CSCI 4140
Photonic Quantum Computers

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Introduction

- Photons are of considerable interest to quantum computing:
  - Operate at room temperature
  - Don’t have a charge, immune to most noise
  - Extremely long coherence time
  - Can move over long distances in fibre optics
- This is the technology that could be put into a desktop or laptop computer
- Potentially much cheaper than ion trap and superconducting qubits

Introduction

- Photons can behave like qubits, but not easily in energy levels, have been some attempts, but it’s quite difficult
- Other properties that can be used:
  - Spin – this is a pure quantum property common to all elementary particles
  - Polarization – at the quantum and macro level, well understood optical property
- There are other approaches, but are more fringe

Introduction

- Just starting to see demonstrations of photonic quantum computers
- There are two main approaches that we will examine
- KLM protocol – named after Knill, Laflamme and Milburn who created it, this is a universal quantum computer
- There are many challenges to constructing a programmable computer with this approach
- Boson Sampling – not a universal quantum computer, but solves a number of important problems
- Can be constructed, close to commercialization

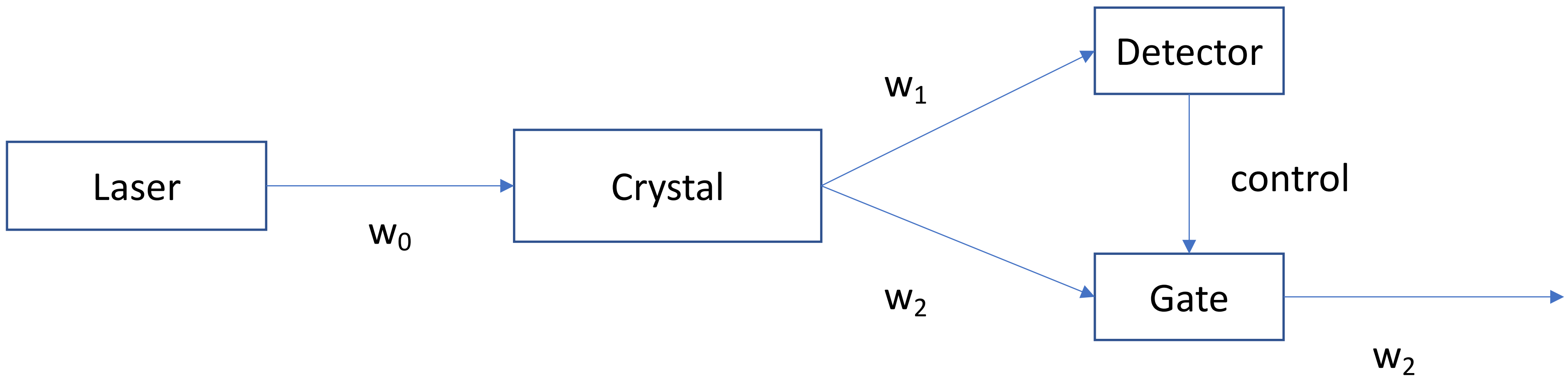
KLM Protocol

- The main attraction of KLM is that it uses standard optical components, which are well understood:
  - Mirror – standard high reflectance mirror
  - Beam splitter – splits a beam of light into two
  - Phase shifter – changes the phase of the photon
- There are unitary matrices for each of these devices
- At the macro level, these are standard optical components, can buy off the shelf

KLM Protocol

- Unfortunately, they are relatively large, need to construct much smaller versions, possible with photonic chips
- One of the challenges with KLM is a source of uniform photons, in particular a source of single photons
- A laser will produce a stream of relatively uniform photons, but not a single photon
- There is a solution to this problem, a laser shot through a nonlinear crystal can produce two photons, their frequencies sum to the original photon frequency
- We can use one to control a gate that lets the other one pass

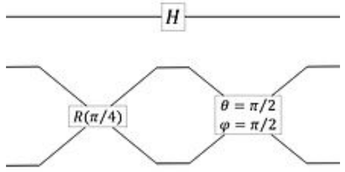
KLM Protocol – Spontaneous Parametric Down-Conversion (SPDC)



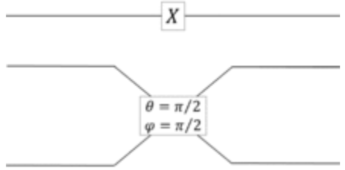
$$w_0 = w_1 + w_2$$

KLM Protocol

- We can use some of the standard optical components to produce some of the standard quantum gates
- A Hadamard is a mirror followed by a beam splitter

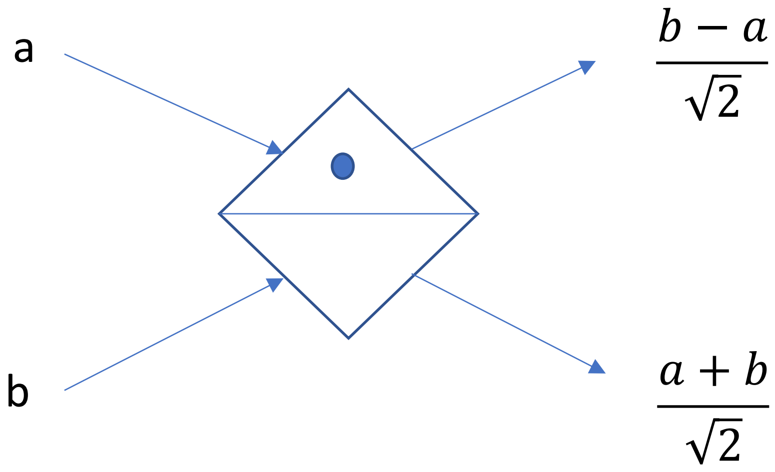


- An X gate is produced by a beam splitter



KLM Protocol

- A 50/50 beam splitter can be used to produce superposition

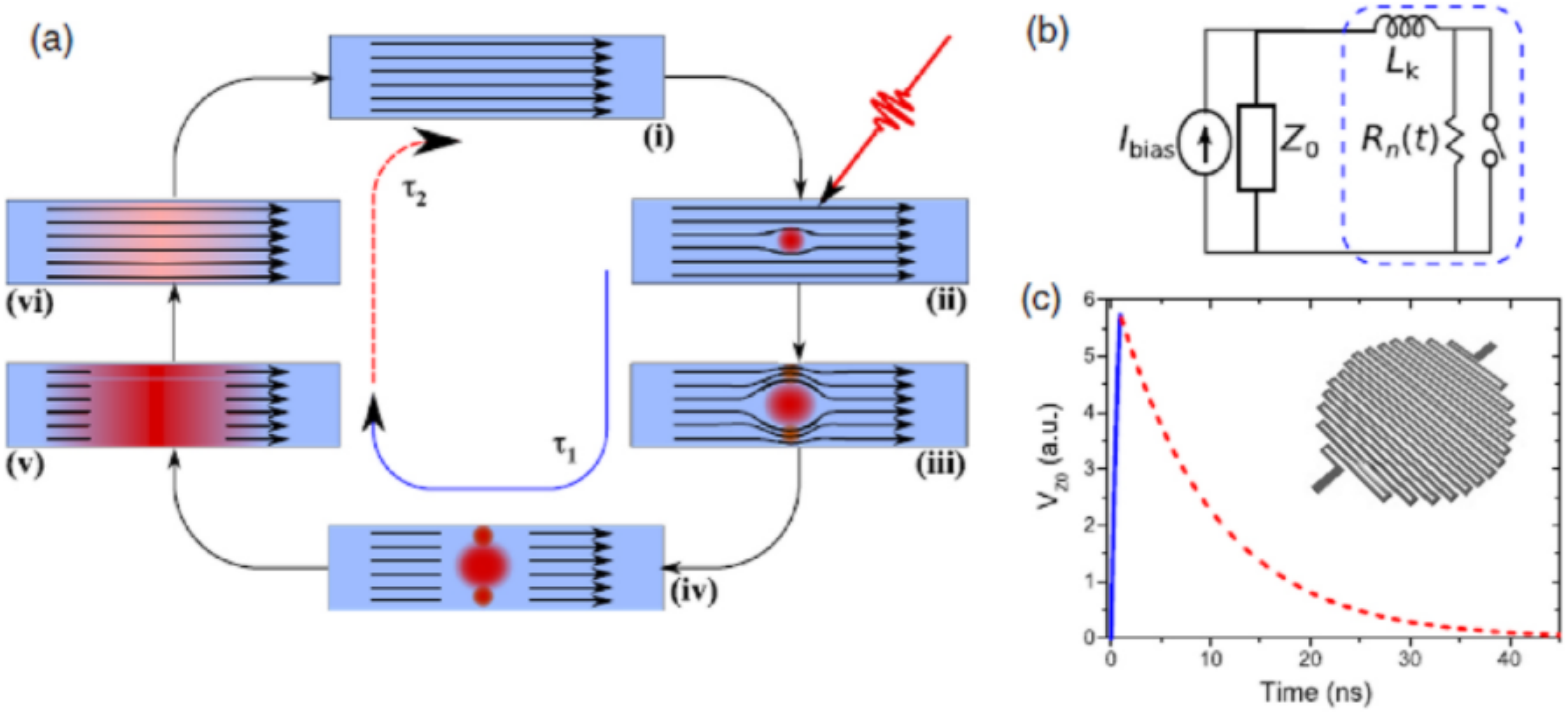


KLM Protocol

- The next problem is detecting single photons, there are several technologies that can be used for this
- Single photon avalanche diodes (SPAD) are commercial devices for detected single photons and counting photons
- Superconducting nanowires are another approach which is becoming popular
- The photon strikes parallel nanowires which are displaced as a result
- This changes their electrical properties, which can be measured
- This requires very low temperatures

KLM Protocol

Figure 3:



KLM Protocol

- While this is a promising technology there is a major problem
- The optical elements need to be arranged for each computation
- This needs to be done under program control, which is a problem
- Unlike our previous two technologies, gates are a real physical resource, don't have an unlimited supply of them
- Photon generation can be slow, not the fastest approach to quantum computing
- Could possibly change the parameters of these devices, but this would require some for of on chip technology

Boson Sampling

- This is a totally different approach to quantum computing
- There are no qubits, and no gates
- There is some thought that boson sampling is more powerful than other types of quantum computers
- General boson sampling appears to be universal, will also examine Gaussian boson sampling, which is not universal
- Bosons are a class of elementary particles, and the ones that we will be interested in are photons

Boson Sampling

- This is an area that quickly gets quite mathematical, will try to avoid the harder parts
- Start with photons that can be in one of m modes, think of a mode as a state of a photon
- Each mode has a creation operator $a_i^\dagger$ with an inverse a_i
- Start with n photons distributed over m modes, m=O(n^2)
- Our input state can be written as

$$\begin{aligned} |\psi_{\text{in}}\rangle &= |1_1, \ldots, 1_n, 0_{n+1}, \ldots, 0_m\rangle \\ &= \hat{a}_1^\dagger \ldots \hat{a}_n^\dagger |0_1, \ldots, 0_m\rangle , \end{aligned}$$

Boson Sampling

- We can then apply a unitary operator U, with m x m matrix $\widehat{U}$ to the input state:

$$\hat{U}\hat{a}_i^\dagger\hat{U}^\dagger = \sum_{j=1}^{m} U_{i,j}\hat{a}_j^\dagger,$$

- This unitary can be implemented by the linear optical elements that we have already seen
- The output will be the superposition of some number of configurations, where each configuration has n photons distributed over its m modes

Boson Sampling

- We can write the output state as:

$$|\psi_{\text{out}}\rangle = \sum_S \gamma_S \, |n_1^{(S)}, \ldots, n_m^{(S)}\rangle \,,$$

- Where $n_i^{(S)}$ is the number of photon in mode i of configuration S
- Here γ_S is the amplitude of configuration S and the probability of configuration S is $P_S = |\gamma_S|^2$
- The power of this technique comes from the expression for γ_S

Boson Sampling

- We have:

$$\gamma_S = \frac{\mathrm{Per}(U_S)}{\sqrt{n_1^{(S)}! \ldots n_m^{(S)}!}},$$

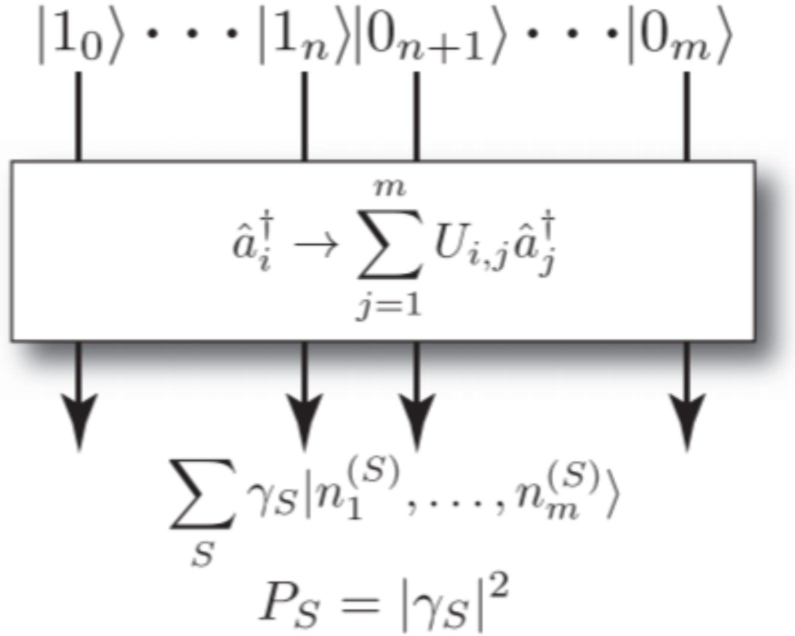
- Where U_S is an n x n submatrix of U and $Per(U_S)$ is the permanent of U_S
- The permanent is given by

$$\mathrm{Per}(B) = \sum_{\sigma \in S_n} \prod_{i=1}^{n} b_{i,\sigma(i)}$$

Boson Sampling

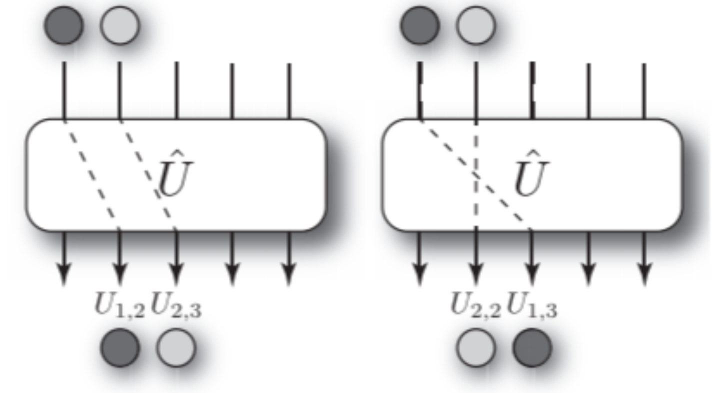
- S_n is the set of all permutations of the first n integers
- Why is this important?
- The permanent of a matrix requires exponential computation, in its most general form it is one of the hardest known computational problems
- Boson sampling is computing this permanent efficiently, which makes it a very powerful computational technique
- The process is summarized on the following slide

Boson Sampling



Boson Sampling

- Example: with two photons that are two possible paths through U



- Which give the following

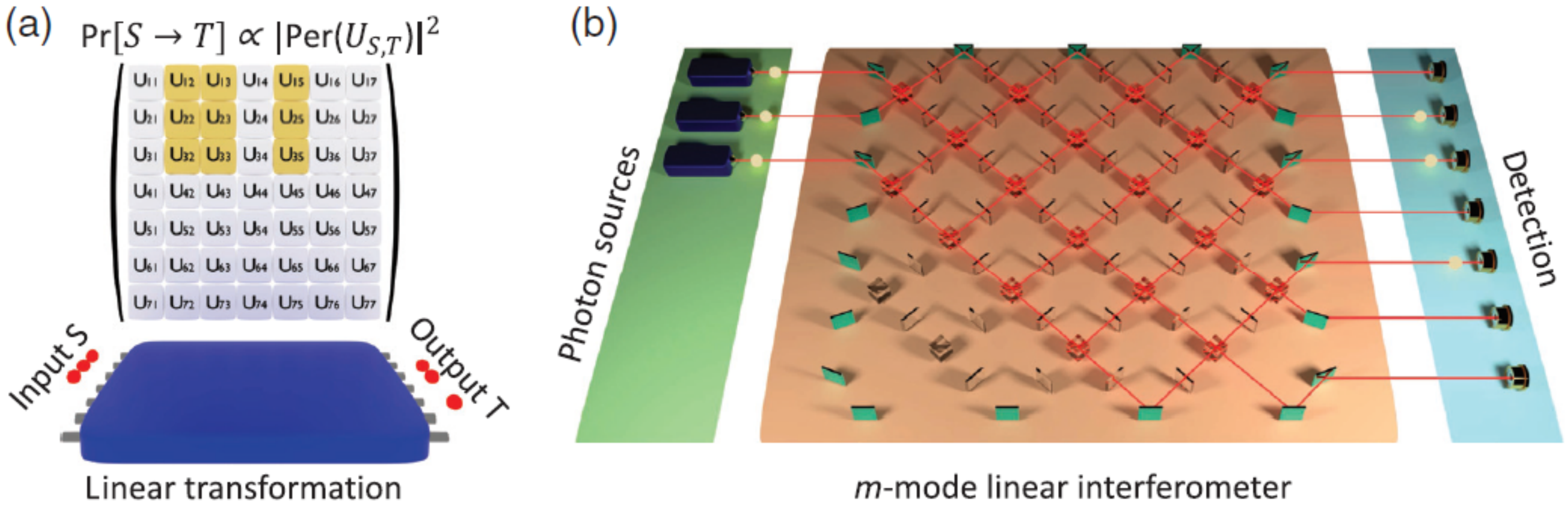
$$\begin{aligned}\gamma_{\{2,3\}} &= \underbrace{U_{1,2}U_{2,3}}_{\text{walkers don't swap}} + \underbrace{U_{1,3}U_{2,2}}_{\text{walkers swap}} \\ &= \mathrm{Per}\begin{bmatrix} U_{1,2} & U_{2,2} \\ U_{1,3} & U_{2,3} \end{bmatrix},\end{aligned}$$

Boson Sampling

- We construct the computation in the following way:

1. Prepare an n photon m mode input state, with each mode having zero or one photon. This state could have a photon in the first n modes
2. Construct an m mode interferometer representing U. This can be done with standard optical components
3. Photon detectors on every output. Only need to detect at most one photon per detector

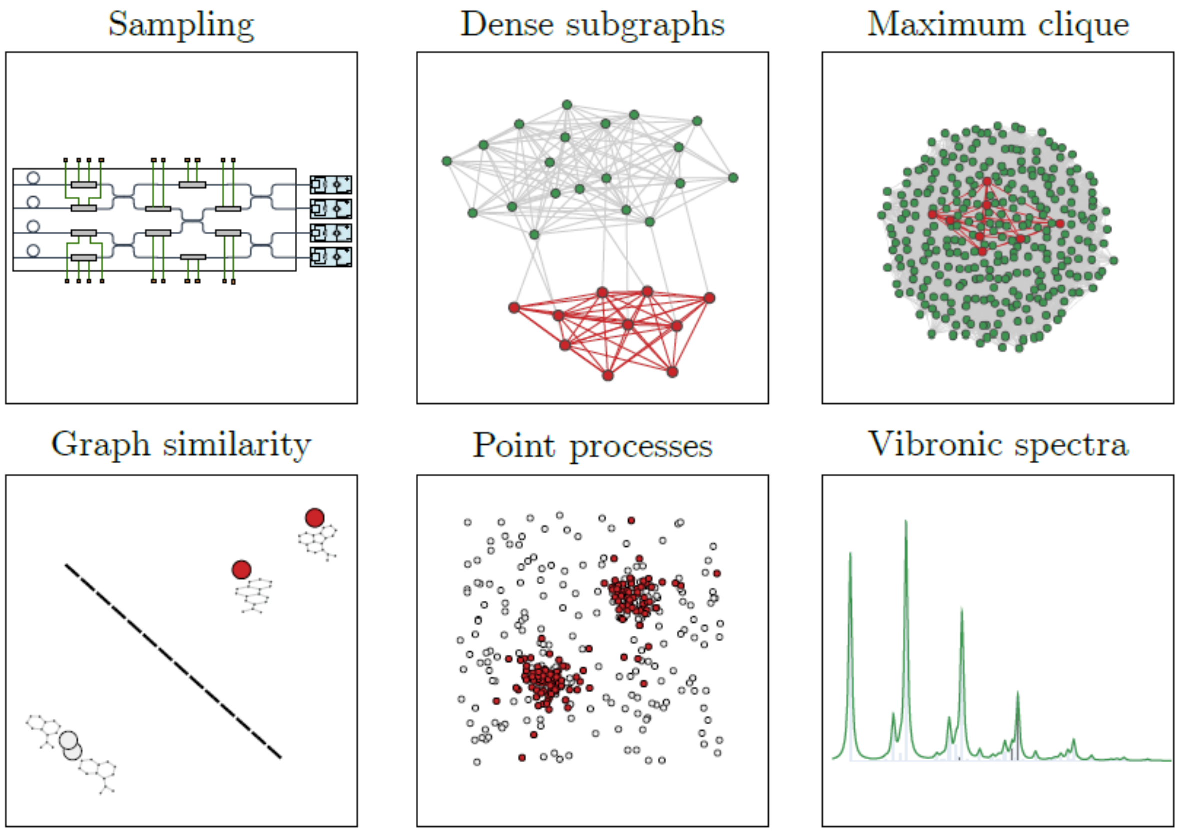
Boson Sampling



Gaussian Boson Sampling

- Boson sampling is quite general and it can be hard to construct hardware for it
- Gaussian Boson Sampling, GBS, is simpler, but not as powerful
- It is not universal, but it does solve a number of interesting problems
- In addition, a local company Xanadu makes hardware for GBS and has made it available on the net
- The mathematics is somewhat difficult, so will just give a brief overview of it

Problems Solved by GBS



Gaussian Boson Sampling

- Again we have modes, which they call qumodes, and the basis modes are |0>, |1>, |2>, … which are called Fock states, where |n> is a Fock state with n photons
- We can represent the state of the system by a Wigner function W(p,q), where p and q are called the position and momentum vectors
- What’s a Wigner function?
- In quantum mechanics we can’t know p and q exactly simultaneously, the Wigner function is a probability distribution for these values

Gaussian Boson Sampling

- The trick is to make the Wigner function a Gaussian distribution, with means the two m vectors p and q and a 2m x 2m covariance matrix V
- We prepare a Gaussian multi mode distribution, apply a unitary U and then measure the results in the Fock basis:

$$\begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_m \end{pmatrix} \longrightarrow \begin{pmatrix} a'_1 \\ a'_2 \\ \vdots \\ a'_m \end{pmatrix} = \boldsymbol{U} \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_m \end{pmatrix} ,$$

- This can be done in hardware

Gaussian Boson Sampling

- Like general boson sampling the probability of observing state S is given by

$$\Pr(S) = \frac{1}{\sqrt{\det(\boldsymbol{Q})}} \frac{\mathrm{Haf}(\boldsymbol{\mathcal{A}}_S)}{s_1! s_2! \cdots s_m!},$$

$$\begin{aligned} \boldsymbol{Q} &:= \boldsymbol{\Sigma} + \mathbb{1}/2, \\ \boldsymbol{\mathcal{A}} &:= \boldsymbol{X}\left(\mathbb{1} - \boldsymbol{Q}^{-1}\right), \\ \boldsymbol{X} &:= \left[\begin{smallmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{smallmatrix}\right]. \end{aligned}$$

- Where Σ is the covariance matrix

Gaussian Boson Sampling

- Where Haf() is the hafnian defined as:

$$\mathrm{Haf}(\boldsymbol{\mathcal{A}}) = \sum_{\pi \in \mathrm{PMP}} \prod_{(i,j) \in \pi} \mathcal{A}_{ij},$$

- This is related to the permanent in the following way

$$\mathrm{Haf}\begin{pmatrix} 0 & \boldsymbol{C} \\ \boldsymbol{C}^T & 0 \end{pmatrix} = \mathrm{Per}(\boldsymbol{C}).$$

- Since we know that the permanent is very hard to compute the hafnian is as well, thus the power of GBS

Gaussian Boson Sampling

- The question now is how do we program such a device?
- The aim of the program is to obtain a sample of the probability distribution described earlier, multiple runs will approximate the probability distribution
- The quantum computer has a fixed number of optical elements, squeezers and interferometers
- These elements are parameterized, so programming the device is the process of determining these parameters

Gaussian Boson Sampling

- Start with the symmetric matrix A that characterizes the probability distribution
- Perform a Takagi-Autonne decomposition to obtain

$$\boldsymbol{A} = \boldsymbol{U}\operatorname{diag}(\lambda_1, \lambda_2, \ldots, \lambda_m)\boldsymbol{U}^T,$$

- Where $0 \leq \lambda_i < 1$, if this isn't possible scale the matrix by a constant c, that satisfies the criterion
- The λ_i are used to compute the parameters for the squeezers, and U is used to compute the parameters for the interferometer

Gaussian Boson Sampling

- The number of photons required $\bar{n}$ is given by

$$\bar{n} = \sum_{i=1}^{M} \frac{\lambda_i^2}{1-\lambda_i^2}.$$

- This results in a sampling from the distribution

$$\Pr(S) \propto c^k \frac{|\mathrm{Haf}(\boldsymbol{A}_S)|^2}{s_1! \ldots s_m!},$$

- Where k is the sum of the s_i

Gaussian Boson Sampling

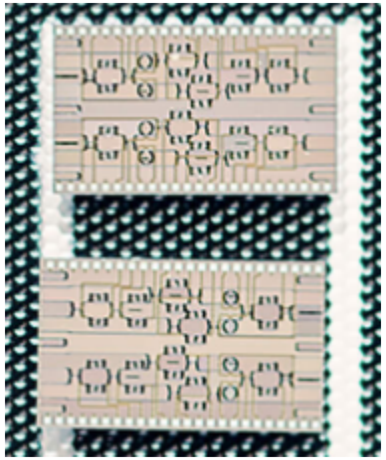
- Graph problems is one of the main application areas for GBS
- Given a graph G, with vertices V and edges E, a symmetric matrix A can be constructed in the following way

$$A_{ij} = \begin{cases} w_{ij} \text{ if } (i,j) \in E \\ 0 \text{ otherwise}, \end{cases}$$

- Where the w_{ij} are the edge weights
- This is the starting point of graph algorithms

Gaussian Boson Sampling

- Xanadu has produced hardware and software for GBS



- Most of the hardware runs at room temperature, only the photon detectors require extreme cooling

Gaussian Boson Sampling

- They have a software toolkit Strawberry Fields that can be used to program their hardware
- Again a Python library, supports traditional gate level quantum programming, plus GBS
- Has special library modules for standard problems that can be solved with GBS
- Have put one of their quantum computers in the cloud with limited access
- Have summer 16 week residency positions, see their website

Summary

- Briefly examined photon based approaches to quantum computing
- KLM approach based on standard optical components, universal but not clear how it can be programmed
- Boson sampling appears to be one of the most powerful quantum computing techniques, again it is hard to implement
- Gaussian boson sampling can be implemented and hardware exists for it
- Unfortunately, it isn’t universal, but does solve a number of important problems