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CSCI 4140U

Laboratory Three

Quantum Key Distribution

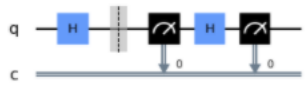
First, change the length of the bit sequence that is generated and observe the change in efficiency. The length is the variable n, set on the second line of code. Try values of 100, 200, 500 and 1000 and record the efficiency. How long of a sequence do you expect you will need to get close to 50% efficiency? Record your results in your laboratory report in the form of a table.

For n = 100

In [53]:
```
np.random.seed(seed=3)
n = 100

alice_bits = randint(2, size=n)
#print("alice_bits = " + str(alice_bits))

alice_bases = randint(2, size=n)
#print("alice_bases = " + str(alice_bases))
message = encode_message(alice_bits, alice_bases)

eve_bases = randint(2, size=n)
intercepted_message = measure_message(message, eve_bases)
#print("intercepted_message = " + str(intercepted_message))

bob_bases = randint(2, size=n)
#print("bob_bases = " + str(bob_bases))
bob_results = measure_message(message, bob_bases)
#print("bob_results = " + str(bob_results))

alice_key = remove_garbage(alice_bases, bob_bases, alice_bits)
#print("alice_key = " + str(alice_key))
bob_key = remove_garbage(alice_bases, bob_bases, bob_results)
#print("bob_key = " + str(bob_key))

alice_key = remove_garbage(alice_bases, bob_bases, alice_bits)
print("alice_key = " + str(alice_key))
bob_key = remove_garbage(alice_bases, bob_bases, bob_results)
print("bob_key = " + str(bob_key))

sample_size = 15

bit_selection = randint(n, size=sample_size)
bob_sample = sample_bits(bob_key, bit_selection)
print("  bob_sample = " + str(bob_sample))
alice_sample = sample_bits(alice_key, bit_selection)
print("alice_sample = "+ str(alice_sample))

if bob_sample != alice_sample:
    print("Eve's interference was detected.")
else:
    print("Eve went undetected!")

if bob_sample == alice_sample:
    print("key is safe")
else:
    print("key is compromized")
print("Efficiency: "+str(len(bob_key)/n))

print("key length = %i" % len(alice_key))
message[0].draw('mpl')
```

```
alice_key = [0, 0, 1, 0, 0, 0, 1, 1, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 1, 0, 0, 1, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 1, 0,
0, 1, 0, 1, 1, 1, 1, 0, 1, 0, 1, 1, 1, 0, 1, 0, 1]
bob_key = [1, 1, 1, 1, 1, 0, 0, 1, 1, 0, 0, 0, 0, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 1, 0, 1, 0, 1, 1, 0, 0, 0, 0, 1, 0, 0,
0, 1, 1, 1, 1, 1, 1, 0, 0, 0, 0, 1, 0, 1, 1, 0, 0]
  bob_sample = [1, 1, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 1, 1]
alice_sample = [1, 1, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0]
Eve's interference was detected.
key is compromized
Efficiency: 0.37
key length = 37
```

Out[53]:

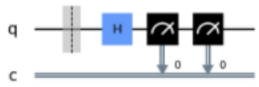


For n = 500

In [52]:
```
np.random.seed(seed=3)
n = 500

alice_bits = randint(2, size=n)
#print("alice_bits = " + str(alice_bits))

alice_bases = randint(2, size=n)
#print("alice_bases = " + str(alice_bases))
message = encode_message(alice_bits, alice_bases)

eve_bases = randint(2, size=n)
intercepted_message = measure_message(message, eve_bases)
#print("intercepted_message = " + str(intercepted_message))

bob_bases = randint(2, size=n)
#print("bob_bases = " + str(bob_bases))
bob_results = measure_message(message, bob_bases)
#print("bob_results = " + str(bob_results))

alice_key = remove_garbage(alice_bases, bob_bases, alice_bits)
#print("alice_key = " + str(alice_key))
bob_key = remove_garbage(alice_bases, bob_bases, bob_results)
#print("bob_key = " + str(bob_key))

sample_size = 15

bit_selection = randint(n, size=sample_size)
bob_sample = sample_bits(bob_key, bit_selection)
print("  bob_sample = " + str(bob_sample))
alice_sample = sample_bits(alice_key, bit_selection)
print("alice_sample = "+ str(alice_sample))

if bob_sample != alice_sample:
    print("Eve's interference was detected.")
else:
    print("Eve went undetected!")

if bob_sample == alice_sample:
    print("key is safe")
else:
    print("key is compromized")
print("Efficiency: "+str(len(bob_key)/n))

print("key length = %i" % len(alice_key))
message[0].draw('mpl')
```

```
  bob_sample = [1, 1, 0, 1, 0, 1, 1, 1, 0, 1, 0, 1, 0, 1, 1]
alice_sample = [1, 0, 0, 1, 0, 0, 1, 0, 1, 1, 0, 0, 0, 0, 0]
Eve's interference was detected.
key is compromized
Efficiency: 0.46
key length = 230
```

Out[52]:



For n = 1000

In [64]:
```
np.random.seed(seed=3)
n = 1000

alice_bits = randint(2, size=n)
#print("alice_bits = " + str(alice_bits))

alice_bases = randint(2, size=n)
#print("alice_bases = " + str(alice_bases))
message = encode_message(alice_bits, alice_bases)

eve_bases = randint(2, size=n)
intercepted_message = measure_message(message, eve_bases)
#print("intercepted_message = " + str(intercepted_message))

bob_bases = randint(2, size=n)
#print("bob_bases = " + str(bob_bases))
bob_results = measure_message(message, bob_bases)
#print("bob_results = " + str(bob_results))

alice_key = remove_garbage(alice_bases, bob_bases, alice_bits)
#print("alice_key = " + str(alice_key))
bob_key = remove_garbage(alice_bases, bob_bases, bob_results)
#print("bob_key = " + str(bob_key))

sample_size = 15

bit_selection = randint(n, size=sample_size)
bob_sample = sample_bits(bob_key, bit_selection)
print("  bob_sample = " + str(bob_sample))
alice_sample = sample_bits(alice_key, bit_selection)
print("alice_sample = "+ str(alice_sample))

if bob_sample != alice_sample:
    print("Eve's interference was detected.")
else:
    print("Eve went undetected!")

if bob_sample == alice_sample:
    print("key is safe")
else:
    print("key is compromized")
print("Efficiency: "+str(len(bob_key)/n))

print("key length = %i" % len(alice_key))
message[0].draw('mpl')
```

```
  bob_sample = [1, 0, 1, 0, 1, 0, 0, 1, 0, 1, 0, 1, 1, 1, 1]
alice_sample = [1, 0, 1, 0, 1, 0, 0, 1, 1, 0, 0, 1, 0, 1, 1]
Eve's interference was detected.
key is compromized
Efficiency: 0.485
key length = 485
```

Out[64]:

For n = 1200

In [66]:
```
np.random.seed(seed=3)
n = 1400

alice_bits = randint(2, size=n)
#print("alice_bits = " + str(alice_bits))

alice_bases = randint(2, size=n)
#print("alice_bases = " + str(alice_bases))
message = encode_message(alice_bits, alice_bases)

eve_bases = randint(2, size=n)
intercepted_message = measure_message(message, eve_bases)
#print("intercepted_message = " + str(intercepted_message))

bob_bases = randint(2, size=n)
#print("bob_bases = " + str(bob_bases))
bob_results = measure_message(message, bob_bases)
#print("bob_results = " + str(bob_results))

alice_key = remove_garbage(alice_bases, bob_bases, alice_bits)
#print("alice_key = " + str(alice_key))
bob_key = remove_garbage(alice_bases, bob_bases, bob_results)
#print("bob_key = " + str(bob_key))

sample_size = 15

bit_selection = randint(n, size=sample_size)
bob_sample = sample_bits(bob_key, bit_selection)
print("  bob_sample = " + str(bob_sample))
alice_sample = sample_bits(alice_key, bit_selection)
print("alice_sample = "+ str(alice_sample))

if bob_sample != alice_sample:
    print("Eve's interference was detected.")
else:
    print("Eve went undetected!")

if bob_sample == alice_sample:
    print("key is safe")
else:
    print("key is compromized")
print("Efficiency: "+str(len(bob_key)/n))

print("key length = %i" % len(alice_key))
message[0].draw('mpl')
```

```
  bob_sample = [0, 1, 0, 0, 1, 0, 1, 0, 0, 0, 1, 0, 0, 1, 0]
alice_sample = [1, 1, 0, 1, 0, 0, 0, 0, 0, 1, 1, 0, 1, 0, 0]
Eve's interference was detected.
key is compromized
Efficiency: 0.4928571428571429
key length = 690
```

Out[66]:

Second, in the video lecture we observed that when the message was intercepted there was only a small number of bits that were different in the sample. Change the seed of the random number on the first line of code and observe how the number of different bits in the sample changes. Try five different seed values and record the number of different bits. Record your results in your laboratory report in the form of a table.

For seed = 0

In [69]:
```
np.random.seed(seed=0)
n = 100

alice_bits = randint(2, size=n)
#print("alice_bits = " + str(alice_bits))

alice_bases = randint(2, size=n)
#print("alice_bases = " + str(alice_bases))
message = encode_message(alice_bits, alice_bases)

eve_bases = randint(2, size=n)
intercepted_message = measure_message(message, eve_bases)
#print("intercepted_message = " + str(intercepted_message))

bob_bases = randint(2, size=n)
#print("bob_bases = " + str(bob_bases))
bob_results = measure_message(message, bob_bases)
#print("bob_results = " + str(bob_results))

alice_key = remove_garbage(alice_bases, bob_bases, alice_bits)
#print("alice_key = " + str(alice_key))
bob_key = remove_garbage(alice_bases, bob_bases, bob_results)
#print("bob_key = " + str(bob_key))

sample_size = 15

bit_selection = randint(n, size=sample_size)
bob_sample = sample_bits(bob_key, bit_selection)
print("  bob_sample = " + str(bob_sample))
alice_sample = sample_bits(alice_key, bit_selection)
print("alice_sample = "+ str(alice_sample))
```

```
  bob_sample = [1, 0, 0, 0, 1, 1, 1, 1, 0, 0, 0, 1, 0, 1, 1]
alice_sample = [0, 0, 0, 1, 0, 1, 0, 1, 1, 1, 0, 1, 0, 0, 1]
```

For seed = 1

In [70]:
```
np.random.seed(seed=1)
n = 100

alice_bits = randint(2, size=n)
#print("alice_bits = " + str(alice_bits))

alice_bases = randint(2, size=n)
#print("alice_bases = " + str(alice_bases))
message = encode_message(alice_bits, alice_bases)

eve_bases = randint(2, size=n)
intercepted_message = measure_message(message, eve_bases)
#print("intercepted_message = " + str(intercepted_message))

bob_bases = randint(2, size=n)
#print("bob_bases = " + str(bob_bases))
bob_results = measure_message(message, bob_bases)
#print("bob_results = " + str(bob_results))

alice_key = remove_garbage(alice_bases, bob_bases, alice_bits)
#print("alice_key = " + str(alice_key))
bob_key = remove_garbage(alice_bases, bob_bases, bob_results)
#print("bob_key = " + str(bob_key))

sample_size = 15

bit_selection = randint(n, size=sample_size)
bob_sample = sample_bits(bob_key, bit_selection)
print("  bob_sample = " + str(bob_sample))
alice_sample = sample_bits(alice_key, bit_selection)
print("alice_sample = "+ str(alice_sample))
```

```
  bob_sample = [1, 0, 0, 1, 0, 1, 0, 0, 0, 1, 1, 0, 0, 0, 1]
alice_sample = [0, 0, 1, 0, 0, 1, 0, 1, 1, 1, 1, 0, 0, 0, 0]
```

For seed = 2

In [71]:
```
np.random.seed(seed=2)
n = 100

alice_bits = randint(2, size=n)
#print("alice_bits = " + str(alice_bits))

alice_bases = randint(2, size=n)
#print("alice_bases = " + str(alice_bases))
message = encode_message(alice_bits, alice_bases)

eve_bases = randint(2, size=n)
intercepted_message = measure_message(message, eve_bases)
#print("intercepted_message = " + str(intercepted_message))

bob_bases = randint(2, size=n)
#print("bob_bases = " + str(bob_bases))
bob_results = measure_message(message, bob_bases)
#print("bob_results = " + str(bob_results))

alice_key = remove_garbage(alice_bases, bob_bases, alice_bits)
#print("alice_key = " + str(alice_key))
bob_key = remove_garbage(alice_bases, bob_bases, bob_results)
#print("bob_key = " + str(bob_key))

sample_size = 15

bit_selection = randint(n, size=sample_size)
bob_sample = sample_bits(bob_key, bit_selection)
print("  bob_sample = " + str(bob_sample))
alice_sample = sample_bits(alice_key, bit_selection)
print("alice_sample = "+ str(alice_sample))
```

```
  bob_sample = [1, 1, 0, 0, 1, 1, 0, 0, 0, 0, 0, 1, 1, 1, 0]
alice_sample = [0, 0, 0, 0, 1, 1, 1, 0, 1, 0, 1, 1, 0, 0, 0]
```

For seed = 4

In [72]:
```
np.random.seed(seed=4)
n = 100

alice_bits = randint(2, size=n)
#print("alice_bits = " + str(alice_bits))

alice_bases = randint(2, size=n)
#print("alice_bases = " + str(alice_bases))
message = encode_message(alice_bits, alice_bases)

eve_bases = randint(2, size=n)
intercepted_message = measure_message(message, eve_bases)
#print("intercepted_message = " + str(intercepted_message))

bob_bases = randint(2, size=n)
#print("bob_bases = " + str(bob_bases))
bob_results = measure_message(message, bob_bases)
#print("bob_results = " + str(bob_results))

alice_key = remove_garbage(alice_bases, bob_bases, alice_bits)
#print("alice_key = " + str(alice_key))
bob_key = remove_garbage(alice_bases, bob_bases, bob_results)
#print("bob_key = " + str(bob_key))

sample_size = 15

bit_selection = randint(n, size=sample_size)
bob_sample = sample_bits(bob_key, bit_selection)
print("  bob_sample = " + str(bob_sample))
alice_sample = sample_bits(alice_key, bit_selection)
print("alice_sample = "+ str(alice_sample))
```

```
  bob_sample = [0, 0, 1, 1, 0, 1, 1, 1, 1, 1, 0, 1, 1, 0, 0]
alice_sample = [0, 1, 1, 1, 0, 0, 0, 1, 1, 1, 0, 1, 1, 1, 1]
```

For seed = 5

In [73]:
```
np.random.seed(seed=5)
n = 100

alice_bits = randint(2, size=n)
#print("alice_bits = " + str(alice_bits))

alice_bases = randint(2, size=n)
#print("alice_bases = " + str(alice_bases))
message = encode_message(alice_bits, alice_bases)

eve_bases = randint(2, size=n)
intercepted_message = measure_message(message, eve_bases)
#print("intercepted_message = " + str(intercepted_message))

bob_bases = randint(2, size=n)
#print("bob_bases = " + str(bob_bases))
bob_results = measure_message(message, bob_bases)
#print("bob_results = " + str(bob_results))

alice_key = remove_garbage(alice_bases, bob_bases, alice_bits)
#print("alice_key = " + str(alice_key))
bob_key = remove_garbage(alice_bases, bob_bases, bob_results)
#print("bob_key = " + str(bob_key))

sample_size = 15

bit_selection = randint(n, size=sample_size)
bob_sample = sample_bits(bob_key, bit_selection)
print("  bob_sample = " + str(bob_sample))
alice_sample = sample_bits(alice_key, bit_selection)
print("alice_sample = "+ str(alice_sample))
```

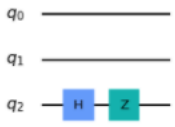
```
  bob_sample = [0, 0, 1, 0, 1, 0, 1, 0, 0, 0, 1, 1, 1, 0, 1]
alice_sample = [0, 0, 0, 1, 1, 0, 0, 0, 1, 1, 0, 1, 1, 1, 0]
```

Balanced Functions

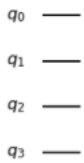
In [96]:
```
import numpy as np

# importing Qiskit
from qiskit import IBMQ, BasicAer
from qiskit.providers.ibmq import least_busy
from qiskit import QuantumCircuit, execute

# import basic plot tools
from qiskit.visualization import plot_histogram
from qiskit_textbook.widgets import dj_widget

dj_widget(size="small", case="balanced")

n = 3

const_oracle = QuantumCircuit(n+1)

output = np.random.randint(2)
if output == 1:
    const_oracle.x(n)
const_oracle.draw('mpl')
```

H⊗ⁿ | Oracle | Clear

$|00\rangle = |00\rangle$

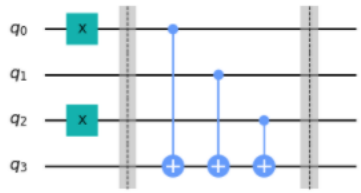


Out[96]:



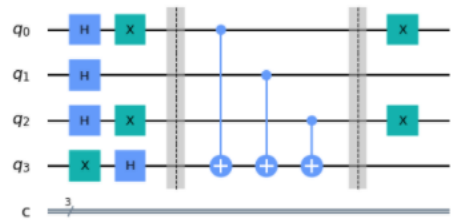
In [97]:
```
#PART NEEDED
balanced_oracle = QuantumCircuit(n+1)
b_str = "101"

for qubit in range(len(b_str)):
    if b_str[qubit] == '1':
        balanced_oracle.x(qubit)
balanced_oracle.barrier()



for qubit in range(n):
    balanced_oracle.cx(qubit, n)

balanced_oracle.barrier()
balanced_oracle.draw('mpl')
```

Out[97]:



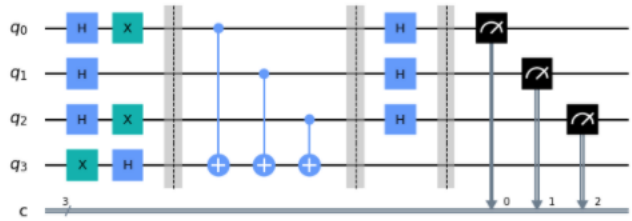
In [105]:
```
for qubit in range(len(b_str)):
    if b_str[qubit] == '1':
        balanced_oracle.x(qubit)
balanced_oracle.draw('mpl')

dj_circuit = QuantumCircuit(n+1, n)

# Apply H-gates
for qubit in range(n):
    dj_circuit.h(qubit)

# Put qubit in state |->
dj_circuit.x(n)
dj_circuit.h(n)

# Add oracle
dj_circuit += balanced_oracle
dj_circuit.draw('mpl')
```

Out[105]:



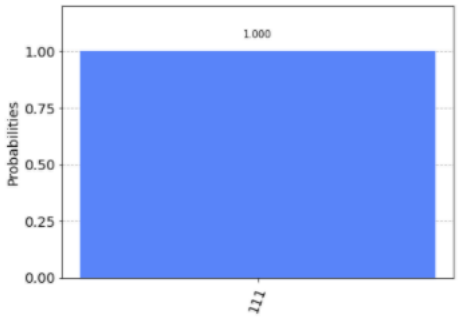
In [98]:
```
#PART NEEDED
dj_circuit = QuantumCircuit(n+1, n)

# Apply H-gates
for qubit in range(n):
    dj_circuit.h(qubit)

# Put qubit in state |->
dj_circuit.x(n)
dj_circuit.h(n)

# Add oracle
dj_circuit += balanced_oracle

# Repeat H-gates
for qubit in range(n):
    dj_circuit.h(qubit)
dj_circuit.barrier()

# Measure
for i in range(n):
    dj_circuit.measure(i, i)

# Display circuit
dj_circuit.draw('mpl')
```

Out[98]:



In [81]:
```
backend = BasicAer.get_backend('qasm_simulator')
shots = 1
results = execute(dj_circuit, backend=backend, shots=shots, memory=True).result()
answer = results.get_counts()

plot_histogram(answer)
```

Out[81]:



Deutsch-Josza ORACLE

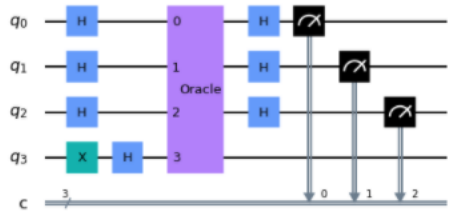
In [82]:
```
def dj_oracle(case, n):

    oracle_qc = QuantumCircuit(n+1)


    if case == "balanced":

        b = np.random.randint(1,2**n)

        b_str = format(b, '0'+str(n)+'b')

        for qubit in range(len(b_str)):
            if b_str[qubit] == '1':
                oracle_qc.x(qubit)

        for qubit in range(n):
            oracle_qc.cx(qubit, n)

        for qubit in range(len(b_str)):
            if b_str[qubit] == '1':
                oracle_qc.x(qubit)

    if case == "constant":

        output = np.random.randint(2)
        if output == 1:
            oracle_qc.x(n)

    oracle_gate = oracle_qc.to_gate()
    oracle_gate.name = "Oracle"
    return oracle_gate
```

Deutsch-Josza ALGORITHM

In [83]:
```
def dj_algorithm(oracle, n):
    dj_circuit = QuantumCircuit(n+1, n)
    dj_circuit.x(n)
    dj_circuit.h(n)

    for qubit in range(n):
        dj_circuit.h(qubit)

    dj_circuit.append(oracle, range(n+1))

    for qubit in range(n):
        dj_circuit.h(qubit)

    for i in range(n):
        dj_circuit.measure(i, i)

    return dj_circuit
```

In [99]:
```
n = 3
oracle_gate = dj_oracle('balanced', n)
dj_circuit = dj_algorithm(oracle_gate, n)
dj_circuit.draw('mpl')
```

Out[99]:



In [100]:
```
backend = BasicAer.get_backend('qasm_simulator')
results = execute(dj_circuit, backend=backend, shots=1, memory=True).result()
answer = results.get_counts()
plot_histogram(answer)
```

Out[100]: