CS6220

Self-Test

2017-09-12

1 Bayes' Rule

$$P(rain) = 73/365 = 0.2 \Rightarrow P(notrain) = 0.8$$

$$p(predictRain|rain) = 0.7$$

$$p(predictRain|notrain) = 0.3$$

$$\begin{aligned}
P(rain|predictRain) &= \frac{P(rain)P(predictRain|rain)}{P(predictRain)} \\
&= \frac{P(predictRain|rain)P(rain)}{P(predictRain|rain)P(rain) + P(predictRain|notrain)P(notrain)} \\
&= \frac{0.7 * 0.2}{0.7 * 0.2 + 0.3 * 0.8} \\
&= 0.37
\end{aligned}$$

2 Probability Distributions

$$A = \begin{bmatrix} A_{11} & A_{21} \\ A_{21} & A_{22} \end{bmatrix}$$

$$p(x) = \begin{cases} \int_0^x 4t \mathrm{d}t & \text{for} \quad 0 \le x \le 1/2 \\ \int_0^x (-4t+4) \mathrm{d}t & \text{for} \quad 1/2 \le x \le 1 \end{cases}$$

$$p(x) = \begin{cases} 2x^2 + C_1 & \text{for} \quad 0 \le x \le 1/2 \\ -2x^2 + 4x + C_2 & \text{for} \quad 1/2 \le x \le 1 \end{cases}$$

Assume the entire mass is $\in [0, 1]$

$$\begin{aligned} P(0) &= 0 \\ P(1) &= 1 \end{aligned}$$

Plug back into the equation and we get

$$\begin{aligned} C_1 &= 0 \\ C_2 &= -1 \end{aligned}$$

3 Discrete Expectation

$$\begin{aligned} E(x) &= \frac{1}{2} * 1 + \frac{1}{10} * 2 + \frac{1}{10} * 3 + \frac{1}{10} * 4 + \frac{1}{10} * 5 + \frac{1}{10} * 6 \\ &= 2.5 \end{aligned}$$

4 Expectation Properties

$$\begin{aligned} Var[x] &= E[(x-\mu)^2] \\ &= E[(x - E(x))^2] \\ &= E[x^2 - 2xE(x) + (E(x))^2] \\ &= E[x^2] - 2E(x)E(x) + (E(x))^2 \\ &= E[x^2] - (E(x))^2 \end{aligned}$$

5 Matrices/Linear Equations

a.

$$\begin{bmatrix} 2 & 1 & 1 \\ 4 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix} x = \begin{bmatrix} 3 \\ 10 \\ -2 \end{bmatrix}$$

b.

$$\begin{bmatrix}4 & 2 & 2\\4 & 0 & 2\\2 & 2 & 0\end{bmatrix} x = \begin{bmatrix}6\\10\\-2\end{bmatrix} \Rightarrow \begin{bmatrix}0 & 2 & 0\\4 & 0 & 2\\4 & 4 & 0\end{bmatrix} x = \begin{bmatrix}-4\\10\\-4\end{bmatrix} \Rightarrow$$

$$\begin{bmatrix}0 & 2 & 0\\4 & 0 & 2\\4 & 0 & 0\end{bmatrix} x = \begin{bmatrix}-4\\10\\4\end{bmatrix} \Rightarrow \begin{bmatrix}0 & 2 & 0\\0 & 0 & 2\\4 & 0 & 0\end{bmatrix} x = \begin{bmatrix}-4\\6\\4\end{bmatrix}$$

$$s = \begin{bmatrix}1\\-2\\3\end{bmatrix}$$

c.

$$\begin{bmatrix}3\\10\\-2\end{bmatrix} = \begin{bmatrix}2\\4\\2\end{bmatrix} - 2\begin{bmatrix}1\\0\\2\end{bmatrix} + 3\begin{bmatrix}1\\2\\0\end{bmatrix}$$

6 Matrices

a.

$$\begin{aligned} det(A) &= 1*4*4+2*3*1+3*1*3-1*4*3+3*3*1-4*1*2 \\ &= 2 \end{aligned}$$

b.

$$\begin{bmatrix}0 & 2 & 0\\1 & 4 & 3\\0 & 1 & -1\end{bmatrix} x \Rightarrow \begin{bmatrix}0 & 2 & 0\\1 & 4 & 3\\0 & 0 & 2\end{bmatrix}$$

As the matrix have no rows of all 0, therefore the matrix is invertible

c. As the matrix is invertible, the rank equals the number of rows which is 3

7 Matrices

$$|A-\lambda I| = \begin{vmatrix}3-\lambda & 6\\1 & 4-\lambda\end{vmatrix}$$

$$for \lambda = 1 : |A-\lambda I| = \begin{vmatrix}2 & 6\\1 & 3\end{vmatrix}\begin{vmatrix}v1\\v2\end{vmatrix} = \begin{vmatrix}0\\0\end{vmatrix}$$

So we don't even need to plug in 6, the answer is b

$$\begin{vmatrix}-3\\1\end{vmatrix}$$

8 MATRICES

p = 0

$$\|x\|_0 = 4 \qquad \text{(number of non-zero elements)}$$

p = 1

$$\begin{aligned}\|x\|_1 = \sum_{i=1}^{n} |x_i| &= |2| + |1| + |-4| + |-2| \\ &= 9\end{aligned}$$

p = 2

$$\begin{aligned}\|x\|_2 = (\sum_{i=1}^{n} |x_i|)^{1/2} &= \sqrt{|2|^2 + |1|^2 + |-4|^2 + |-2|^2} \\ &= 5\end{aligned}$$

$p = \infty$

$$\begin{aligned}\|x\|_\infty = \lim_{p\to\infty} (\sum_{i=1}^{p} |x_i|)^{1/p} &= \max_i |x_i|_\infty \\ &= 4\end{aligned}$$