

## Feedback — Lecture 11 Quiz

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You submitted this quiz on **Fri 1 Jul 2016 1:16 AM CEST**. You got a score of **3.50** out of **8.00**. However, you will not get credit for it, since it was submitted past the deadline.

## Question 1

For a stochastic binary unit in a Boltzmann Machine, the probability of the unit turning on, i.e.  $P(s = 1)$ , is defined as  $P(s = 1) = \frac{1}{1 + \exp(-\Delta E/T)}$ . Here,  $\Delta E$  is the *energy gap*, i.e. the energy when the unit is off, minus the energy when the unit is on.  $T$  is the *temperature*.

If  $\Delta E = -3$ , then:

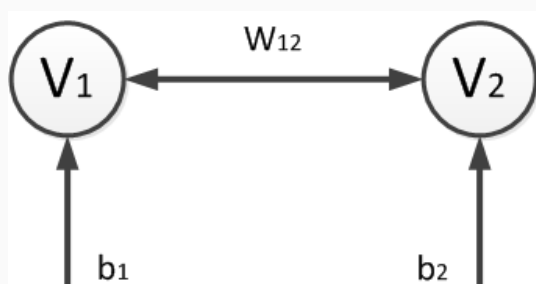
Your Answer	Score	Explanation
<input checked="" type="radio"/> $P(s = 1)$ increases when $T$ increases.	✓ 1.00	
<input type="radio"/> $P(s = 1)$ decreases when $T$ increases.		
Total	1.00 / 1.00	

## Question Explanation

At  $T = 1$ ,  $P(s = 1)$  is 0.05, i.e. fairly close to 0. Increasing the temperature brings that probability closer to 0.5, i.e. it increases it.

## Question 2

The Hopfield network shown below has two visible units:  $V_1$  and  $V_2$ . It has a connection between the two units, and each unit has a bias.



Let  $W_{12} = -10$ ,  $b_1 = 1$ , and  $b_2 = 1$  and the initial states of  $V_1 = 0$  and  $V_2 = 0$ .

If the network always updates both units simultaneously, then what is the lowest energy value that it will encounter (given those initial states)?

If the network always updates the units one at a time, i.e. it alternates between updating  $V_1$  and updating  $V_2$ , then what is the lowest energy value that it will encounter (given those initial states)?

Write those two numbers with a comma between them. For example, if you think that the answer to that first question is 4, and that the answer to the second question is -7, then write this: **4, -7**

You entered:

0,-1

Your Answer	Score	Explanation
0,-1	✓ 1.00	
Total	1.00 / 1.00	

**Question Explanation**

From the initial state, both units will want to turn on.

If we update both of them at the same time, then both will turn on, leading to a configuration with energy 8. Next, both units will want to turn off, bringing us back to the initial state, which has energy 0. We'll only ever alternate between those two states, so the lowest energy we'll see is 0.

If we update one unit, say  $V_1$ , first, then it will turn on. Now we're in a state with energy -1. From that state, neither unit will want to change its state, so we'll stay in that state forever.

**Question 3**

This question is about Boltzmann Machines, a.k.a. a stochastic Hopfield networks. Recall from the lecture that when we pick a new state  $s_i$  for unit  $i$ , we do so in a stochastic way:  $p(s_i = 1) = \frac{1}{1 + \exp(-\Delta E/T)}$ , and  $p(s_i = 0) = 1 - p(s_i = 1)$ .

Here,  $\Delta E$  is the *energy gap*, i.e. the energy when the unit is off, minus the energy when the unit is on.  $T$  is the *temperature*.

We can run our system with any temperature that we like, but the most commonly used temperatures are 0 and 1.

When we want to explore the configurations of a Boltzmann Machine, we initialize it in some starting configuration, and then repeatedly choose a unit at random, and pick a new state for it, using the probability formula described above.

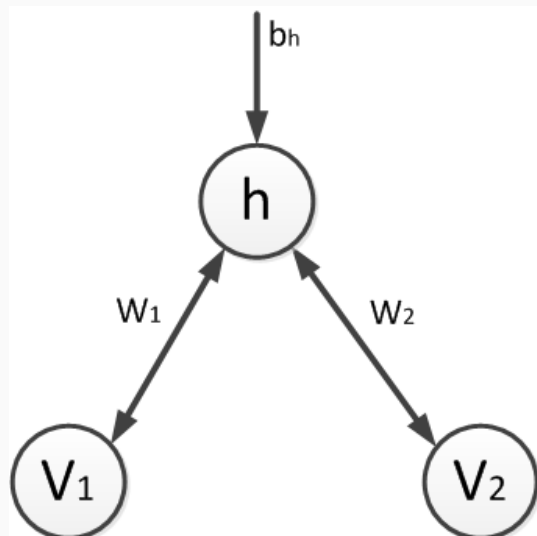
Consider two small Boltzmann Machines with 10 units, with the same weights, but with different temperatures. One, the "cold" one, has temperature 0. The other, the "warm" one, has temperature 1. We run both of them for 1000 iterations (updates), as described above, and then we look at the configuration that we end up with after those 1000 updates.

Which of the following statements are true? (Note that an "energy minimum" is what could also reasonably be called a "local energy minimum")

Your Answer	Score	Explanation
<input checked="" type="checkbox"/> The cold one is more likely to end in an energy minimum than the warm one.	✓ 0.50	When a Boltzmann Machine reaches an energy minimum, then if it's cold it will stay there. If it's warm, it might move away from it.
<input checked="" type="checkbox"/> If the cold one is exponentially unfortunate, it could end up in a configuration that's not an energy minimum.	✓ 0.50	If the random choice of which unit to update is always the same, then there won't be much progress towards an energy minimum. However, that's very unlikely.
<input checked="" type="checkbox"/> For the cold one, $P(s_i = 1)$ can be any value between 0 and 1, depending on the weights.	✗ 0.00	For the cold one, $P(s_i = 1)$ is always either 0 or 1 (or one might say it could be 0.5 when the energy gap is 0). It can never be something like $1/3$ .
<input checked="" type="checkbox"/> The warm one could end up in a configuration that's not an energy minimum.	✓ 0.50	The warm one could end up anywhere, because it's truly stochastic.

## Question 4

The Boltzmann Machine shown below has two visible units  $V_1$  and  $V_2$ , and one hidden unit  $h$ .



Let  $W_1 = 3$ ,  $W_2 = -1$ , and  $b_h = -2$ .

What is  $P(V_1 = 1, V_2 = 0)$ ? Write your answer with at least three digits after the decimal point.

*Hint: if you feel confused about the purpose of a hidden unit, revisit the lecture videos.*

You entered:

-1

Your Answer

Score

Explanation

-1

✗

0.00

Total

0.00 / 2.00

### Question Explanation

This Boltzmann Machine has eight configurations. Each has an energy, and from the energies you can calculate the probabilities. To get the requested probability, we add up two probabilities:

$P(V_1 = 1, V_2 = 0) = P(h = 0, V_1 = 1, V_2 = 0) + P(h = 1, V_1 = 1, V_2 = 0)$ . In detail, it's as follows.

$$E(h = 0, V_1 = 0, V_2 = 0) = 0$$

$$E(h = 0, V_1 = 0, V_2 = 1) = 0$$

$$E(h = 0, V_1 = 1, V_2 = 0) = 0$$

$$E(h = 0, V_1 = 1, V_2 = 1) = 0$$

$$E(h = 1, V_1 = 0, V_2 = 0) = 2$$

$$E(h = 1, V_1 = 0, V_2 = 1) = 2 + 1$$

$$E(h = 1, V_1 = 1, V_2 = 0) = 2 + -3$$

$$E(h = 1, V_1 = 1, V_2 = 1) = 2 + -3 + 1$$

Thus,

$$\sum_s \exp(-E(s)) = \exp(0) + \exp(0) + \exp(0) + \exp(0) + \exp(-2) + \exp(-3) + \exp(1) + \exp(0) \approx 7.903404.$$

Now we can convert the energies into probabilities. We're only interested in two probabilities, as mentioned above.

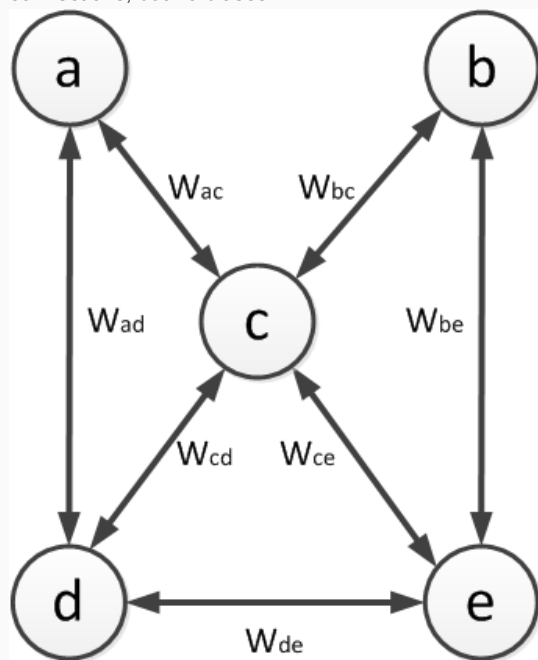
$$P(h = 0, V_1 = 1, V_2 = 0) \approx \frac{\exp(0)}{7.903404} \approx 0.1265$$

$$P(h = 1, V_1 = 1, V_2 = 0) \approx \frac{\exp(1)}{7.903404} \approx 0.3440$$

That makes for a total of about 0.4705.

## Question 5

The figure below shows a Hopfield network with five binary threshold units:  $a$ ,  $b$ ,  $c$ ,  $d$ , and  $e$ . The network has many connections, but no biases.



Let  $W_{ac} = W_{bc} = 1$ ,  $W_{ce} = W_{cd} = 2$ ,  $W_{be} = -3$ ,  $W_{ad} = -2$ , and  $W_{de} = 3$ .

What is the configuration that has the lowest energy? What is the configuration that has the second lowest energy (considering all configurations, not just those that are energy minima)?

A configuration consists of a state for each unit. Write "1" for a unit that's on, and "0" for a unit that's off. To describe a configuration, first write the state of unit  $a$ , then the state of unit  $b$ , etc. For example, if you want to describe the configuration where units  $a$  and  $d$  are on and the other units are off, then write 10010. For this question you have to describe two configurations, and write them with a comma in between. For example, if you think that the lowest energy configuration is the one where only units  $a$  and  $d$  are on, and that the second lowest energy configuration is the one where only units  $b$ ,  $d$ , and  $e$  are on, then you should write this: **10010, 01011**

You entered:

00100

Your Answer

Score

Explanation

00100

✗

0.00

Total

0.00 / 2.00

### Question Explanation

You can simply try all 32 configurations to find the answer, or you can do some clever elimination, such as "if we want low energy, then unit  $c$  will definitely be on, because it only has positive connections."

