This theorem is the extended part of the master's theorem. If running time in a recurrence equation has a part of log n, then, by this theorem, we can solve the problem.

$T(n) = aT(n/b) + n^k * \log^p n$,

where $a \geq 1$, $b \geq 1$, $k \geq 0$, p is any real number.

If $a > b^k$, then $T(n) = \theta(n^{\log_b a})$

If $a = b^k$,

if $p > -1$, then $T(n) = \theta(n^k \log^{p+1} n)$.

if $p = -1$, then $T(n) = \theta(n^k \log\log n)$. **//Note: $\log \text{lon } n \neq \log^2 n$**

if $p < -1$, then $T(n) = \theta(n^k)$.

If $a < b^k$,

if $p \geq 0$, then $T(n) = \theta(n^k \log^p n)$.

if $p < 0$, then $T(n) = O(n^k)$.

Example 1 – Find the time complexity of $T(n) = 2T(n/2) + n/\log^2 n$.

Ans-

Apply Extended master's theorem.

In this equation, $a = 2$, $b = 2$, $k = 1$ and $p = -2$

Now, Case 2 applies because $a = b^k$

-> $a = 2$, $b^k = 2^1 = 2$

Value of p is -2, means it should be in the case of $p < -1$.

-> $T(n) = \theta(n^k)$.

-> $T(n) = \theta(n^1)$.

Example 2 – Find the time complexity of $T(n) = 2T(n/2) + n/\log^{0.5} n$.

Ans-

Apply Extended master's theorem.

In this equation, $a = 2$, $b = 2$, $k = 1$ and $p = -0.5$

Now, Case 2 applies because $a = b^k$

-> $a = 2$, $b^k = 2^1 = 2$

Value of p is -0.5, means it should be in the case of $p > -1$.

-> $T(n) = \theta(n^k \log^{p+1} n)$.

-> $T(n) = \theta(n^1 \log^{1-0.5} n)$.

-> $T(n) = \theta(n * \log^{0.5} n)$.

Example 3 – Find the time complexity of $T(n) = 8T(n/2) + n^2 \log^2 n$.

Ans-

Apply Extended master's theorem.

In this equation, $a = 8$, $b = 2$, $k = 2$ and $p = 2$

Now, Case 2 applies because $a > b^k$

-> $a = 8$, $b^k = 2^2 = 4$

Only one case applies

-> $T(n) = \theta(n^\wedge \log_b a)$.

-> $T(n) = \theta(n^\wedge \log_2 8)$.

-> $T(n) = \theta(n^3)$.

Example 4 – Find the time complexity of $T(n) = T(n/2) + n^2\log^2 n$.

Ans-

Apply Extended master's theorem.

In this equation, $a = 1$, $b = 2$, $k = 2$ and $p = 2$

Now, Case 3 applies because $a < b^k$

-> $a = 1$, $b^k = 2^2 = 4$

Value of p is 2, means it should be in the case of $p \geq 0$.

-> $T(n) = \theta(n^k\log^p n)$.

-> $T(n) = \theta(n^2\log^2 n)$.

I think you understand the extended master's theorem very well. But the tricky part is that it can also solve the master's theorem problem.

Q6 – Find the time complexity of $T(n) = T(n/2) + n$.

Ans-

Apply Extended master's theorem.

In this equation, $a = 1$, $b = 2$, $k = 1$ and $p = 0$

Now, Case 3 applies because $a < b^k$

-> $a = 1$, $b^k = 2^1 = 2$

Value of p is 0, means it should be in the case of $p \geq 0$.

-> $T(n) = \theta(n^k\log^p n)$.

-> $T(n) = \theta(n^1)$.

Q7 – Find the time complexity of $T(n) = 2T(n/2) + 1$.

Ans-

Apply Extended master's theorem.

In this equation, $a = 2$, $b = 2$, $k = 0$ and $p = 0$

Now, Case 2 applies because $a = b^k$

-> $a = 2$, $b^k = 2^0 = 1$

Value of p is -1, means it should be in the case of $p > -1$.

-> $T(n) = \theta(n^{\log_b a})$.

-> $T(n) = \theta(n^1)$.

Q8 – Find the time complexity of T(n) = 2T(n/2) + n.

Ans-

Apply Extended master's theorem.

In this equation, $a = 2$, $b = 2$, $k = 1$ and $p = 0$

Now, Case 2 applies because $a = b^k$

-> $a = 2$, $b^k = 2^1 = 2$

Value of p is 0, means it should be in the case of $p > -1$.

-> $T(n) = \theta(n^k \log^{p+1} n)$.

-> $T(n) = \theta(n^* \log n)$.

Q9 – Find the time complexity of T(n) = T(n/2) + log n.

Ans-

Apply Extended master's theorem.

In this equation, $a = 1$, $b = 2$, $k = 0$ and $p = 1$

Now, Case 2 applies because $a = b^k$

-> $a = 1$, $b^k = 2^0 = 1$

Value of p is 1, means it should be in the case of $p > -1$.

-> $T(n) = \theta(n^k \log^{p+1} n)$.

-> $T(n) = \theta(n^0 \log^{1+1} n)$.

-> $T(n) = \theta(\log^2 n)$.

You can check the answer by iterative method.

Q10 – Find the time complexity of T(n) = 2T(n/2) + n/log n.

Ans-

Apply Extended master's theorem.

In this equation, $a = 2$, $b = 2$, $k = 1$ and $p = -1$

Now, Case 2 applies because $a = b^k$

-> $a = 2$, $b^k = 2^1 = 2$

Value of p is -1, means it should be in the case of $p > -1$.

-> $T(n) = \theta(n^k loglogn)$.

-> $T(n) = \theta(n^1 loglogn)$.

-> $T(n) = \theta(n*log^2n) = \theta(n*logn*logn)$.

You can check the answer by iterative method.

I will give you some other examples so that you can understand easily.