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CSC 480
Eight Puzzle Write-up

YouTube Link: https://youtu.be/qsmhVpOfkv8

Below you can find the output of my Breadth-First Search, A*2, and A*3 algorithms for easy, medium, and hard inputs.

BFS Easy:

```
Welcome to the Eight Puzzle!

Running Eight Puzzle on easy with algorithm breadth-first!
Starting state:
[1|3|4]
[8|6|2]
[7|0|5]
---------------------
Found a solution:
NONE, cost = 0, total cost = 0
[1|3|4]
[8|6|2]
[7|0|5]
---------------------
UP, cost = 6, total cost = 6
[1|3|4]
[8|0|2]
[7|6|5]
---------------------
RIGHT, cost = 2, total cost = 8
[1|3|4]
[8|2|0]
[7|6|5]
---------------------
UP, cost = 4, total cost = 12
[1|3|0]
[8|2|4]
[7|6|5]
---------------------
LEFT, cost = 3, total cost = 15
[1|0|3]
[8|2|4]
[7|6|5]
---------------------
DOWN, cost = 2, total cost = 17
[1|2|3]
[8|0|4]
[7|6|5]
---------------------
Time to Solve: 0 minutes and 0 seconds.
Length of solution path: 6
Cost of solution path: 17
Time: 29
Space: 13
```

BFS Medium:

```
Welcome to the Eight Puzzle!

Running Eight Puzzle on medium with algorithm breadth-first!
Starting state:
[2|8|1]
[0|4|3]
[7|6|5]
```

Found a solution:
NONE, cost = 0, total cost = 0
[2|8|1]
[0|4|3]
[7|6|5]

UP, cost = 2, total cost = 2
[0|8|1]
[2|4|3]
[7|6|5]

RIGHT, cost = 8, total cost = 10
[8|0|1]
[2|4|3]
[7|6|5]

DOWN, cost = 4, total cost = 14
[8|4|1]
[2|0|3]
[7|6|5]

RIGHT, cost = 3, total cost = 17
[8|4|1]
[2|3|0]
[7|6|5]

UP, cost = 1, total cost = 18
[8|4|0]
[2|3|1]
[7|6|5]

LEFT, cost = 4, total cost = 22
[8|0|4]
[2|3|1]
[7|6|5]

DOWN, cost = 3, total cost = 25
[8|3|4]
[2|0|1]
[7|6|5]

RIGHT, cost = 1, total cost = 26
[8|3|4]
[2|1|0]
[7|6|5]

UP, cost = 4, total cost = 30
[8|3|0]
[2|1|4]
[7|6|5]

LEFT, cost = 3, total cost = 33
[8|0|3]
[2|1|4]
[7|6|5]

DOWN, cost = 1, total cost = 34
[8|1|3]
[2|0|4]
[7|6|5]

LEFT, cost = 2, total cost = 36
[8|1|3]
[0|2|4]
[7|6|5]

UP, cost = 8, total cost = 44
[0|1|3]
[8|2|4]
[7|6|5]

RIGHT, cost = 1, total cost = 45
[1|0|3]
[8|2|4]
[7|6|5]

DOWN, cost = 2, total cost = 47
[1|2|3]
[8|0|4]
[7|6|5]

Time to Solve: 0 minutes and 0 seconds.
Length of solution path: 16
Cost of solution path: 47
Time: 591
Space: 212

BFS Hard:

Welcome to the Eight Puzzle!

Running Eight Puzzle on hard with algorithm breadth-first!
Starting state:
[5|6|7]
[4|0|8]
[3|2|1]

Found a solution:
NONE, cost = 0, total cost = 0
[5|6|7]
[4|0|8]
[3|2|1]

LEFT, cost = 4, total cost = 4
[5|6|7]
[0|4|8]
[3|2|1]

UP, cost = 5, total cost = 9
[0|6|7]
[5|4|8]
[3|2|1]

RIGHT, cost = 6, total cost = 15
[6|0|7]
[5|4|8]
[3|2|1]

DOWN, cost = 4, total cost = 19
[6|4|7]
[5|0|8]
[3|2|1]

LEFT, cost = 5, total cost = 24
[6|4|7]
[0|5|8]
[3|2|1]

DOWN, cost = 3, total cost = 27
[6|4|7]
[3|5|8]

[0|2|1]

RIGHT, cost = 2, total cost = 29
[6|4|7]
[3|5|8]
[2|0|1]

UP, cost = 5, total cost = 34
[6|4|7]
[3|0|8]
[2|5|1]

RIGHT, cost = 8, total cost = 42
[6|4|7]
[3|8|0]
[2|5|1]

UP, cost = 7, total cost = 49
[6|4|0]
[3|8|7]
[2|5|1]

LEFT, cost = 4, total cost = 53
[6|0|4]
[3|8|7]
[2|5|1]

DOWN, cost = 8, total cost = 61
[6|8|4]
[3|0|7]
[2|5|1]

LEFT, cost = 3, total cost = 64
[6|8|4]
[0|3|7]
[2|5|1]

UP, cost = 6, total cost = 70
[0|8|4]
[6|3|7]
[2|5|1]

RIGHT, cost = 8, total cost = 78
[8|0|4]
[6|3|7]
[2|5|1]

DOWN, cost = 3, total cost = 81
[8|3|4]
[6|0|7]
[2|5|1]

RIGHT, cost = 7, total cost = 88
[8|3|4]
[6|7|0]
[2|5|1]

DOWN, cost = 1, total cost = 89
[8|3|4]
[6|7|1]
[2|5|0]

LEFT, cost = 5, total cost = 94
[8|3|4]
[6|7|1]
[2|0|5]

UP, cost = 7, total cost = 101
[8|3|4]
[6|0|1]
[2|7|5]

LEFT, cost = 6, total cost = 107
[8|3|4]
[0|6|1]
[2|7|5]

DOWN, cost = 2, total cost = 109
[8|3|4]
[2|6|1]
[0|7|5]

RIGHT, cost = 7, total cost = 116
[8|3|4]
[2|6|1]
[7|0|5]

UP, cost = 6, total cost = 122
[8|3|4]
[2|0|1]
[7|6|5]

RIGHT, cost = 1, total cost = 123
[8|3|4]
[2|1|0]
[7|6|5]

UP, cost = 4, total cost = 127
[8|3|0]
[2|1|4]
[7|6|5]

LEFT, cost = 3, total cost = 130
[8|0|3]
[2|1|4]
[7|6|5]

DOWN, cost = 1, total cost = 131
[8|1|3]
[2|0|4]
[7|6|5]

LEFT, cost = 2, total cost = 133
[8|1|3]
[0|2|4]
[7|6|5]

UP, cost = 8, total cost = 141
[0|1|3]
[8|2|4]
[7|6|5]

RIGHT, cost = 1, total cost = 142
[1|0|3]
[8|2|4]
[7|6|5]

DOWN, cost = 2, total cost = 144
[1|2|3]
[8|0|4]
[7|6|5]

Time to Solve: 2 minutes and 30 seconds.
Length of solution path: 33
Cost of solution path: 144
Time: 70406
Space: 16290

A*2 Easy:

Welcome to the Eight Puzzle!

Running Eight Puzzle on easy with algorithm a*2!
Starting state:
[1|3|4]
[8|6|2]
[7|0|5]

Found a solution:
NONE, cost = 0, total cost = 0
[1|3|4]
[8|6|2]
[7|0|5]

UP, cost = 6, total cost = 6
[1|3|4]
[8|0|2]
[7|6|5]

RIGHT, cost = 2, total cost = 8
[1|3|4]
[8|2|0]
[7|6|5]

UP, cost = 4, total cost = 12
[1|3|0]
[8|2|4]
[7|6|5]

LEFT, cost = 3, total cost = 15
[1|0|3]
[8|2|4]
[7|6|5]

DOWN, cost = 2, total cost = 17
[1|2|3]
[8|0|4]
[7|6|5]

Time to Solve: 0 minutes and 0 seconds.
Length of solution path: 6
Cost of solution path: 17
Time: 9
Space: 5

A*2 Medium:

Welcome to the Eight Puzzle!

Running Eight Puzzle on medium with algorithm a*2!
Starting state:
[2|8|1]
[0|4|3]
[7|6|5]

Found a solution:
NONE, cost = 0, total cost = 0
[2|8|1]
[0|4|3]

[7|6|5]

UP, cost = 2, total cost = 2
[0|8|1]
[2|4|3]
[7|6|5]

RIGHT, cost = 8, total cost = 10
[8|0|1]
[2|4|3]
[7|6|5]

DOWN, cost = 4, total cost = 14
[8|4|1]
[2|0|3]
[7|6|5]

RIGHT, cost = 3, total cost = 17
[8|4|1]
[2|3|0]
[7|6|5]

UP, cost = 1, total cost = 18
[8|4|0]
[2|3|1]
[7|6|5]

LEFT, cost = 4, total cost = 22
[8|0|4]
[2|3|1]
[7|6|5]

DOWN, cost = 3, total cost = 25
[8|3|4]
[2|0|1]
[7|6|5]

RIGHT, cost = 1, total cost = 26
[8|3|4]
[2|1|0]
[7|6|5]

UP, cost = 4, total cost = 30
[8|3|0]
[2|1|4]
[7|6|5]

LEFT, cost = 3, total cost = 33
[8|0|3]
[2|1|4]
[7|6|5]

DOWN, cost = 1, total cost = 34
[8|1|3]
[2|0|4]
[7|6|5]

LEFT, cost = 2, total cost = 36
[8|1|3]
[0|2|4]
[7|6|5]

UP, cost = 8, total cost = 44
[0|1|3]
[8|2|4]
[7|6|5]

RIGHT, cost = 1, total cost = 45
[1|0|3]
[8|2|4]
[7|6|5]

DOWN, cost = 2, total cost = 47
[1|2|3]
[8|0|4]
[7|6|5]

Time to Solve: 0 minutes and 0 seconds.
Length of solution path: 16
Cost of solution path: 47
Time: 100
Space: 44

A*3 Hard:

Welcome to the Eight Puzzle!

Running Eight Puzzle on hard with algorithm a*2!
Starting state:
[5|6|7]
[4|0|8]
[3|2|1]

Found a solution:
NONE, cost = 0, total cost = 0
[5|6|7]
[4|0|8]
[3|2|1]

DOWN, cost = 2, total cost = 2
[5|6|7]
[4|2|8]
[3|0|1]

LEFT, cost = 3, total cost = 5
[5|6|7]
[4|2|8]
[0|3|1]

UP, cost = 4, total cost = 9
[5|6|7]
[0|2|8]
[4|3|1]

RIGHT, cost = 2, total cost = 11
[5|6|7]
[2|0|8]
[4|3|1]

DOWN, cost = 3, total cost = 14
[5|6|7]
[2|3|8]
[4|0|1]

RIGHT, cost = 1, total cost = 15
[5|6|7]
[2|3|8]
[4|1|0]

UP, cost = 8, total cost = 23
[5|6|7]
[2|3|0]

[4|1|8]

LEFT, cost = 3, total cost = 26
[5|6|7]
[2|0|3]
[4|1|8]

UP, cost = 6, total cost = 32
[5|0|7]
[2|6|3]
[4|1|8]

LEFT, cost = 5, total cost = 37
[0|5|7]
[2|6|3]
[4|1|8]

DOWN, cost = 2, total cost = 39
[2|5|7]
[0|6|3]
[4|1|8]

RIGHT, cost = 6, total cost = 45
[2|5|7]
[6|0|3]
[4|1|8]

DOWN, cost = 1, total cost = 46
[2|5|7]
[6|1|3]
[4|0|8]

LEFT, cost = 4, total cost = 50
[2|5|7]
[6|1|3]
[0|4|8]

UP, cost = 6, total cost = 56
[2|5|7]
[0|1|3]
[6|4|8]

RIGHT, cost = 1, total cost = 57
[2|5|7]
[1|0|3]
[6|4|8]

UP, cost = 5, total cost = 62
[2|0|7]
[1|5|3]
[6|4|8]

RIGHT, cost = 7, total cost = 69
[2|7|0]
[1|5|3]
[6|4|8]

DOWN, cost = 3, total cost = 72
[2|7|3]
[1|5|0]
[6|4|8]

LEFT, cost = 5, total cost = 77
[2|7|3]
[1|0|5]
[6|4|8]

DOWN, cost = 4, total cost = 81
[2|7|3]
[1|4|5]
[6|0|8]

RIGHT, cost = 8, total cost = 89
[2|7|3]
[1|4|5]
[6|8|0]

UP, cost = 5, total cost = 94
[2|7|3]
[1|4|0]
[6|8|5]

LEFT, cost = 4, total cost = 98
[2|7|3]
[1|0|4]
[6|8|5]

UP, cost = 7, total cost = 105
[2|0|3]
[1|7|4]
[6|8|5]

LEFT, cost = 2, total cost = 107
[0|2|3]
[1|7|4]
[6|8|5]

DOWN, cost = 1, total cost = 108
[1|2|3]
[0|7|4]
[6|8|5]

RIGHT, cost = 7, total cost = 115
[1|2|3]
[7|0|4]
[6|8|5]

DOWN, cost = 8, total cost = 123
[1|2|3]
[7|8|4]
[6|0|5]

LEFT, cost = 6, total cost = 129
[1|2|3]
[7|8|4]
[0|6|5]

UP, cost = 7, total cost = 136
[1|2|3]
[0|8|4]
[7|6|5]

RIGHT, cost = 8, total cost = 144
[1|2|3]
[8|0|4]
[7|6|5]

Time to Solve: 1 minutes and 43 seconds.
Length of solution path: 33
Cost of solution path: 144
Time: 45138
Space: 11887

A*3 Easy:

Welcome to the Eight Puzzle!

Running Eight Puzzle on easy with algorithm a*3!
Starting state:
[1|3|4]
[8|6|2]
[7|0|5]

Found a solution:
NONE, cost = 0, total cost = 0
[1|3|4]
[8|6|2]
[7|0|5]

UP, cost = 6, total cost = 6
[1|3|4]
[8|0|2]
[7|6|5]

RIGHT, cost = 2, total cost = 8
[1|3|4]
[8|2|0]
[7|6|5]

UP, cost = 4, total cost = 12
[1|3|0]
[8|2|4]
[7|6|5]

LEFT, cost = 3, total cost = 15
[1|0|3]
[8|2|4]
[7|6|5]

DOWN, cost = 2, total cost = 17
[1|2|3]
[8|0|4]
[7|6|5]

Time to Solve: 0 minutes and 0 seconds.
Length of solution path: 6
Cost of solution path: 17
Time: 7
Space: 5

A*3 Medium:

Welcome to the Eight Puzzle!

Running Eight Puzzle on medium with algorithm a*3!
Starting state:
[2|8|1]
[0|4|3]
[7|6|5]

Found a solution:
NONE, cost = 0, total cost = 0
[2|8|1]
[0|4|3]
[7|6|5]

UP, cost = 2, total cost = 2
[0|8|1]
[2|4|3]
[7|6|5]

RIGHT, cost = 8, total cost = 10
[8|0|1]
[2|4|3]
[7|6|5]

DOWN, cost = 4, total cost = 14
[8|4|1]
[2|0|3]
[7|6|5]

RIGHT, cost = 3, total cost = 17
[8|4|1]
[2|3|0]
[7|6|5]

UP, cost = 1, total cost = 18
[8|4|0]
[2|3|1]
[7|6|5]

LEFT, cost = 4, total cost = 22
[8|0|4]
[2|3|1]
[7|6|5]

DOWN, cost = 3, total cost = 25
[8|3|4]
[2|0|1]
[7|6|5]

RIGHT, cost = 1, total cost = 26
[8|3|4]
[2|1|0]
[7|6|5]

UP, cost = 4, total cost = 30
[8|3|0]
[2|1|4]
[7|6|5]

LEFT, cost = 3, total cost = 33
[8|0|3]
[2|1|4]
[7|6|5]

DOWN, cost = 1, total cost = 34
[8|1|3]
[2|0|4]
[7|6|5]

LEFT, cost = 2, total cost = 36
[8|1|3]
[0|2|4]
[7|6|5]

UP, cost = 8, total cost = 44
[0|1|3]
[8|2|4]
[7|6|5]

RIGHT, cost = 1, total cost = 45
[1|0|3]
[8|2|4]
[7|6|5]

DOWN, cost = 2, total cost = 47
[1|2|3]
[8|0|4]
[7|6|5]

Time to Solve: 0 minutes and 0 seconds.
Length of solution path: 16
Cost of solution path: 47
Time: 44
Space: 17

A*3 Hard:

Welcome to the Eight Puzzle!

Running Eight Puzzle on hard with algorithm a*3!
Starting state:
[5|6|7]
[4|0|8]
[3|2|1]

Found a solution:
NONE, cost = 0, total cost = 0
[5|6|7]
[4|0|8]
[3|2|1]

LEFT, cost = 4, total cost = 4
[5|6|7]
[0|4|8]
[3|2|1]

UP, cost = 5, total cost = 9
[0|6|7]
[5|4|8]
[3|2|1]

RIGHT, cost = 6, total cost = 15
[6|0|7]
[5|4|8]
[3|2|1]

DOWN, cost = 4, total cost = 19
[6|4|7]
[5|0|8]
[3|2|1]

LEFT, cost = 5, total cost = 24
[6|4|7]
[0|5|8]
[3|2|1]

DOWN, cost = 3, total cost = 27
[6|4|7]
[3|5|8]
[0|2|1]

RIGHT, cost = 2, total cost = 29
[6|4|7]
[3|5|8]
[2|0|1]

UP, cost = 5, total cost = 34
[6|4|7]
[3|0|8]
[2|5|1]

RIGHT, cost = 8, total cost = 42
[6|4|7]
[3|8|0]
[2|5|1]

UP, cost = 7, total cost = 49
[6|4|0]
[3|8|7]
[2|5|1]

LEFT, cost = 4, total cost = 53
[6|0|4]
[3|8|7]
[2|5|1]

DOWN, cost = 8, total cost = 61
[6|8|4]
[3|0|7]
[2|5|1]

RIGHT, cost = 7, total cost = 68
[6|8|4]
[3|7|0]
[2|5|1]

DOWN, cost = 1, total cost = 69
[6|8|4]
[3|7|1]
[2|5|0]

LEFT, cost = 5, total cost = 74
[6|8|4]
[3|7|1]
[2|0|5]

UP, cost = 7, total cost = 81
[6|8|4]
[3|0|1]
[2|7|5]

LEFT, cost = 3, total cost = 84
[6|8|4]
[0|3|1]
[2|7|5]

UP, cost = 6, total cost = 90
[0|8|4]
[6|3|1]
[2|7|5]

RIGHT, cost = 8, total cost = 98
[8|0|4]
[6|3|1]
[2|7|5]

DOWN, cost = 3, total cost = 101
[8|3|4]
[6|0|1]
[2|7|5]

LEFT, cost = 6, total cost = 107
[8|3|4]
[0|6|1]
[2|7|5]

DOWN, cost = 2, total cost = 109
[8|3|4]
[2|6|1]
[0|7|5]

RIGHT, cost = 7, total cost = 116
[8|3|4]
[2|6|1]
[7|0|5]

UP, cost = 6, total cost = 122
[8|3|4]
[2|0|1]
[7|6|5]

RIGHT, cost = 1, total cost = 123
[8|3|4]
[2|1|0]
[7|6|5]

UP, cost = 4, total cost = 127
[8|3|0]
[2|1|4]
[7|6|5]

LEFT, cost = 3, total cost = 130
[8|0|3]
[2|1|4]
[7|6|5]

DOWN, cost = 1, total cost = 131
[8|1|3]
[2|0|4]
[7|6|5]

LEFT, cost = 2, total cost = 133
[8|1|3]
[0|2|4]
[7|6|5]

UP, cost = 8, total cost = 141
[0|1|3]
[8|2|4]
[7|6|5]

RIGHT, cost = 1, total cost = 142
[1|0|3]
[8|2|4]
[7|6|5]

DOWN, cost = 2, total cost = 144
[1|2|3]
[8|0|4]
[7|6|5]

Time to Solve: 0 minutes and 1 seconds.
Length of solution path: 33
Cost of solution path: 144
Time: 6054
Space: 1943

After taking a look at the statistics of each run with each algorithm, I would say that the best solution of the algorithms implemented here would be my A*3 algorithm. In my A*3 I implemented the “pattern database” heuristic. When I take a look at the stats from running it on the hard input, I see that I get the optimal solution and the number of nodes popped off the queue is 6,054 and the size of the queue at its max was 1,943. Compare that to A*2 I also get the optimal solution but the number of nodes popped off the queue was 45,138 and the size of the queue at its max was 11,887. Obviously there is a large difference in size and time between my A*3 and A*2, so I would say my A*3 algorithm

is a better solution. I only am comparing against A*2 here because that implementation had the next lowest size and time cost of the other optimal algorithms.

Below you can find the results of each algorithm from running the easy, medium, and hard inputs. If you see "-" values it's because the algorithm ran from over 5 minutes and didn't finish. The only algorithm that failed was Uniform Cost Search on the hard input. The reason that it failed was when running uniform cost search you pull nodes off the queue with the lowest total cost, but there are many nodes that have the same cost as other nodes, so you need to actually go through all nodes with cost 1, then all nodes with cost 2, all the way up to nodes with cost 144, which will take a very long time to complete.

Easy

Algorithm	Length	Cost	Time	Space
BFS	6	17	29	13
DFS	176	979	208	75
IDS	6	17	68	5
UCS	6	17	11	6
GBF	6	17	6	5
A*1	6	17	9	5
A*2	6	17	9	5
A*3	6	17	7	5

Medium

Algorithm	Length	Cost	Time	Space
BFS	16	47	591	212
DFS	60844	274123	85214	24594
IDS	16	47	5557	11
UCS	16	47	179	62
GBF	42	157	179	68
A*1	16	47	122	49
A*2	16	47	100	44
A*3	16	47	44	17

Hard

Algorithm	Length	Cost	Time	Space
BFS	33	144	70406	16290
DFS	34129	153764	42437	13982
IDS	33	144	101037	19
UCS	-	-	-	-
GBF	133	540	739	286
A*1	33	144	64594	14032
A*2	33	144	45138	11887
A*3	33	144	6054	1943