Stein's Method for Policy Gradients Methods in Deep Reinforcement Learning

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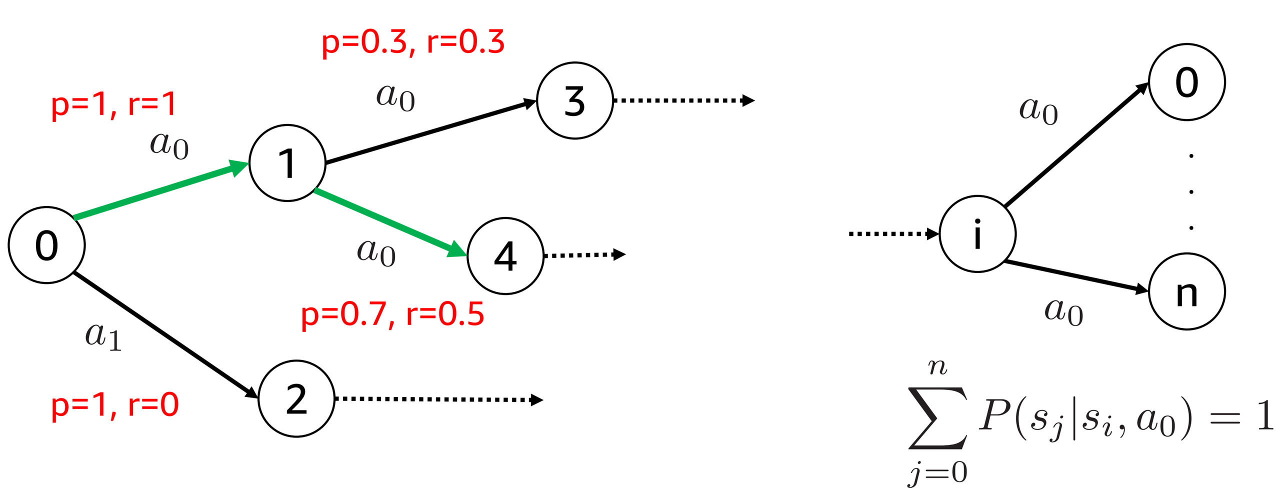
Policy Optimization and Surrogate Loss Using on Advantage Function

Markov Decision Process (MDP)

$$\begin{aligned} \mathcal{S} \quad & \text{Set of states} \\ \mathcal{A} \quad & \text{Set of actions} \\ P : \mathcal{S} \times \mathcal{A} \times \mathcal{S} \to \mathbb{R} \quad & \text{Transition probability distribution} \\ c : \mathcal{S} \to \mathbb{R} \quad & \text{Cost function} \\ \rho_0 : \mathcal{S} \to \mathbb{R} \quad & \text{Initial state distribution} \end{aligned}$$



$$\sum_{j=0}^{n} P(s_j | s_i, a_0) = 1$$

Stochastic Policy

Goal: Obtain a policy to restrict the behavior of the MDP to obtain a desired behavior.

$$\pi : \mathcal{S} \times \mathcal{A} \to [0, 1]$$

Desired Behavior: Minimize (maximize) expected discounted cumulative cost (reward).

$$\eta(\pi) = \mathbb{E}_{\mathcal{S},\mathcal{A}} \left[\sum_{t=0}^{\infty} \gamma^t c(s_t) \right]$$

Policy Optimization and Advantage Function

Express the expected reward of another policy in terms of the advantage over the current policy, accumulated over timesteps.

$$A_\pi(s, a) = Q_{s,a} - V_\pi(s)$$

Q-function is the future cumulative discounted reward is from s take action a

Value function is the future cumulative discounted reward is from s take any action

Any policy update that has non-positive expected advantage at every state will reduce the expected cumulative reward or keep it constant when the advantage is zero at every state.

$$\pi \to \tilde{\pi} \Rightarrow \eta(\tilde{\pi}) = \eta(\pi) + \sum_s \rho_{\tilde{\pi}} \sum_a \tilde{\pi}(a|s) A_\pi(s, a)$$

Difficult to optimize directly → Local approximation using visitation frequency from the old policy to update to the new policy.

$$L(\tilde{\pi}) = \eta(\pi) + \sum_s \rho_\pi \sum_a \tilde{\pi}(a|s) A_\pi(s, a)$$

Conservative policy iteration update provides explicit lower bound on the improvement from the old policy to the new one.

$$\pi_{new}(a|s) = (1 - \alpha)\pi_{old}(a|s) + \alpha \pi'(a|s), \quad \pi' = arg \min_{\pi'} L_{\pi_{old}}(\pi')$$

$$\eta(\pi_{new}) \leq L_{\pi_{old}}(\pi_{new}) + \frac{2\epsilon\gamma}{(1-\gamma)^2}\alpha^2, \quad \alpha, \gamma \in [0, 1] \qquad (1)$$

Stein's Method for Policy Optimization

Goal: Kernelized Stein Discrepancy (KSD) to bound step size in policy update to robustly allow large policy updates.

Stein's method is a general theoretical tool for bounding differences between distributions. The score function is independent of normalization.

$$s_p(x) = \nabla x \log p(x) = \frac{\nabla_x p(x)}{p(x)}$$

KSD for policy update given a positive definite kernel and reproducing kernel Hilbert space (RKHS) has a closed form solution.

$$\mathbb{S}(\pi_{new}, \pi_{old}) = \mathbb{E}_{x,x' \ \pi_{new}} \approx \frac{1}{n(n-1)} \sum_{i \neq j} \kappa_{\pi_{old}} = trace(\mathcal{A}^x_{\pi_{old}} \mathcal{A}^x_{\pi_{old}} k(x, x'))$$

Policy Optimization and KL divergence

Lower bound on the improvement update shown in Equation (1) can be written in terms of total variation divergence.

$$\alpha = D^{max}_{TV}(\pi_{old}, \pi_{new}) \qquad D_{TV}(p||q) = \frac{1}{2}\sum_i |p_i - q_i|$$

Using the relation between total variation divergence and KL divergence, obtain a bound in terms of maximum KL divergence.

$$\eta(\pi_{new}) \leq L_{\pi_{old}}(\pi_{new}) + \frac{2\epsilon\gamma}{(1-\gamma)^2} \underbrace{D^{max}_{KL}(\pi_{old}) || \pi_{new})}_{\text{Replaced } \alpha \text{ in Equation (1)}}$$

Fisher divergence (FD) vs Stein:

- The total variance divergence squared is the FD given the score function.
- KSD is a smoothed version of Fisher divergence with a kernel.

KL divergence vs Stein:

- FD is the derivative of KL when variables are perturbed by i.i.d. Gaussian.
- KSD is the derivative of KL when variables are perturbed by smooth functions in RKHS.

Observations:

- Policy optimization methods using REINFORCE and Fisher or KL divergence metrics have been shown to converge on complex problems.
- When policy parameters are perturbed by smooth functions in RKHS as in REINFORCE, then policy optimization problem can be solved without the need to find the correct normalization factor.

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ICML Workshop, Stein's Method, February 25-March 1, 2019, Los Angeles, CA USA