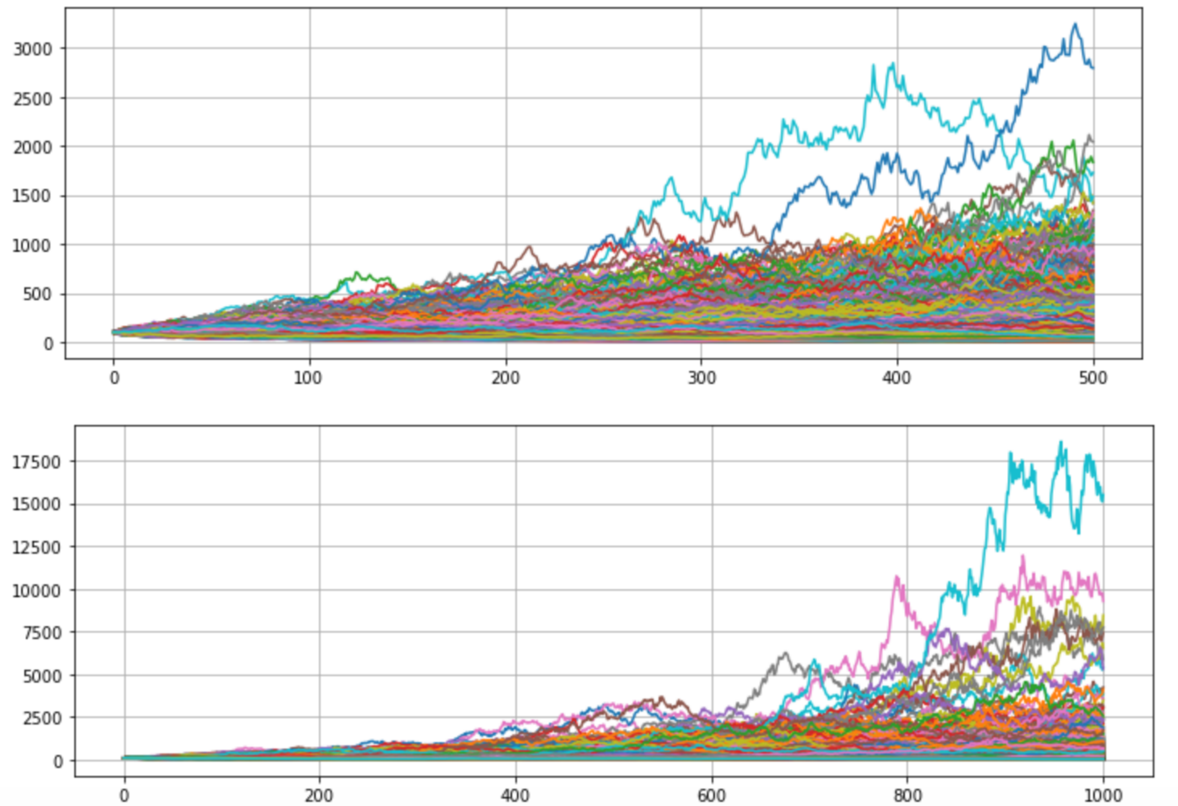


Figure 3: when the number of steps=500 and 1000

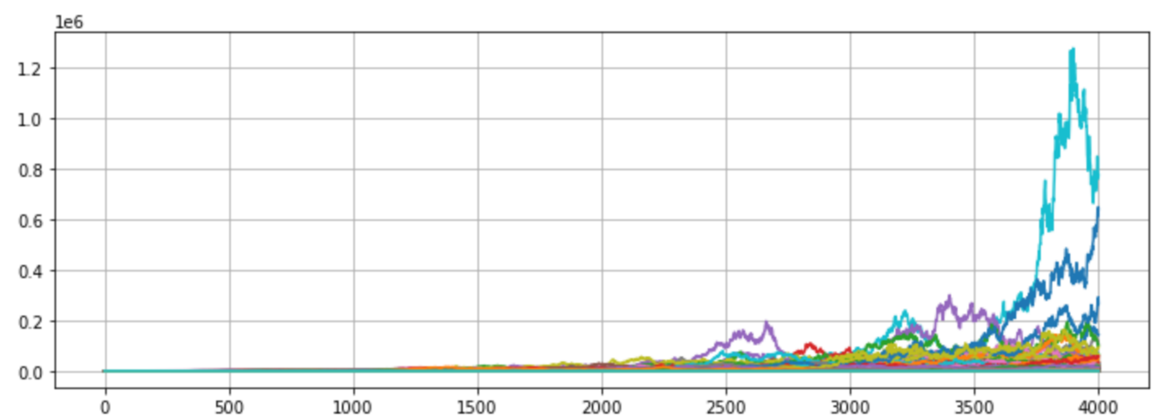
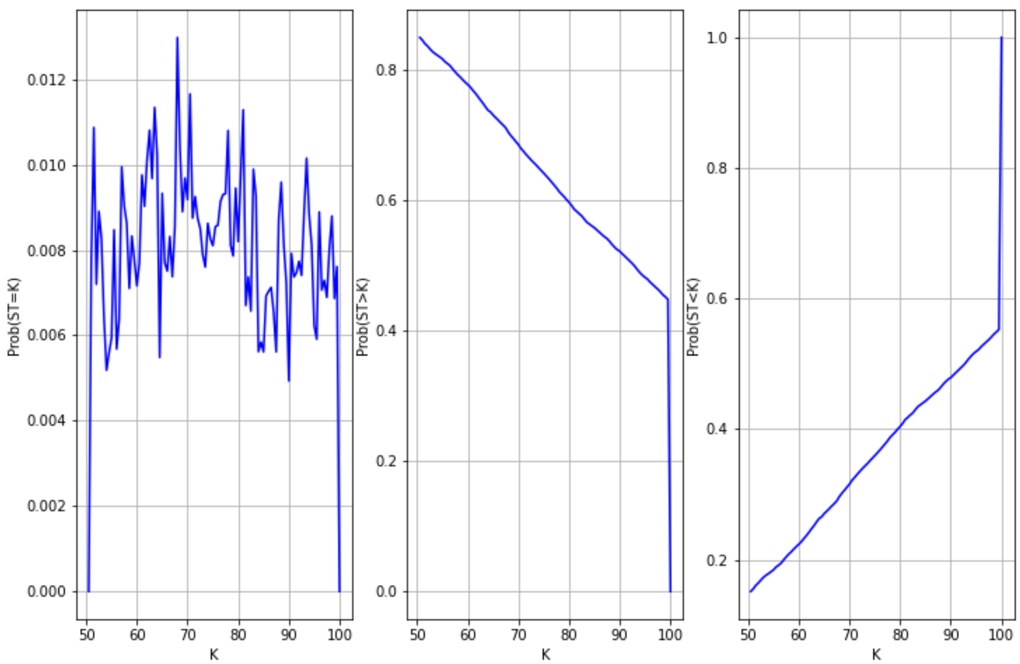


Figure 4: when the number of steps=4000

3 Part 3

3.1 The probability functions of strike price at maturity

the figure below illustrates the culumative probability and the density probability:



4 Part 4

4.1 The pricipal of delta hedging

Delta is the slope (first derivative) of the underlying curve. A delta hedge protects only against small movements in the price of the underlying. An example of a delta hedge is when buying a put, which gives a negative delta and positive gamma, and then buy enough of the underlying to zero out the total delta.

This hedge does not protect against larger movements of the underlying. When the underlying moves, the non-zero gamma will change the delta, causing us to need to re-hedge.

$$Delta(\Delta) = \frac{\partial C(t)}{\partial S(t)} = \frac{\partial}{\partial S(t)}[S(t)N(d1)] - e^{r(T-t)}K\frac{\partial}{\partial S(t)}[N(d2)]$$

$$\phi(d2) = \frac{S(t)e^{r(T-t)}}{K}$$

4.2 The result of delta hedging

when the simulation number of paths equals to 5000, the strategy of trading will be :

the amount of option A: short by one unit

the amount of Stock holding: 0.644767279 unit

the amount of bank account (or Bond) holding: -40.22844954763912 unit

the delta is: 0.644767

the gamma is: 0.006583

and the 'k' will be zero due to delta-nuetral-hedging.

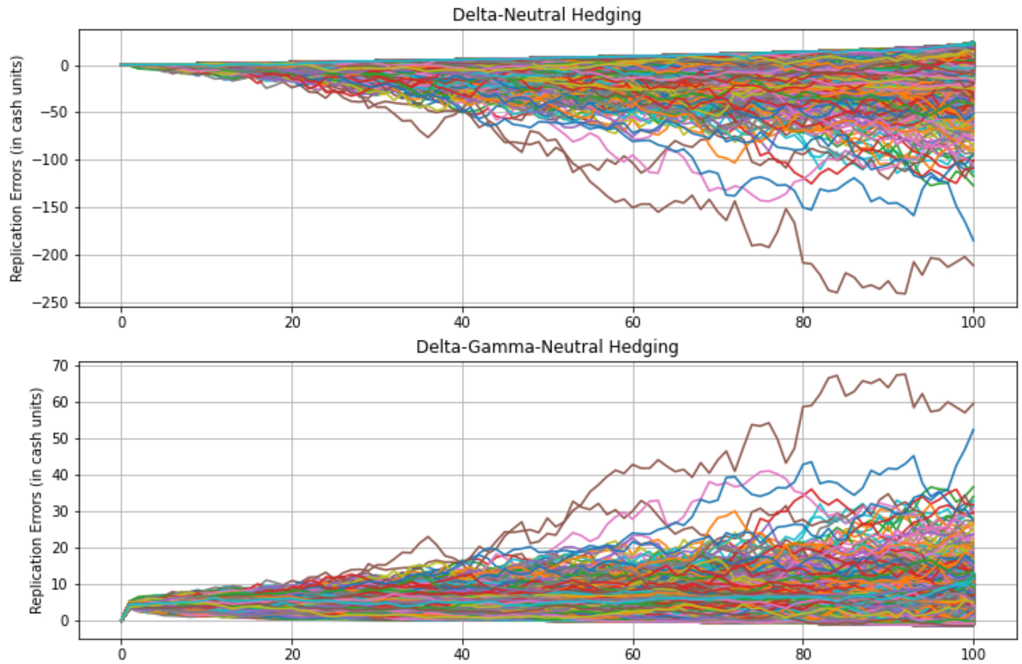


Figure 5: the replication error of delta and delta-gamma hedging strategy

4.3 The pricipal of delta-gamma hedging

Gamma is the second derivative of the P&L/underlying curve. A gamma hedge protects only against small movements of gamma; gamma will move when either the underlying or its implied volatility move. An example of a gamma hedge is when we buy a put, which gives us negative delta and positive gamma, then sell a call to zero out our gamma but give the even more negative delta. This exposes us to large movements of the underlying, so we will likely want to then buy enough of the underlying to zero out delta.

A gamma hedge does not protect against larger movements of gamma, because the put and call each have non-zero "speed".

$$Gamma(\Gamma) = \frac{\partial^2 C(t)}{\partial^2 S(t)} = \frac{\partial}{\partial}(\frac{\partial C(t)}{S_t}) = \frac{\partial}{\partial S_t}(N(d_1))$$

$$= \phi(d_1)\frac{1}{S_t}\frac{1}{\sigma\sqrt{T-t}}$$

where the $\phi(x) = \frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$

4.4 The result of delta-gamma hedging

when the simulation number of paths equals to 5000, the strategy of trading will be :

the amount of option A: short by one unit

the amount of Stock holding :-0.10586423018360214 unit

the amount of bank account (or Bond) holding: 3.1464404826842234

the delta is: 0.624414

the gamma is: 0.005476

therefore, using the delta-gamma hedging the change of delta((first derivative) of the P&L/underlying curve) is useful.

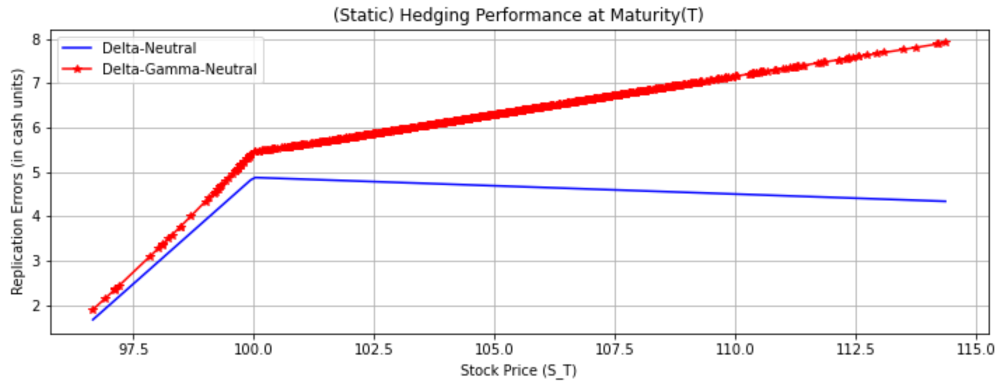
and the 'k' (the mount of option B used to hedging the change in the) is: 1.2021371188598209 unit

4.5 Analysis of replication Error

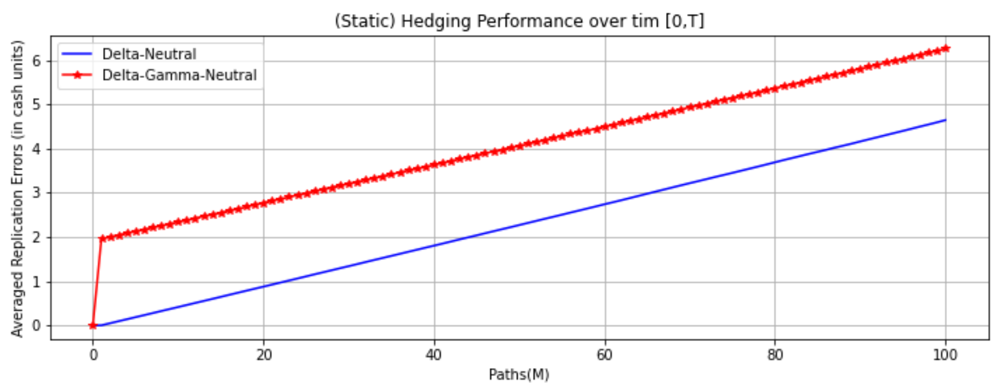
When the sigma=0.02 (assuming the cash unit is $):

This is the situation when the market is almost risk free and under this particular condition, the plot of simulated stock price is much more converged over all paths and without obvious fluctuated and volatile pricing paths showing.

The plot of replication error of delta hedging strategy and delta-gamma hedging strategy are similar under the stock price changing. They all increased rapidly when stock price lower than 100$ which is the strike price and then the replication error of delta-gamma hedging strategy slow the speed of increase while the replication error of delta hedging strategy starts to fall when stock price hits 100$.

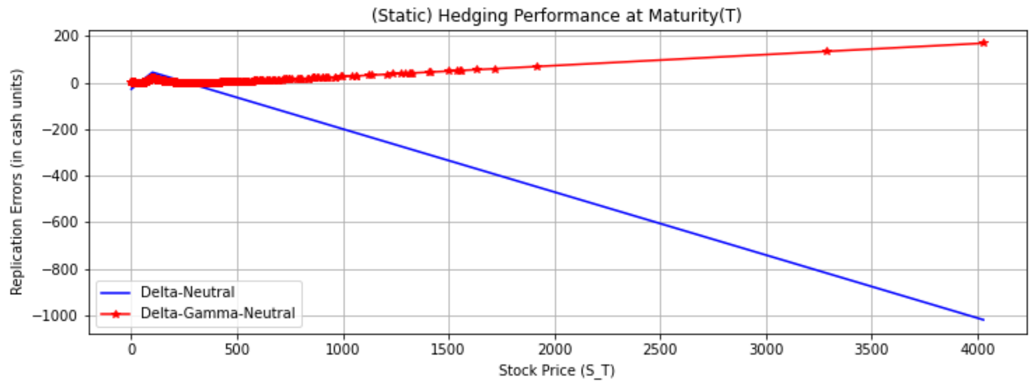


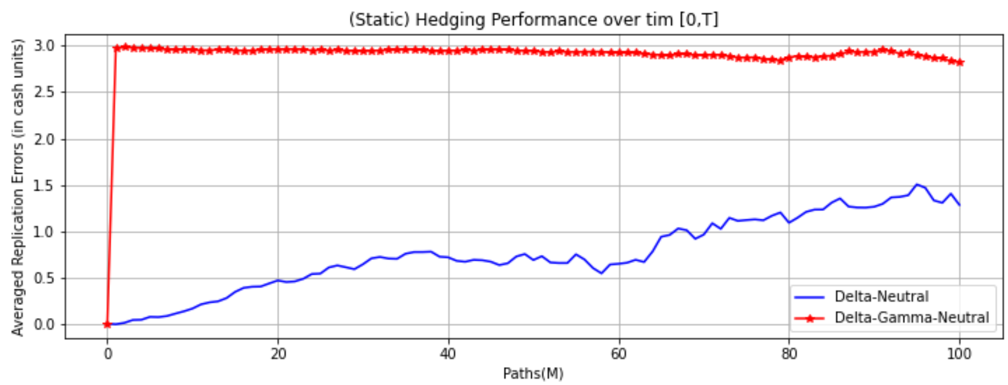
The average replication error's pattern of delta hedging strategy and delta-gamma hedging strategy are almost the same under nearly risk free assumption which indicates that due to the uncertainty of the market is low so the change of stock price (Δ)and the rate of the change (Γ)should be close due to the sigma (volatility) is low ($\Downarrow$ is the figure).



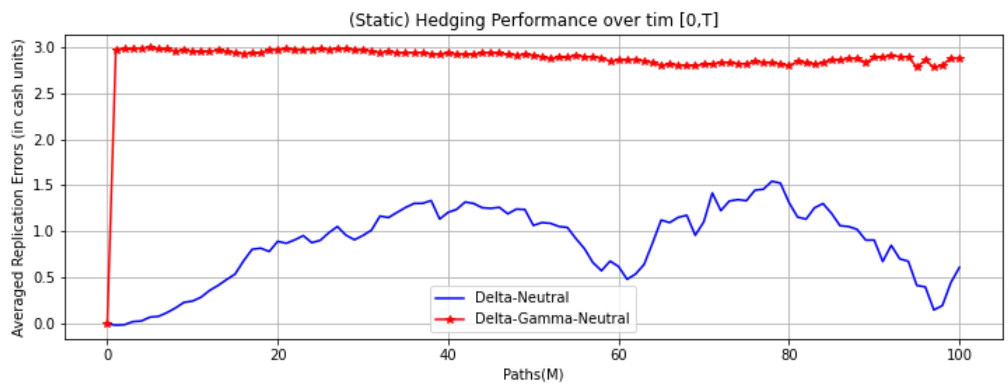
When the sigma=0.4 (assuming the cash unit is \$):
There will be a spike of replication error of both delta hedging strategy and delta-gamma hedging strategy when the stock hits 100 but over all the delta-gamma hedging strategy's replication error is much more stable and smaller than the one of delta hedging strategy. For average replication error, with the number of simulation paths increased, the average replication error for delta-gamma hedging is stable around 4\$ while the average replication error for delta hedging strategy is fluctuated but the level is lower which no

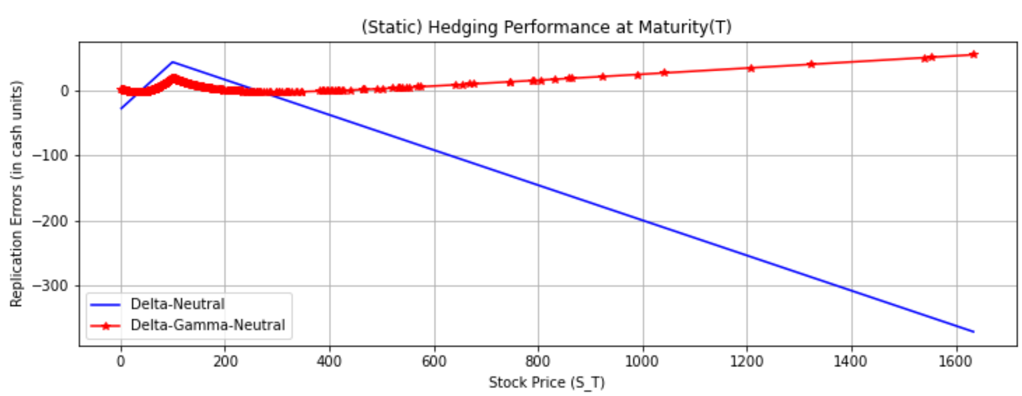
more than 2$ (plots are displayed below ⇓).





When the sigma=0.8 (assuming the cash unit is $):
This might be a extreme case due to very risky market volatility setting, and in this market there will be a spike of replication error of both delta hedging strategy and delta-gamma hedging strategy when the stock around 100 but the delta hedging strategy's replication error is far more large compare to the delta-gamma hedging strategy. Moreover, the average replication error of delta hedging strategy is more fluctuated when sigma equals to 0.4. However, the average replication error of delta-gamma hedging is 3$ which is smaller compares to the situation under sigma=0.4.


(Static) Hedging Performance over tim [0,T]
Averaged Replication Errors (in cash units)
Paths(M)
Delta-Neutral
Delta-Gamma-Neutral
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0
20
40
60
80
100


(Static) Hedging Performance at Maturity(T)
Replication Errors (in cash units)
Stock Price (S_T)
Delta-Neutral
Delta-Gamma-Neutral
0
−100
−200
−300
0
200
400
600
800
1000
1200
1400
1600