For office use only	Team Control Number	For office use only
T1 ________	**66240**	F1 ________
T2 ________		F2 ________
T3 ________	Problem Chosen	F3 ________
T4 ________	**B**	F4 ________

2017
MCM/ICM
Summary Sheet

Merge After Toll

Summary

This paper aims to establish a new toll plaza design, which brings about improvement in accident prevention, throughput and cost control.

We at first model traffic before entering the tollbooths with the queueing theory, however, our focus and constructive work is the design of the fan-in area after the toll barrier.

With advancement of automated technologies, it is very likely and beneficial for to accommodate them into future transportation system design, even 100 per cent automation is possible. To simplify the model, we assume an extreme future situation where there only exist self-driving vehicles. Self-driving vehicles, in our assumptions, can make decisions to maximize systematic efficiency. We use a network flow model to describe the merge pattern. Each vertice contains the throughput information while lines only affect computation of time consumption. Then we can translate the problem into minimum-cost maximum-flow question. Here, cost is referred to time.

We make innovative assumptions about lanes (including definition of "main" and "secondary" lanes) and merging rules, which enable our model closer to reality. For example, a car can merge into adjacent lanes at any point before lanes converging.

We also build a penalty function to strike a balance between robustness and construction cost and finally give our solution about the shape and size of our design, whose key factors are times of lanes merging and length of each "secondary" lane.

In addition, tests are carried out to answer three questions listed in the problem:

- Simulations in both light and heavy traffic confirm the robustness of our model.
- By establishing a complementary model of a system without self-driving vehicles, and comparing performance of our solution under the two extreme systems (all-automated and none-automated system), we can verify universality and stability of our solution. In other words, we don't need to make big changes as the degree of vehicle automation increases.
- Since the mix of different types of tollbooths only affects traffic before entering, we give a supplementary model to reflect these changes but without any change of our solution.

In conclusion, our plaza design is better than designs in common use in light of larger throughput and better accident prevention with reasonable construction costs.

A Letter to New Jersey Turnpike Authority

Dear New Jersey Turnpike Authority:

First of all, thank you for your dedication to the safe and efficient movement of people and goods over two of the busiest toll roads in the United States âĂŞ the New Jersey Turnpike and the Garden State Parkway. We feel honored to give suggestions to you for the toll plaza design to help you build a safe, reliable and modern toll road system.

Now let me give you a brief introduction about our models.

We apply classic queueing theory to describe the pattern that cars entering tollbooths. Basic assumptions concerning the distribution of cars' arriving and leaving tollbooths allow us to describe the accumulation of cars waiting for open tollbooths thus deriving the expected total waiting time and the passing pattern of given number of cars in the system.

Automated highway systems could make driving easier and safer, improve travel time reliability and possibly reduce congestion and energy consumption. As automated technologies continue to advance, it is very likely and beneficial for you to accommodate them into New Jersey future transportation system design.

Therefore, we imagine a future picture that there only exist automated (self-driving) vehicles, which make decisions for systematic optimization. Under three reasonable assumptions and designed a penalty function, we use the "lattice-liked" simulation to give our final solution about the shape and size of our design, whose key factors are times of lanes merging and length of each "secondary" lane.

Our solution can be depicted as the following picture:

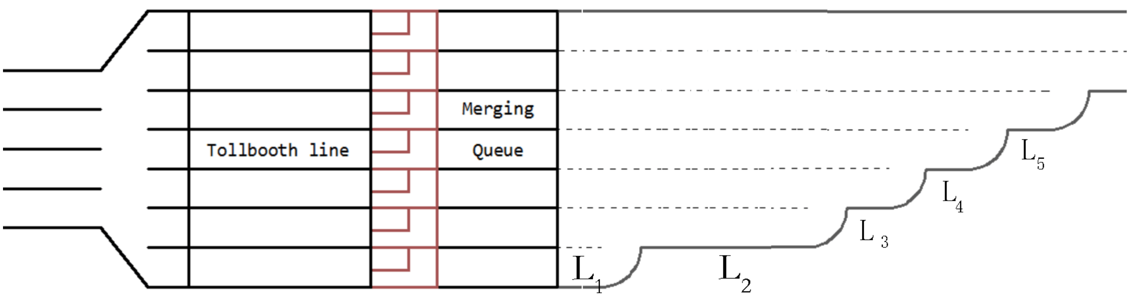


Figure 1: Illustration of pattern designing

For given B, L and construction budget, we suggest that B lanes change into L lanes, following the merge pattern illustrated in **Figure 1**. Considering real-life traffic rules, we can separate the procedure of combination into multiple steps, each of them can be depicted as the rightmost lane merged into its adjacent left lane. After attempts of optimization, we come to conclusion that the *second* merge should take as long distance as feasible.

We also test robustness of our solution by observing its performance under light and heavy traffic respectively. By the way, our so-called performance is measured by throughput of the system, which remains relatively stable under different traffic condition.

In addition, we compare our ideally automated model with the situation where no self-driving vehicle exists. Simulation results suggest only slight differences, so we can make conclusion that our design is always applicable with development of automated highway systems.

Our toll plaza design is distinguished for its stable performance and systematically efficient merging pattern. We also consider the fact that a car can merge into adjacent lanes at any point before lanes converging and make specific assumptions to be closer to reality. Our simulations use both computer-generated data and realistic data of a Shanghai-Nanjing high-speed toll, and construction costs per mile is derived from the New Jersey Department of Transportation website, all the above points make our model more convincing.

In conclusion, we have a strong believe that our model can enhance the efficiency of New Jersey Turnpike toll plaza transportation system and adapt to the future trend of vehicle automation.

Thank you for your time!

Yours sincerely.
Team 66240

Contents

1 Statement of the problem

Our main task is to establish a toll plaza model, which determines the shape, size and merging pattern of the area following the toll barrier, in order to improve systematic efficiency of the tollway. We have taken three key factors into consideration: accident prevention, throughput and construction cost. We try to find an optimal solution to strike a balance among the above goals and make further simulation under different circumstances. More specifically, we test our model by changing traffic conditions, proportion of autonomous (self-driving) vehicles, and mix of three kinds of tollbooths respectively.

2 Background

Background As we known, tollways only account for a small proportion of highways in the US, more specifically, less than 4% of the total mileage of the expressways. And this explains why Americans often call their highway as "freeway". However, the current trend shows that tollways has been increasing gradually. (Electronic Toll Collection Global Study, see [1])

Some researchers have made significant contribution to designs of toll plazas which can improve the systematic efficiency. Albert E. Schaufler focuses on the toll plaza design including the geometry of lanes, lane configuration, congestion management and so on. [2] Nico M.van Dijk and Mark D. Hermans execute queueing and simulation study for the design to configure the types of toll booths with multiple payment functionalities and determine the number of toll booths for each type. [3] Pratelli A, Schoen F. present their research progress in developing a toll station layout optimisation methodology based on toll plaza circulation safety analysis. [4]Perry R F, Gupta S M. do experiments using a simulation model of a typical toll plaza with varying mixes of vehicle types and tollbooth allocations provided data for four output measures: vehicle volume, average queue length, average waiting time, and tollbooth utilization. Multiple linear regression analysis was then applied to fit response surfaces for each measure. The response surfaces provided the optimum tollbooth allocation for a given mix of vehicles (see [5]).

In addition, automated transportation systems involve the application of control systems and information technologies to guide a vehicle that was once controlled manually. Automated highway systems could make driving easier and safer, improve travel time reliability and possibly reduce congestion and energy consumption. Automated transportation systems have been proposed in a number of ways and at varying degrees; certain models require lane segregation that involves active coordination between the vehicle and infrastructure, while others include reliance on only vehicle to vehicle communication using wireless technologies. Currently, a prototype has been developed in Texas, designed by the Texas Transportation Institute (TTI), which is envisioned as a privately-owned and operated system, built through a public private partnership. As automated technologies continue to advance, it is very likely that future transportation system design will need to accommodate them (see [6]).

3 Assumptions

3.1 Assumption for queueing model

(1) Poisson distribution: The distribution of arrival time follows such distribution:

$$P(n_{t+\Delta t} - n_t = 1) = \mu\Delta t + o(\Delta t)$$
$$P(n_{t+\Delta t} - n_t \geq 2) = o(\Delta t);$$

(2) Exponential distribution: The distribution of service time follows such distribution:

$$P(n_{t+\Delta t} - n_t = -1) = n_t\lambda\Delta t + o(\Delta t)$$
$$P(n_{t+\Delta t} - n_t \leq -2) = o(\Delta t);$$

(3) Asynchrony: A car will voluntarily choose an empty tollbooth and will join the shortest queue if no empty tollbooth is available.

3.2 Assumption for merge pattern

(1) Lane change rule: A car can merge into adjacent lanes at any time. For convenience, we view its merging pattern as discrete, that is to say, a car is only allowed to change lanes every time it passes a unit length.

(2) Unilateral rule: Since that in reality cases, highway lanes are arranged like **figure 8**. Therefore, we consider the left lane (facing the direction of driving) is a straight line and it will not converge in other lines.

(3) Upper limit of traffic flow rule: Each lane has a fixe upper limit of traffic flow and the limit will not affected by cars' merging behavior.

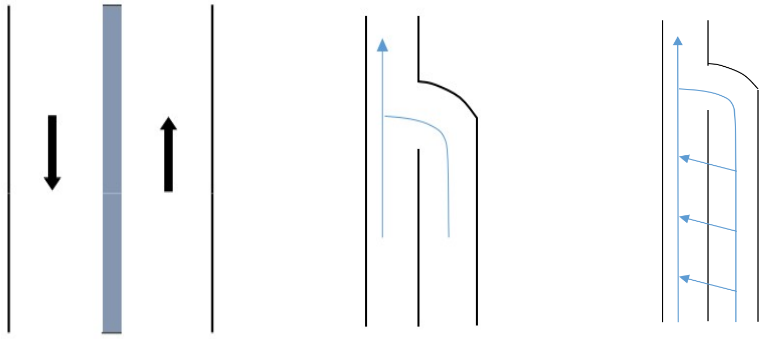
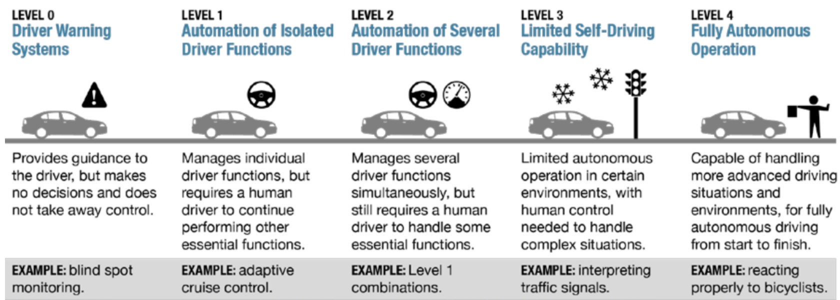


Figure 2: Illustration for Unilateral rule

3.3 Assumptions on self-driving vehicles

Automated Vehicles are defined as those that use onboard sensing technologies, rather than intervehicle communications, to provide the vehicle and driver with a greater awareness of the surroundings, thereby enabling the automation of one or more driver functions through artificial intelligence, machine learning, machine vision, and computer processing. [7]

In this paper, we assume that all self-driving vehicles follow the optimal strategy which optimizes overall effects on traffic condition. However, non-self-drivers would always be selfish and shortsighted.



Vehicle-to-Vehicle	Vehicle-to-Infrastructure	Vehicle-to-Other
• Cooperative Adaptive Cruise Control. • Do-Not-Pass Warnings. • Intersection Movement Assistance. • Queue Warnings. • Lane Change Warnings.	• Curve Speed Warnings. • Roadway Surface Condition Warnings. • Transit Signal Priority. • Red Light Violation Warnings.	• Reduced Speed Work Zone Warnings. • Notification of Pedestrians in Signalized Crosswalks. • Transit Stop Requests/Alerts.

(Source: Cambridge Systematics, Inc.)

Figure 3: Potential connected vehicles

4 Notations

notations	*description*
n_t	number of vehicle arrival in time t
k	total amount of vehicle in system
B	number of tollbooths
L	number of lanes after merging
μ	arrival rate
λ	service rate
π_k	probability of exactly k cars in steady state
$\boldsymbol{\pi} = (\pi_1, \pi_2, \ldots)^T$	distribution vector in steady state
$T(k)$	expected total waiting time in case of k cars in system
η	utilization rate of tollbooths
τ	duration of receiving service
α	degree of congestion
$\mathscr{L} = (L_1, L_2, \ldots)$	configuration of merge pattern
Φ	total throughput for the system within one time step
K	limitation on traffic volume for each vertice per time step
$\mathcal{A}$	a mapping from configuration of merge pattern to its vulnerability

Table 1: Table of notations

5 Models

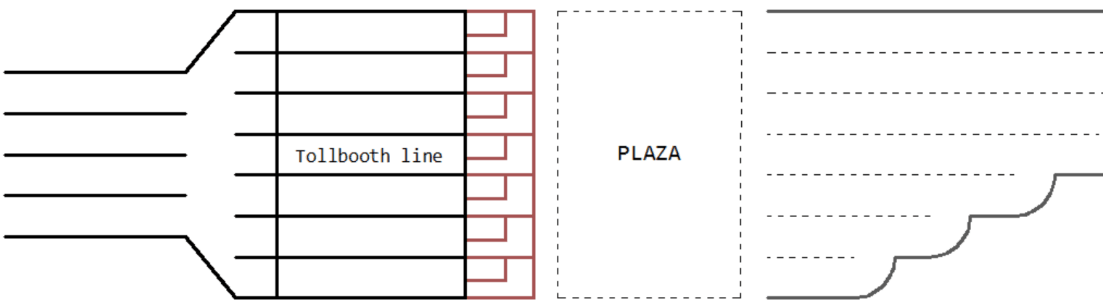


Figure 4: Sketch map of scenario

5.1 Queueing model before tollbooth

In this part, we introduce *Queueing theory* (see [8]) to conduct analysis on vehicles' arrival and entrance of tollbooths.

5.1.1 Model for vehicle arrival

We assume that vehicles arrive at highway tollbooths stochastically. That is to say, distribution of arrival intervals, as well as characteristic parameters (e.g. expectations and variances) varies as time passes. One of the most commonly implemented distribution is *Poisson distribution*. Under such assumption, intervals of vehicle arrival can be viewed as random sequence under negative exponential distribution. Hence, the probability of arrival of exactly n highway vehicles during time interval Δt can be viewed as

$$\mathsf{Poisson}^{\lambda\Delta t}(n) = \frac{(\lambda\Delta t)^n}{n!}e^{-\lambda\Delta t}$$

where λ is average vehicle arrival per unit of time.

5.1.2 Model for queueing system

Vehicles arriving at tollbooths pay at the tollbooth and leave afterwards. When it takes time to charge tolls, queueing happens. Within such scenario, procedure of vehicle arrival, rules of queueing, and service organization can be viewed as integral queueing system.

(1) Poisson distribution: The distribution of arrival time follows such distribution:

$$P(n_{t+\Delta t} - n_t = 1) = \mu\Delta t + o(\Delta t)$$
$$P(n_{t+\Delta t} - n_t \geq 2) = o(\Delta t);$$

(2) Exponential distribution: The distribution of service time follows such distribution:

$$P(n_{t+\Delta t} - n_t = -1) = n_t\lambda\Delta t + o(\Delta t)$$
$$P(n_{t+\Delta t} - n_t \leq -2) = o(\Delta t);$$

(3) Asynchrony: A car will voluntarily choose an empty tollbooth and will join the shortest queue if no empty tollbooth is available.

The number of cars is discrete. Also, we assume that different cars do not arrive/leave at the same time. Therefore, for any time interval from t to $t + \Delta t$, at most one car arrives/leaves a tollbooth and the arrival and leave of the cars are independent.

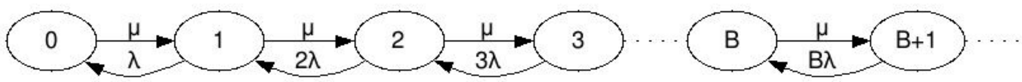


Figure 5: Underlying Markov chain

Suppose $\boldsymbol{\pi} = (\pi_1, \pi_2, \ldots)^T$ is the steady state of this system, then we have

$$\begin{cases} \pi_0 \cdot \mu & = \pi_1 \cdot \lambda \\ \pi_1 \cdot \mu & = \pi_2 \cdot 2\lambda \\ & \vdots \\ \pi_{B-1} \cdot \mu & = \pi_B \cdot B\lambda \\ \pi_B \cdot \mu & = \pi_{B+1} \cdot B\lambda \\ & \vdots \\ \pi_k \cdot \mu & = \pi_{k+1} \cdot B\lambda \ \ (\forall k > B) \\ & \vdots \end{cases}$$

and that

$$\sum_{i=1}^{\infty} \pi_i = 1.$$

When the number of cars in the system is less than the number of tollbooths, that is to say, $K < B$. Then, the probability of another car coming $P(n_{t+\Delta t} = k+1 | n_t = k) = \Delta t \cdot \mu$; the probability of one car leaving $P(n_{t+\Delta t} = k-1 | n_t = k) = \Delta t \cdot \lambda \cdot k$.

When the number of cars in the system is not less than the number of tollbooths, that is to say, $K \geq B$. Then, the probability of another car coming $P(n_{t+\Delta t} = k+1 | n_t = k) = \Delta t \cdot \mu$; the probability of one car leaving $P(n_{t+\Delta t} = k-1 | n_t = k) = \Delta t \cdot \lambda \cdot B$.

After certain derivation, we know the probability that there exists exact n vehicles in queue is

$$\pi_n = \begin{cases} \frac{1}{n!}(\frac{\mu}{\lambda})^n \pi_0, & n \leq B \\ \frac{1}{B!}(\frac{\mu}{\lambda})^B \pi_0 (\frac{\mu}{B\lambda})^{n-B}, & n > B \end{cases},$$

and that the probability for a empty queue is

$$\pi_0 = \frac{1}{\sum_{i=1}^{B} \frac{1}{i!}(\frac{\mu}{\lambda})^i + \frac{1}{B!}(\frac{\mu}{\lambda})^B \frac{\mu}{B\lambda - \mu}}.$$

5.1.3 Evaluation of waiting time

Denoting $T(k)$ as expected total waiting time in case of k cars in system,

$$T(k) = \begin{cases} 0, & k \leq B \\ T(k-1) + \mathbb{E}[\tau] = (k-B) \cdot \frac{1}{B\lambda}, & k > B \end{cases},$$

$$\mathbb{E}[T(k)] = \sum_{k=0}^{\infty} T(k) \cdot \pi_k$$
$$= \sum_{k=B+1}^{\infty} (k-B)\frac{1}{B\lambda}\frac{1}{\sum_{i=1}^{B}\frac{1}{i!}(\frac{\mu}{\lambda})^i + \frac{1}{B!}(\frac{\mu}{\lambda})^B\frac{\mu}{B\lambda-\mu}} \cdot \frac{1}{B!}(\frac{\mu}{\lambda})^B(\frac{\mu}{B\lambda})^{k-B}$$

5.1.4 Utilization of facilities

At any period of time, expected amount of facilities not being utilized is

$$\sum_{k=0}^{B}\frac{B-k}{B}\cdot\pi_k = \sum_{k=0}\frac{B-k}{B}\cdot\frac{1}{k!}\cdot(\frac{\mu}{\lambda})^k\cdot\frac{1}{(\frac{\mu}{\lambda})^B\pi_0(\frac{\mu}{B\lambda})^{n-B}}.$$

and the utilization rate is

$$1-\sum_{k=0}^{B}\frac{B-k}{B}\cdot\pi_k$$

After simulation by setting μ/λ as $11.5, 12.5, 13.5$ respectively, we can draw figures showing relationship between waiting time and tollbooth number.

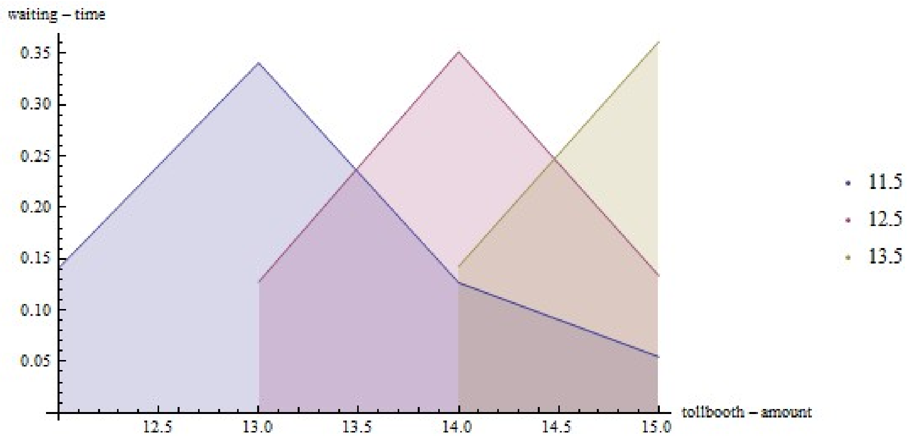


Figure 6: Variation of waiting time corresponding to tollbooth amount ($\mu/\lambda = 11.5, 12.5, 13.5$)

Also, we make some simulation to show relationship between utilization rate of facilities and tollbooth amount, given μ/λ equals $11.5, 12.5, 13.5$ respectively.

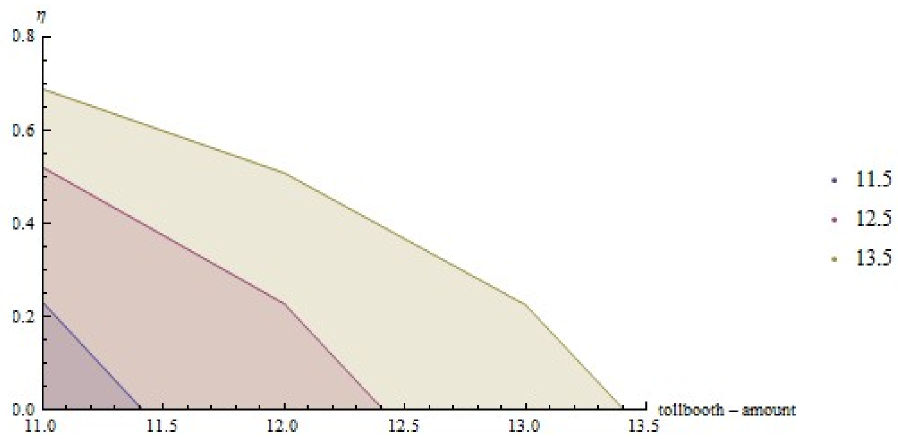


Figure 7: Variation of η corresponding to tollbooth amount ($\mu/\lambda = 11.5, 12.5, 13.5$)

Graphic varies a lot with changes of μ/λ.

5.2 Model for merging

Our goal is to find a solution to optimize its robustness, and try to control construction cost at the same time.

For further analysis, we make following assumptions:

(1) Lane change rule. A car can merge into adjacent lanes at any time. For convenience, we view its merging pattern as discrete, that is to say, a car is only allowed to change lanes every time it passes a unit length.

(2) Unilateral rule. Since that in reality cases, highway lanes are arranged like **Figure 8**. Therefore, we consider the left lane (facing the direction of driving) is a straight line and it will not converge in other lines.

(3) Upper limit of traffic flow rule. Each lane has a fixed upper limit of traffic flow and the limit will not affected by cars' merging behavior.

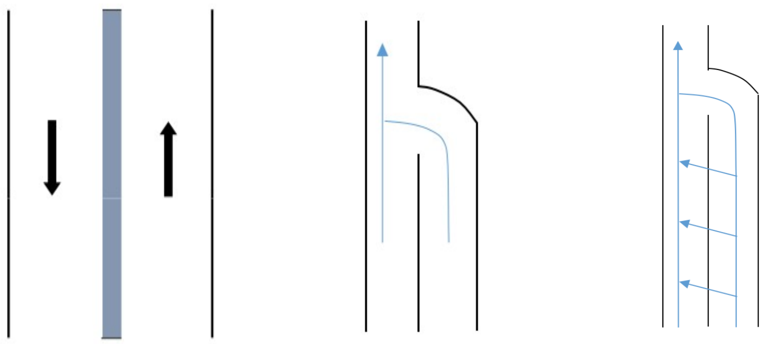


Figure 8: Illustration for Unilateral rule

5.3 Model description

For convenient description, we call the left lane "main lane" and the right one "secondary lane". Then we aim to determine the length of the "secondary lane", which is denoted as L.

As our network depicted, larger L means more opportunities to change lanes (greater robustness), while higher construction costs as well. So our focus is to achieve a balance between robustness and costs by using penalty function and convex optimization.

After that, we consider some more complicated situations. Suppose L_i is length of i^{th} widest component to end of merging zone, as illustrated by **Figure 9**.

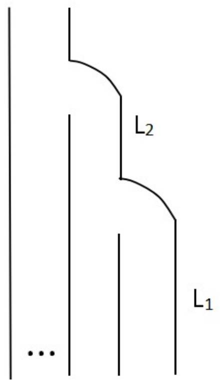


Figure 9: Merge pattern

In this way, we are allowed to present configuration of merge pattern as vector $\mathscr{L} = (L_1, L_2, \ldots)^T$.

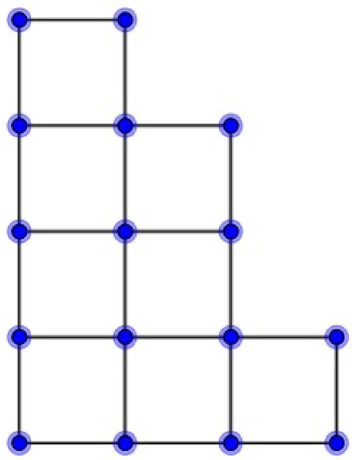


Figure 10: Network derived from assumptions

We can describe an abstract model under our assumptions as a "lattice-liked" network as shown in **Figure 10**. Each vertice stands for one lane interval and each edge represents one possible switching of driver. Vertical edges in this figure represents movement towards current direction for one unit step, and horizontal edges represents merging or switching between lanes. This is easy to convert minimization of total waiting time into *Mincost-maxflow* problem with fixed flow capacity on vertices and fixed cost on each unit of flow passing through any edge.

5.4 Simulation and optimization

In this part, we introduce our work about simulation on throughput and robustness of our model when random accident happens. And then we present our optimal scheme.

5.4.1 Mincost-maxflow and robustness

We define a network flow model, which can be illustrated by **Figure 11**. In this network, *Source* is connected by B edges directed towards B vertices of the bottom layer, and *Sink* is connected by L edges pointed out from L highest layer vertices representing L fan-in lanes.

And all other vertices are connected in the same way as that of **Figure 10**, the cost (i.e. elapse of time) of passing through each edge will be the same, which is identical to any vertical edge and to all horizontal edges. Each vertice has certain flow capacity, which is upper limit of quantity of traffic per time unit.

In this paper, we define robustness and vulnerability as a pair of antonym. Rigid description of definition of robustness concerns model of *Mincost-Maxflow* (see [9]), which regards problem of finding "cheapest" solution to max-flow problem. That is to say, summation of overall time consumption (to all participants) should be minimized, under the restriction that volume of traffic flow (i.e. overall throughput) should be maximized. When random accident happens (each edge will be removed with a same small probability), overall *Mincost-Maxflow* will change. In a topologically badly designed system, this change will be conspicuous. And for well-designed cases it will be much smaller. We define this expected difference caused by random accidents as an indicator of system robustness.

In following parts, we denote the mapping from congestion parameter α, and $\mathscr{L} = (L_1, L_2, \ldots L_{B-L+1})^T$ to the difference of overall *Mincost-Maxflow* as $\mathcal{R} : \mathbb{R}^+ \times \mathbb{Z} \times \mathbb{Z} \times (\mathbb{Z})^* \to \mathbb{R}$. To be more specific,

$$\mathcal{R}(\alpha, B, L, \mathscr{L})$$

denotes expected difference in *Mincost-Maxflow* cost by random accident of removing each edge with certain probability, in the lattice-liked graphic parameterized by $\mathscr{L}$, with B inwards lanes and L outwards lanes.

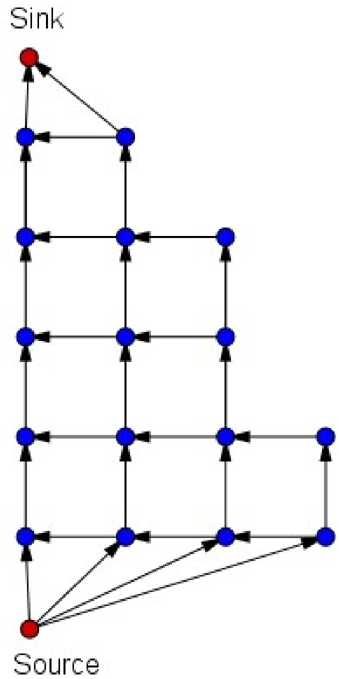


Figure 11: Network flow model

5.4.2 Construction cost per lane mile

NJDOT (New Jersey Department of Transportation) operates and maintains 8,410 lane miles of roadway, 4,078 miles of shoulders and 853 miles of ramps. In order to calculate the average cost per mile to construct, operate and maintain roadways under the NJDOT's jurisdiction, the research team preformed a straightforward calculation by dividing total expenditures related to the administration, construction, operations and maintenance of state roadways under NJDOT jurisdiction by the number of lane miles.

As shown in **Table 2**, on a per lane mile basis, over the five year period, NJDOT spent on average $\$123,755$ on capital construction projects.

According to construction cost trends for highways (NHCCI index), we estimate the annual capital cost per lane mile for our project to be: $\$123755 * 1.0728/1.0850 = \$122363$.

Summary Cost per Lane Mile Estimates for Roadways and Bridges under NJDOT Jurisdiction by Fiscal Year

Cost Per Lane Mile Estimates	2010	2011	2012	2013	2014	Average
Administration, Planning & Research	$7,282	$7,261	$8,491	$9,167	$5,924	$7,625
Capital Construction	$151,756	$131,713	$101,004	$96,305	$137,999	$123,755
Operations & Maintenance	$37,567	$54,468	$47,312	$58,072	$64,465	$52,377
TOTAL	$196,606	$193,442	$156,807	$163,544	$208,388	$183,757

Table 2: Summary cost per lane mile estimates[10]

Year	Quarter	NHCCI Index
2015	December	1.0850
2016	March	1.0728

Table 3: Constriction cost trends for highways

Source: Federal Highway Administration, Office of Highway Policy Information, "National Highway Construction Cost Index (NHCCI)"

5.4.3 Optimization

With technique similar to *Steepest descent method*, we can find optimal asiignment of $\mathscr{L}$ (within the budget boundary). This procedure would be slightly different from that of traditional *Steepest descent Method* since that in our model, all elements of vector $\mathscr{L} \in (\mathbb{Z})^*$ should be integers. After repeated attempts, we come to final conclusion that it should be wiser to invest almost all budgets in prolonging second widest road interval.

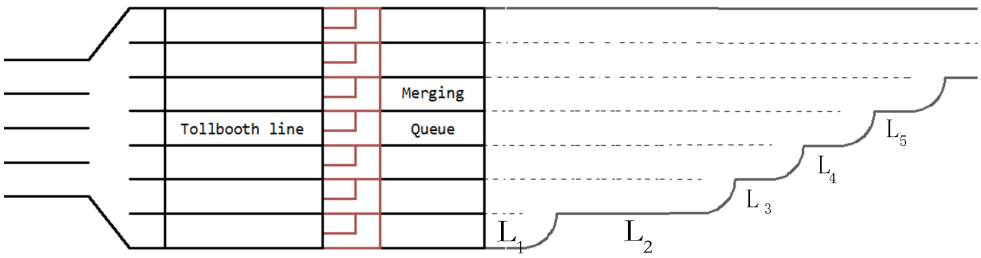


Figure 12: Illustration of pattern designing

6 Sensitivity Analysis

6.1 Testing on different degree of traffic congestion

Sensitivity of a system is the extent to which one accident would cause to soundness of overall system. As traffic volume grows, system would be be more sensitive towards random accidents. Our experimental results are shown in **Figure 13**.

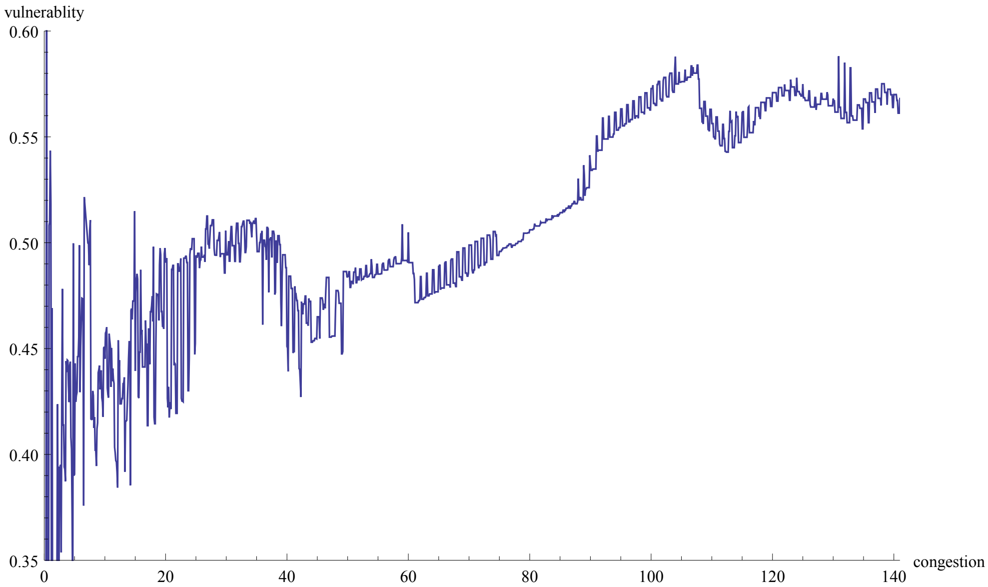


Figure 13: Variation of vulnerability corresponding to degree of congestion

6.2 Testing on non-self-driving and self-driving scenarios

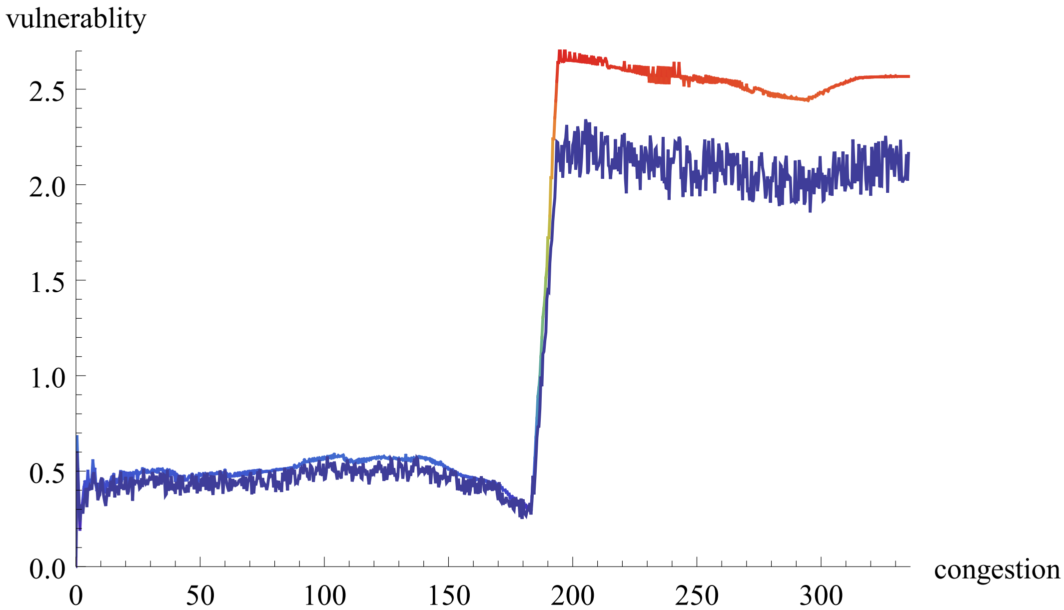


Figure 14: Comparison on vulnerability for two scenarios

By establishing a complementary model of a system without self-driving vehicles, and comparing performance of our solution under the two extreme systems (all-automated and none-automated system), we can verify universality and stability of our solution. In other words, we don't need to make big changes as the degree of vehicle automation increases.

7 Conclusion

In this section, we present conclusion on strengths of our model, as well as certain demonstration on the weakness of our model.

7.1 Strengths

- Our toll plaza design is distinguished for its high robustness and systematically efficient merging pattern.
- We consider the fact that a car can merge into adjacent lanes at any point before lanes converging and make assumptions about "main" and "secondary" lanes (thus asymmetric shape of our design), which are more closer to reality.
- Our simulations use both computer-generated data and realistic data of a Shanghai-Nanjing high-speed toll, and construction costs per mile is derived from the New Jersey Department of Transportation website. Application of these statistical data makes our model more convincing.

7.2 Weaknesses

- We view the plaza after tollbooths as a "buffer" area, so we ignore the length and costs related to it.
- We set a same upper limit of the traffic flow capacity for each vertex, but in reality, road conditions may vary from one another.
- We assume that accidents happen with the same probability at each line in our network model. In real world, though, lanes of the narrower part tend to have higher probability of accidents.

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