CSCI 4140
Linear Algebra
Part Two

Mark Green

Faculty of Science

Ontario Tech

Introduction

- We've seen that matrices are used to represent gates in quantum computers
- From a previous linear algebra course you should recall the following:
  - Matrix-vector multiply
  - Matrix-matrix multiply
  - Inverse matrix
- Examine a few properties of matrices that are important for quantum computing

Introduction

- There are two properties that we would like our matrices (or gates) to have
- First, they must be reversible, they must be their own inverse
- Running the output backwards through the gate should give us the input
- Second, they must map qubits to qubits
- Thinking of a qubit as a vector, they don’t change the length of the vector

Conjugate Transpose

- The key to our matrix classification is the conjugate transpose of a matrix
- You can guess what this does from the name, we take the matrix transpose and then the complex conjugate of all of its elements
- For the matrix A we write the complex conjugate as $A^{\dagger}$
- Example:

$$\begin{pmatrix} 1+1i & 1-1i \\ 2+3i & 3-2i \end{pmatrix}^{\dagger} = \begin{pmatrix} 1-1i & 2-3i \\ 1+1i & 3+2i \end{pmatrix}$$

Matrices

- We say that a matrix is Hermitian if $A = A^\dagger$
- We say that a matrix is Unitary if $A^\dagger = A^{-1}$
- Unitary matrices are important since the preserve the inner product
- This is fairly easy to show given a qubit q and a unitary matrix A
- First we have that $\langle q|q\rangle$=1, now what happens to $q' = Aq$
$$\langle q'|q'\rangle = (Aq)^\dagger Aq = qA^\dagger Aq = \langle q|q\rangle$$
- So a unitary matrix will preserve our qubits

Matrices

- Now if a matrix is both Hermitian and unitary we have the following:
$$A = A^{-1}$$
- This is exactly what we want
- If we run our qubits through the gate backwards we want to apply the inverse matrix, but this is the same as the forward matrix, so everything works out the way we want it

Tensor Product

- The tensor product was used to construct multi qubit states
- The majority of the gates are 1 and 2 qubit gates, which are 2x2 and 4x4 matrices
- How do we apply them to this much larger state vector?
- The answer is the tensor product, which can be applied to matrices the same way that its applied to vectors
- The tensor product is used to construct a matrix that can be applied to the state vector

Tensor Product

- Example:

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \otimes B = \begin{pmatrix} a_{11}B & a_{12}B \\ a_{21}B & a_{21}B \end{pmatrix}$$

- The size of B doesn't matter
- We can build up the complete matrix for the computation using a combination of matrix multiply and tensor product
- This is only practical for fairly small quantum circuits, since the matrices get quite large with the state size