# CSCI 4140
# Quantum Internet

Mark Green

Faculty of Science

Ontario Tech

# Introduction

- With multiple quantum computers there is a desire to have them work together
- In order to do this quantum information must be sent between these computers
- Qubits can be in superimposed state, cannot send this over a standard communications network, it must be sent as qubits
- This brings up the idea of the quantum Internet, connecting quantum computers

# Benefits

- Connecting multiple quantum computers greatly increases their powers
- An n qubit quantum computer has $2^n$ states, connecting two of them will produce $2^{2n}$ states, this is very important scaling
- Blind computing: quantum computer acts as a server, but can’t inspect the data it is running on, data is secure during computation
- Secure communications, can detect when the data has been read

# Problems

- The main problem is we can’t copy qubits, there can only be one copy
- Reliable communications requires the ability to retransmit data if it is lost, but we can’t store a copy to be sent again later
- Qubits can only travel so far before they decay, traditional solution is to use a repeater, but we can’t copy qubits the way that we copy bits
- What do we do with the qubit once it arrives?  If we are not ready to use it, it must be stored

# Solution

- Quantum teleportation is the solution, but there is more to it
- This needs to be done on a large scale
- It also needs to be reliable
- Start with a review of the quantum teleportation algorithm
- Then examine how it can be built out into a network

# Quantum Teleportation

- This is a somewhat interesting algorithm to get your head around, you may need to read through it several times
- Problem: Alice has a qubit $|\psi\rangle$ that she wants to send to Bob, but she only has a classical communications channel that transmits bits
- We have that $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ and both $\alpha$ and $\beta$ are complex
- There is potentially an infinite amount of information here, so sending it over a classical communications channel seems to be impossible
- Entanglement to the rescue!

# Quantum Teleportation

- Assume that sometime in the past Alice and Bob created an entangled pair of qubits:

$$q = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

- Bob takes one qubit and Alice takes the other one
- Alice now has $|\psi\rangle$ and one half of q, Bob has the other half
- The complete state of the system is given by:

$$|\psi\rangle q\rangle = \frac{1}{\sqrt{2}}\big(\alpha|0\rangle(|00\rangle + |11\rangle) + \beta|1\rangle(|00\rangle + |11\rangle)\big)$$

# Quantum Teleportation

- Now Alice applies a CNOT to her qubits giving:

$$\frac{1}{\sqrt{2}}\big(\alpha|0\rangle(|00\rangle + |11\rangle) + \beta|1\rangle(|10\rangle + |01\rangle)\big)$$

- Next Alice applies a Hadamard gate to obtain:

$$\frac{1}{\sqrt{2}}[\alpha(|0\rangle + |1\rangle)(|00\rangle + |11\rangle) + \beta(|0\rangle - |1\rangle)(|10\rangle + |01\rangle)]$$

- We can rewrite this in the following way:

$$\frac{1}{\sqrt{2}}[|00\rangle(\alpha|0\rangle + \beta|1\rangle) + |01\rangle(\alpha|1\rangle + \beta|0\rangle) + |10\rangle(\alpha|0\rangle - \beta|1\rangle) \\ + |11\rangle(\alpha|1\rangle - \beta|0\rangle)]$$

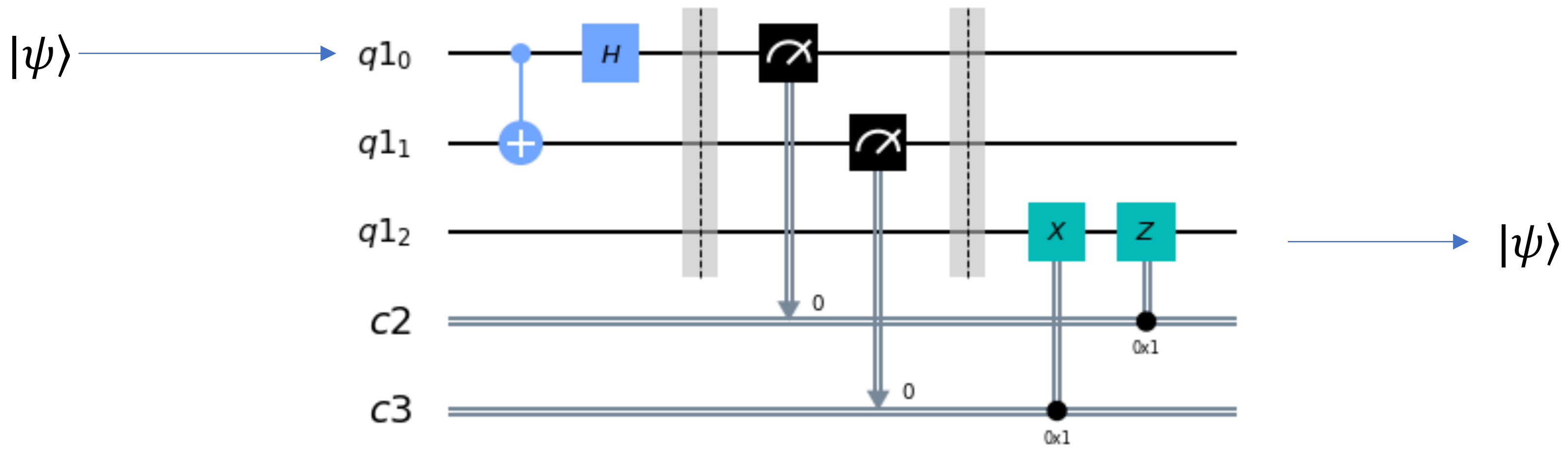
# Quantum Teleportation

- We now see our solution, we measure Alice’s two qubits to get two classical bits
- These two bits are sent to Bob, who can now reconstruct the original $|\psi\rangle$:
  - If the two bits are 00, Bob’s qubit has the correct value
  - If the two bits are 01, Bob applies a X gate to his qubit
  - If the two bits are 10, Bob applies a Z gate to his qubit
  - If the two bits are 11, Bob applies a X gate followed by a Z gate to his qubit

# Quantum Teleportation

- This involves two gates at Alice’s end, a CNOT and a Hadamard
- There are also two gates at Bob’s end, a X and a Z gate
- We have essentially used two bits to move an infinite amount of information in a qubit
- But, we need a previously shared pair of entangled qubits, so we just can’t send a qubit anywhere we like, we must preplan for the exchange

# Quantum Teleportation




The top qubit is the one that is being transmitted, the next two qubits are the entangled pair. The bottom two lines are classical bits

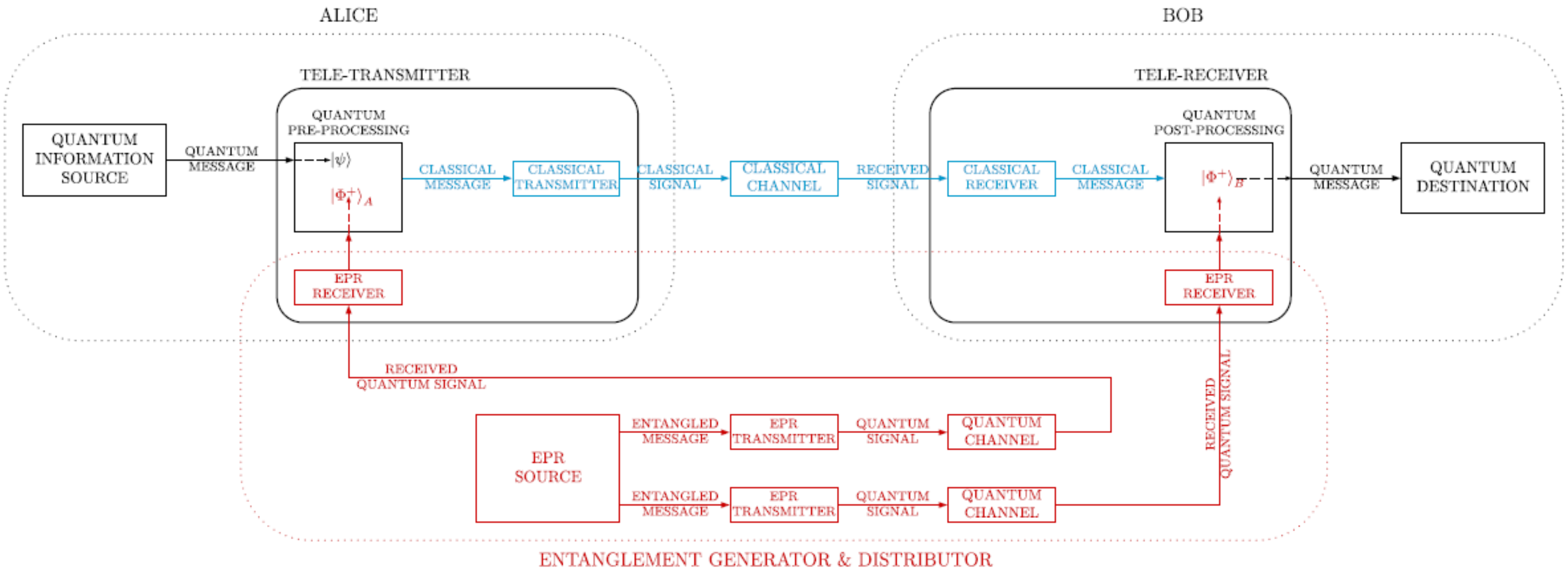
# Quantum Communications

- Quantum teleportation gives us point to point communications
- It requires two classical bits, we already know how to deal with this, we use all our classical communications technology
- Also need a pair of entangled qubits, this is a different story
- Many ways of representing qubits, but photons seems to be the best choice here:
    - Least interaction with the environment
    - Can be transmitted over existing fibre optics

# Quantum Communications

- The photons are quite often called “flying qubits”, since they are ones that are moving
- The following slide shows the general architecture of a point to point quantum communications system
- The top part of the figure is quantum teleportation, plus the classical communications channel
- On receiving a qubit it may need to be stored until its ready to be consumed
- This works well with ion traps that can store qubits for long periods of time

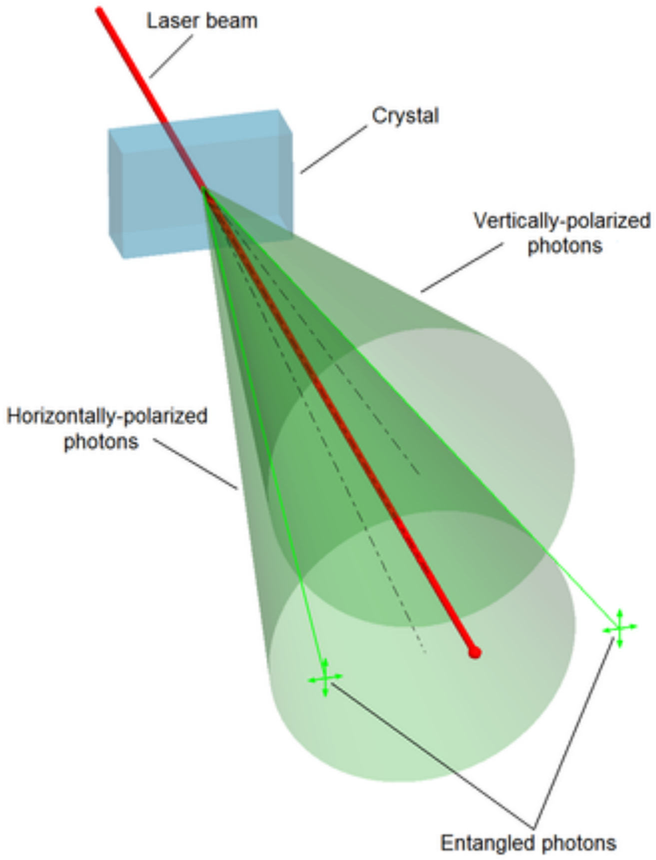
# Quantum Communications



# Quantum Communications

- The interesting part is the quantum channel at the bottom
- First we need a way of generating entangled photons
- We can do this by sending a laser beam through a non-linear crystal that produces pairs of photons
- They exit in two different, but overlapping cones
- One photon will by horizontally polarized and the other vertically polarized
- The entangled photons appear in the overlap region

# Quantum Communications

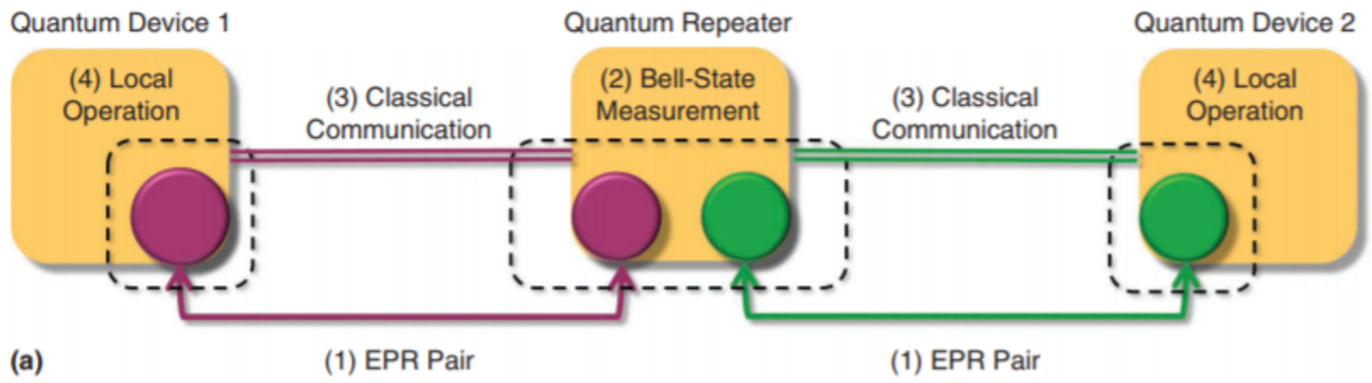


# Quantum Communications

- Where do we put the photon source?
- The ideal placement is between the two end points, gives an equal distance to each point, reducing timing problems
- Photons will loose coherence over time, so shorter length will preserve the quality of the photons
- Photons have been transmitted through free space to satellites
- Over fibre optics they are limited to several kilometers depending upon the quality of the source

# Quantum Communications

- To solve this problem we need to have quantum repeaters
- Problem: we can't copy photons, so a classical repeater can't be used
- We need a more sophisticated approach based on two pairs of entangled qubits



# Quantum Communications

- Notice that device 1 and the repeater share an entangled pair
- The repeater can perform the receiver side of the teleportation algorithm and retrieve the original qubit
- The repeater also shares an entangled pair with device 2
- It can now take the qubit and use the sender side of the teleportation algorithm and send it to device 2
- Note that this doesn’t copy a qubit, since the original qubit is destroyed in the process

# Quantum Communications

- We can use a similar technique to build a quantum router
- Assume that all the routing information is transmitted on the classical channel, we already know how to handle this
- Now instead of having one output, the router has multiple outputs to multiple devices
- It has a separate set of entangled pairs for each of these devices
- The router only needs to examine the routing information to determine the device the qubit is sent to

# Quantum Communications

- A number of experimental quantum communications networks have been constructed, some of them spanning up to 1000 kilometers
- They have been mainly used for QKD, can’t generate quantum pairs fast enough to send regular data
- Key research is on generating entangled pairs faster, approaching the million pairs per second mark, really need several order of magnitude more
- Routing and switching is also being investigated

# Error Model

- Briefly examine an error model that has been used with both quantum computing and quantum communications
- Want to model various forms of decoherence
- Earlier looked at bit flips, but these are 180 degree rotation on the Bloch sphere, which is quite radical
- But, more likely to have smaller rotations, these will have an impact on any state that we measure
- Need to examine a bit more quantum mechanics and then examine the error model

# Error Model

- So far only dealt with pure qubits, they live on the surface of the Bloch sphere and are relatively easy to deal with
- We can also have mixed state qubits, the state of the qubit is a mixture of pure states
- A mixed qubit is a collection of $\{p_i, |\psi_i\rangle\}$ where $\psi_i$ is a pure state and $p_i$is the probability of that state
- In the case of a pure qubit there is only one state with probability 1
- Unfortunately, we can no longer use vectors for states

# Error Model

- Instead our qubit is represented by a density matrix

$$\rho = \sum_i p_i \left|\psi_i\right\rangle \left\langle\psi_i\right| .$$

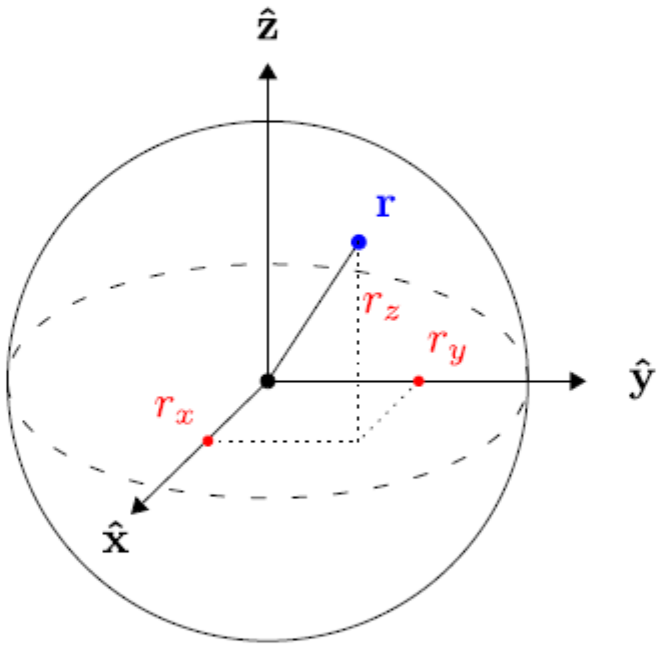
- With a single qubit we are dealing with a 2x2 matrix for the state, given by

$$\rho = \begin{bmatrix} \rho^{00} & \rho^{01} \\ \rho^{10} & \rho^{11} \end{bmatrix} = \frac{1}{2}\left(I + r_x\sigma_x + r_y\sigma_y + r_z\sigma_z\right),$$

- Where we again have our standard Pauli rotations

# Error Model

- We can view $(r_x, r_y, r_z)$ as a point, but now it's inside the Bloch sphere



# Error Model

- We can view decoherence as the qubit interacting with its environment
- We can construct a unitary operator for this, and this leads to a differential equation which describes the time evolution of the density matrix
- We will skip the differential equation and just examine some of its solutions and the impact they have on errors

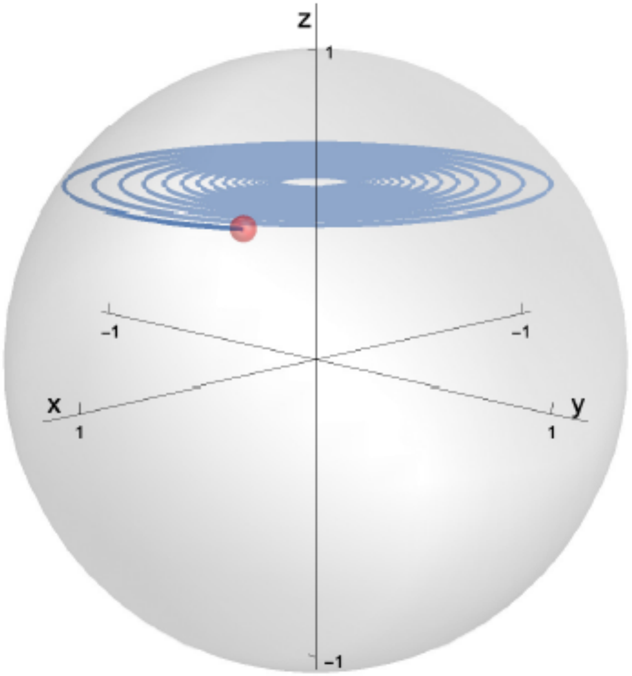
# Error Model

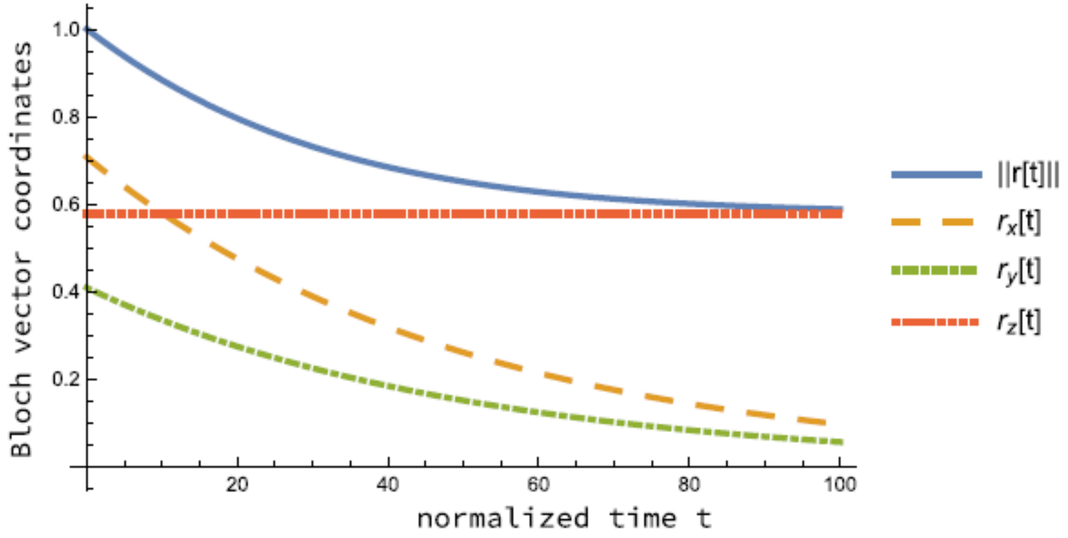
- One common error is phase damping, the loss of quantum information without a loss of energy
- In this case the solution is:

$$
\begin{aligned}
r_x(t) &= r_x(0)e^{-2\gamma_z t},\\
r_y(t) &= r_y(0)e^{-2\gamma_z t},\\
r_z(t) &= r_z(0),
\end{aligned}
$$

- Note that this isn’t symmetric in the three dimensions and the loss increases with time

# Error Model – Phase Damping



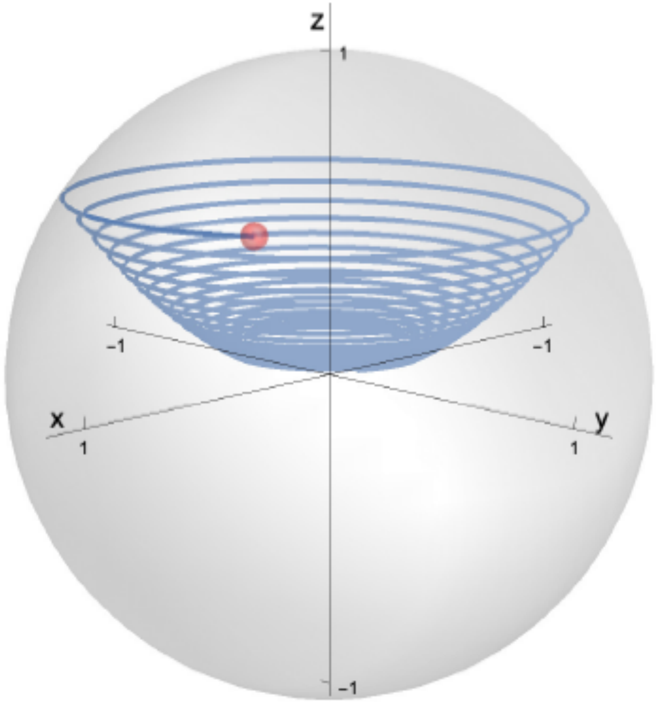
# Error Model

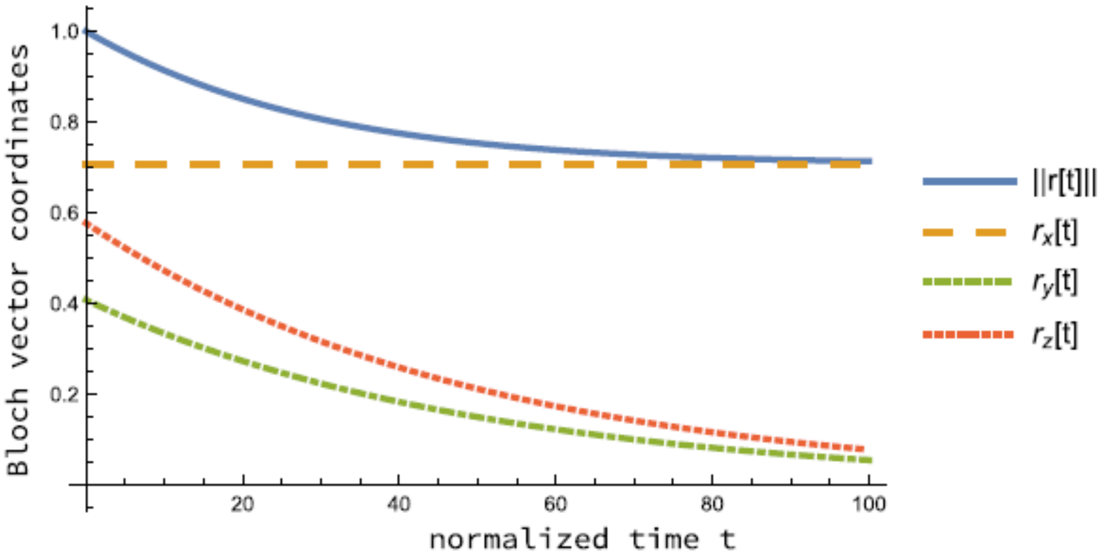
- Another common form of error is y-z damping

$$
\begin{aligned}
r_x(t) &= r_x(0), \\
r_y(t) &= r_y(0)e^{-2\gamma_x t}, \\
r_z(t) &= r_z(0)e^{-2\gamma_x t},
\end{aligned}
$$

- Again we see that this is not symmetric and the loss increases with time

# Error Model – y-z Damping



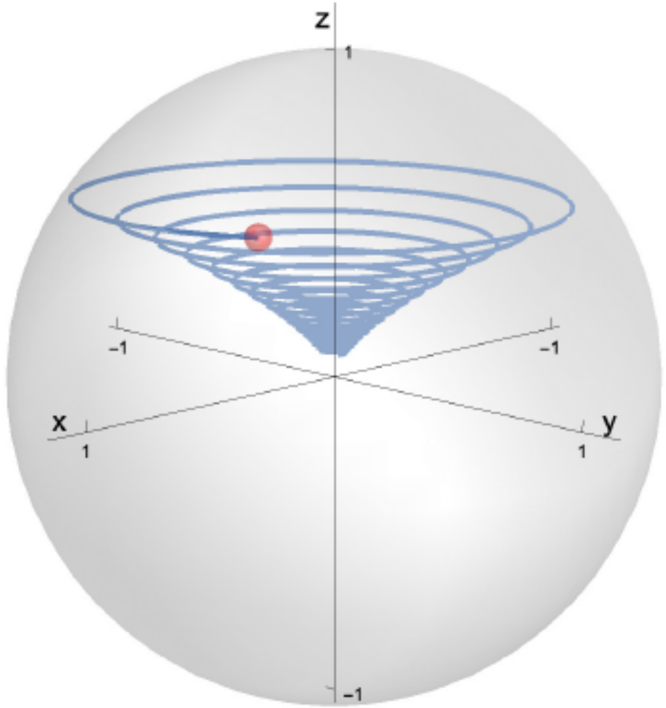
# Error Model

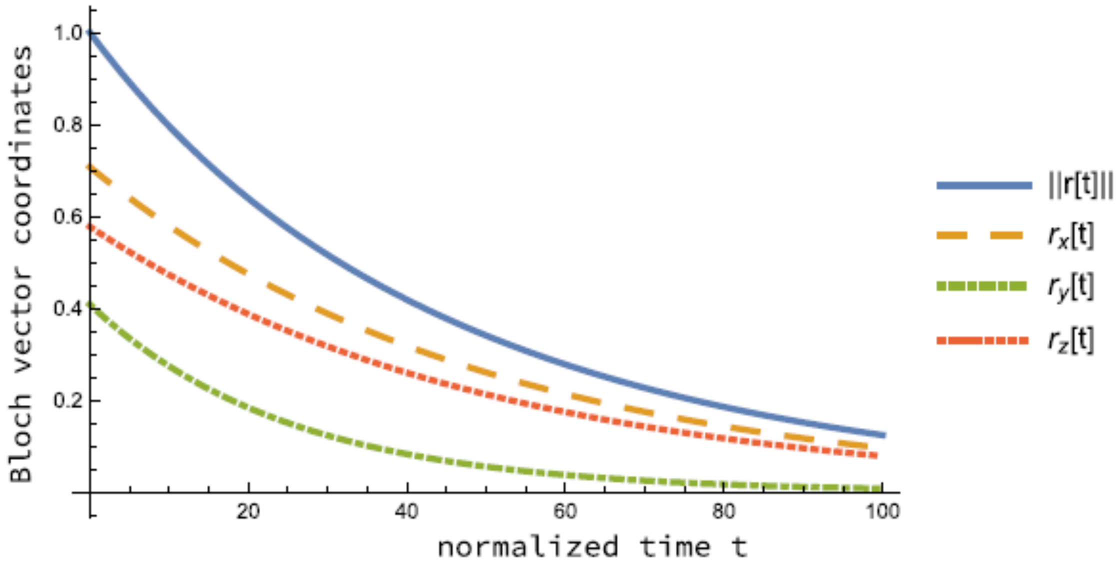
- The final type of error that we will examine is y-z phase damping, which is a combination of the previous two types of errors
- The solution to this is

$$r_x(t) = r_x(0)e^{-2\gamma_z t},$$
$$r_y(t) = r_y(0)e^{-2(\gamma_x+\gamma_z)t},$$
$$r_z(t) = r_z(0)e^{-2\gamma_x t}.$$

- Note that these errors can occur both within the quantum computer and in quantum communications

# Error Model – y-z Phase Damping



# Summary

- Introduced the idea of quantum communications
- Showed how quantum teleportation can be used for this
- Examined the basic ideas behind quantum repeaters and quantum routers
- Introduced a error model and demonstrated its impact on qubits