Homework #4

CSE 446/546: Machine Learning

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B1:

- **Part a:**

  - **I:**
    By applying Euler's rule and $A = \int f(x) + ig(x)dx = \int f(x)dx$, we have following:

$$k_p(\mathbf{x}, \mathbf{x'}) = \mathbb{E}_w \left[e^{iw^T(x-x')} \right], e^{iX} = \cos(w^T(x-x')) + i\sin(w^T(x-x'))$$
$$k_p(\mathbf{x}, \mathbf{x'}) = \mathbb{E}_w \left[\cos(w^T(x-x')) + i\sin(w^T(x-x')) \right]$$
$$k_p(\mathbf{x}, \mathbf{x'}) = \mathbb{E}_w \left[\cos(w^T(x-x')) \right]$$

  - **II:**
    Plug in $z_w(x) = \sqrt{2}\cos(w^T x + b)$, $z_w(x') = \sqrt{2}\cos(w^T x' + b)$, and applying the formula $2\cos(a)\cos(b) = \cos(a+b) + \cos(a-b)$, we have:

$$E_{w,b}\left[z_w(x)z_w(x')\right] = E_{w,b}\left[\sqrt{2}\cos(w^T x + b)\sqrt{2}\cos(w^T x' + b)\right]$$
$$E_{w,b}\left[z_w(x)z_w(x')\right] = E_{w,b}\left[2\cos(w^T x + b)\cos(w^T x' + b)\right]$$
$$E_{w,b}\left[z_w(x)z_w(x')\right] = E_{w,b}\left[\cos(w^T x + b - w^T x' - b) + \cos(w^T x + b + w^T x' + b))\right]$$
$$E_{w,b}\left[z_w(x)z_w(x')\right] = E_{w,b}\left[\cos(w^T(x - x'))\right]$$
$$E_{w,b}\left[z_w(x)z_w(x')\right] = k_p(x, x')$$

  - **III:** Plug in the two equations $z(x)^T = \left[\frac{1}{\sqrt{D}} z_{w_1}(x),\ \frac{1}{\sqrt{D}} z_{w_2}(x),\ \ldots,\ \frac{1}{\sqrt{D}} z_{w_D}(x)\right]$ and $z(x') = \left[\frac{1}{\sqrt{D}} z_{w_1}(x'),\ \frac{1}{\sqrt{D}} z_{w_2}(x'),\ \ldots,\ \frac{1}{\sqrt{D}} z_{w_D}(x')\right]^T$ we have:

$$E_{w,b}\left[z(x)^T z(x')\right] = E_{w,b}\left[\left[\frac{1}{\sqrt{D}} z_{w_1}(x),\ \frac{1}{\sqrt{D}} z_{w_2}(x),\ \ldots,\ \frac{1}{\sqrt{D}} z_{w_D}(x)\right]\left[\frac{1}{\sqrt{D}} z_{w_1}(x'),\ \frac{1}{\sqrt{D}} z_{w_2}(x'),\ \ldots,\ \frac{1}{\sqrt{D}} z_{w_D}(x')\right]^T\right]$$
$$E_{w,b}\left[z(x)^T z(x')\right] = E_{w,b}\left[\left[\frac{1}{D} z^T_{w_1}(x) z_{w_1}(x'),\ \frac{1}{D} z^T_{w_2}(x) z_{w_2}(x'),\ \ldots,\ \frac{1}{D} z^T_{w_D}(x z_{w_D}(x')\right]\right]$$
$$E_{w,b}\left[z(x)^T z(x')\right] = \frac{1}{D}\sum_{1}^{D} z^T_{w_D}(x) z_{w_D}(x')$$
$$E_{w,b}\left[z(x)^T z(x')\right] = k_p(x, x'), \text{By use parts (i) and (ii)}$$

    We can see that z is unbiased.

- **Part b:**

- **Part c:**