CSCI 4140
Eigenvalues and Eigenvectors

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Introduction

- You've probably seen eigenvalues and eigenvectors in a linear algebra course
- We will review them here using our notation and introduce some of the special properties of our matrices
- We basically need to understand them, we don't need to do much math with them
- They will appear in some of our fundamental algorithms

Definition

- The definition of the eigenvalues and eigenvectors of a matrix A come from the following equation:

$$A|v\rangle = \lambda|v\rangle$$

- In this equation λ is a scalar and is called the eigenvalue
- Also |v> is a vector that is called an eigenvector, we will sometimes call this an eigenstate, since we use vectors for states
- Eigenvalues and eigenvectors occur in pairs, an eigenvalue belongs to a particular eigenvector

Definition

- One way of interpreting this equation is that A doesn't change the direction of the vector it only changes its length
- Since we are dealing with state vectors we often want:

$$|\lambda| = 1$$

- In general an eigenvector can be complex
- Eigenvalues can be computed from the following equation:

$$|A - \lambda I| = 0$$

- This gives us a polynomial that is called the characteristic polynomial, the eigenvalues are the roots of this equation

Special Matrices

- Recall our two special kinds of matrices, unitary and Hermitian:

$$U^\dagger U = I$$
$$H = H^\dagger$$

- For unitary matrices U, we can write the matrix as

$$U = \sum_j \lambda_j |v_j><v_j|$$

- Where λ_j are the eigenvalues of U and $|v_j>$ are the eigenvectors of U

Special Matrices

- For a unitary matrix we have:

$$\lambda\lambda^{*}=1$$

- The eigenvalues are complex numbers, in this case we can write them as:

$$e^{ih_j}$$

- Where the h_j are real numbers
- In the case of Hermitian matrices since $H = H^{\dagger}$ the eigenvalues must be real numbers

Summary

- A quick review of eigenvalues and eigenvectors
- Examined the special properties of unitary and Hermitian matrices when it comes the eigenvalues and eigenvectors