Homework #2

CSE 446/546: Machine Learning
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B1:

- **Answer :**

$$||x||_2 \leq ||x||_1: \quad ||x||_2^2 = \sum_{i=1}^{n} |x_i|^2 \leq \left(\sum_{i=1}^{n} |x_i|^2 + 2 \sum_{i \neq j} |x_i||x_j| \right) = ||x||_1^2,$$

$$||x||_\infty \leq ||x||_2: \ ||x||_\infty^2 = (\max_{i=1,\cdots,n} |x_i|)^2 = (\max_{i=1,\cdots,n} |x_i|^2) \leq \sum_{i=1}^{n} |x_i|^2 = ||x||_2^2.$$

$$\text{we have:} \quad \|x\|_\infty \leq \|x\|_2 \leq \|x\|_1.$$

B2:

- **Part a:**

  Firstly, we need to prove $f(x) + g(x)$ is also a convex function. Because f and g are convex, we have:

$$
\begin{aligned}
f(\lambda x + (1-\lambda)y) + g(\lambda x + (1-\lambda)y) &\leq \lambda f(x) + (1-\lambda)f(y) + \lambda g(x) + (1-\lambda)g(y) \\
f(\lambda x + (1-\lambda)y) + g(\lambda x + (1-\lambda)y) &\leq \lambda(f(x) + g(x)) + (1-\lambda)(f(y) + g(y)) \\
f(\lambda x + (1-\lambda)y) + g(\lambda x + (1-\lambda)y) &\leq \lambda f(x) + (1-\lambda)f(y) \\
f(\lambda x + (1-\lambda)y) &\leq \lambda f(x) + (1-\lambda)f(y)
\end{aligned}
$$

  So, $f(x) + g(x)$ is a convex function

  For $\sum_{i=1}^{n} l_i(w) + \lambda||w||$, if it is convex, it should be Triangle Inequality, Absolute Scalability and Non-Negativity.

  $l_i(w)$ is convex for any $x, y \in \mathbb{R}$, so $\sum_{i=1}^{n} l_i(w)$ is convex.

  Because $|| \cdot ||$ is convex by given, $\lambda||w||$ is convex.

  As prove before if $f(x)$ and $g(x)$ are convex, then $f(x) + g(x)$ is convex.

  So, $\sum_{i=1}^{n} l_i(w) + \lambda||w||$ is convex.

- **Part b:**

  We prefer to use loss functions and regularized loss functions because this can ensure every local minima is the global minima for all the training data.

B3:

- **Part :**

- **Part :**