# CSCI 4140
# Phase Kickback

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# Introduction

- Now that we know about eigenvalues and eigenvectors we can explain how phase kickback works
- For any unitary matrix U we have:

$$U = \sum_j \lambda_j |v_j >< v_j|$$

- We also have that:

$$\lambda_j = e^{ih_j}$$

- So we can write the sum as

$$U = \sum_j e^{ih_j} |v_j >< v_j|$$

# Phase Kickback

- Now apply U to any of its eigenvectors, we get back the eigenvector multiplied by its eigenvalue: $e^{ih_j}v_j$
- Now lets created a controlled version of U, we have the following:
$$U(|1\rangle|v_j\rangle = e^{ih_j}|1\rangle|v_j\rangle$$
- The eigenvector is a global phase, so we don’t see it in the computation
- As long as the control qubit is |0> or |1> everything works out okay

# Phase Kickback

- Now consider what happens when the control qubit is a mixture of |0> and |1>

$$(\alpha|0\rangle + \beta|1\rangle)|v_j\rangle \longmapsto \alpha|0\rangle|v_j\rangle + e^{ih_j}\beta|1\rangle|v_j\rangle$$

- We can simplify this to:

$$(\alpha|0\rangle + e^{ih_j}\beta|1\rangle)|v_j\rangle$$

- The eigenvalue now becomes a local phase on the control qubit
- U can be of any size, this gives us a way of extracting a single value from a complex circuit

# Discussion

- But, wait, you’ve only shown this for eigenvectors, what about arbitrary vectors???
- The eigenvectors for a unitary matrix form an orthonormal basis
- Therefor an arbitrary vector can be written as a linear combination of eigenvectors, thus it works for them as well
- The mathematics for this is more complex, you need to keep track of all of the phases, so I just did the simple case

# Summary

- With the use of eigenvalues and eigenvectors we have demonstrated how phase kickback works
- There is not real magic here, just the transfer of information from a group of qubits to another qubit