实验报告

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1 实验题目

```
1   使用Coq证明如下命题(不允许使用搜索策略，不允许使用Classical库)，附上代码和文档(文档中列出每个
    证明步骤的输出截图)
2
3   - Lemma ex1: forall A, ~~~ A -> ~ A.
4   - Lemma ex2: forall A B, A \/ B -> ~ (~ A /\ ~ B).
5   - Lemma ex3: forall T (P:T -> Prop), (~ exists x, P x) -> forall x, ~ P x.
```

2 实验过程

2.1 证明lemma1

2.1.1 源代码

```
1   Lemma ex1 : forall A : Prop, ~~~A -> ~A.
2   Proof.
3     intro h1.
4     intro h2.
5     intro h3.
6     apply h2.
7     intro h4.
8     apply h4.
9     assumption.
10  Qed.
11
```

2.1.2 执行过程

```
Lemma ex1 : forall A : Prop, ~~~A -> ~A.
Proof.
  intro h1.
  intro h2.
  intro h3.
  apply h2.
  intro h4.
  apply h4.
  assumption.
Qed.
```

```
1 goal
_____(1/1)
forall A : Prop, ~ ~ ~ A -> ~ A
```

```
Lemma ex1 : forall A : Prop, ~~~A -> ~A.
Proof.
  intro h1.
  intro h2.
  intro h3.
  apply h2.
  intro h4.
  apply h4.
  assumption.
Qed.
```

```
1 goal
h1 : Prop
_____(1/1)
~ ~ ~ h1 -> ~ h1
```

```
Lemma ex1 : forall A : Prop, ~~~A -> ~A.
Proof.
  intro h1.
  intro h2.
  intro h3.
  apply h2.
  intro h4.
  apply h4.
  assumption.
Qed.
```

```
1 goal
h1 : Prop
h2 : ~ ~ ~ h1
_____(1/1)
~ h1
```

```
Lemma ex1 : forall A : Prop, ~~~A -> ~A.
Proof.
  intro h1.
  intro h2.
  intro h3.
  apply h2.
  intro h4.
  apply h4.
  assumption.
Qed.
```

```
1 goal
h1 : Prop
h2 : ~ ~ ~ h1
h3 : h1
_____(1/1)
False
```

```
Lemma ex1 : forall A : Prop, ~~~A -> ~A.
Proof.
  intro h1.
  intro h2.
  intro h3.
  apply h2.
  intro h4.
  apply h4.
  assumption.
Qed.
```

```
1 goal
h1 : Prop
h2 : ~ ~ ~ h1
h3 : h1
_____(1/1)
~ ~ h1
```

```
Lemma ex1 : forall A : Prop, ~~~A -> ~A.
Proof.
  intro h1.
  intro h2.
  intro h3.
  apply h2.
  intro h4.
  apply h4.
  assumption.
Qed.
```

```
1 goal
h1 : Prop
h2 : ~ ~ ~ h1
h3 : h1
h4 : ~ h1
_____(1/1)
False
```

```
Lemma ex1 : forall A : Prop, ~~~A -> ~A.
Proof.
  intro h1.
  intro h2.
  intro h3.
  apply h2.
  intro h4.
  apply h4.
  assumption.
Qed.
```

```
1 goal
h1 : Prop
h2 : ~ ~ ~ h1
h3 : h1
h4 : ~ h1
_____(1/1)
h1
```

```
Lemma ex1 : forall A : Prop, ~~~A -> ~A.
Proof.
  intro h1.
  intro h2.
  intro h3.
  apply h2.
  intro h4.
  apply h4.
  assumption.
Qed.
```

```
No more goals.
```

2.2 证明lemma2

2.2.1 源代码

```
1   Lemma ex2 : forall A B : Prop, A \/ B -> ~ (~A /\ ~B).
2   Proof.
3     intro h1.
4     intro h2.
5     intro h3.
6     intro h4.
7     destruct h4 as (h41 , h42).
8     destruct h3 as [h31 | h32].
9     - apply h41. assumption.
10    - apply h42. assumption.
11  Qed.
12
```

2.2.2　执行过程

scratch

```
Lemma ex2 : forall A B : Prop, A \/ B -> ~ (~A /\ ~B).
Proof.
  intro h1.
  intro h2.
  intro h3.
  intro h4.
  destruct h4 as (h41 , h42).
  destruct h3 as [h31 | h32].
  - apply h41. assumption.
  - apply h42. assumption.
Qed.
```

```
1 goal
_____(1/1)
forall A B : Prop, A \/ B -> ~ (~ A /\ ~ B)
```

scratch

```
Lemma ex2 : forall A B : Prop, A \/ B -> ~ (~A /\ ~B).
Proof.
  intro h1.
  intro h2.
  intro h3.
  intro h4.
  destruct h4 as (h41 , h42).
  destruct h3 as [h31 | h32].
  - apply h41. assumption.
  - apply h42. assumption.
Qed.
```

```
1 goal
h1 : Prop
_____(1/1)
forall B : Prop, h1 \/ B -> ~ (~ h1 /\ ~ B)
```

scratch

```
Lemma ex2 : forall A B : Prop, A \/ B -> ~ (~A /\ ~B).
Proof.
  intro h1.
  intro h2.
  intro h3.
  intro h4.
  destruct h4 as (h41 , h42).
  destruct h3 as [h31 | h32].
  - apply h41. assumption.
  - apply h42. assumption.
Qed.
```

```
1 goal
h1, h2 : Prop
_____(1/1)
h1 \/ h2 -> ~ (~ h1 /\ ~ h2)
```

scratch

```
Lemma ex2 : forall A B : Prop, A \/ B -> ~ (~A /\ ~B).
Proof.
  intro h1.
  intro h2.
  intro h3.
  intro h4.
  destruct h4 as (h41 , h42).
  destruct h3 as [h31 | h32].
  - apply h41. assumption.
  - apply h42. assumption.
Qed.
```

```
1 goal
h1, h2 : Prop
h3 : h1 \/ h2
_____(1/1)
~ (~ h1 /\ ~ h2)
```

scratch

```
Lemma ex2 : forall A B : Prop, A \/ B -> ~ (~A /\ ~B).
Proof.
  intro h1.
  intro h2.
  intro h3.
  intro h4.
  destruct h4 as (h41 , h42).
  destruct h3 as [h31 | h32].
  - apply h41. assumption.
  - apply h42. assumption.
Qed.
```

```
1 goal
h1, h2 : Prop
h3 : h1 \/ h2
h4 : ~ h1 /\ ~ h2
_____(1/1)
False
```

```
Lemma ex2 : forall A B : Prop, A \/ B -> ~ (~A /\ ~B).
Proof.
  intro h1.
  intro h2.
  intro h3.
  intro h4.
  destruct h4 as (h41 , h42).
  destruct h3 as [h31 | h32].
  - apply h41. assumption.
  - apply h42. assumption.
Qed.
```

```
1 goal
h1, h2 : Prop
h3 : h1 \/ h2
h41 : ~ h1
h42 : ~ h2
_____(1/1)
False
```

```
Lemma ex2 : forall A B : Prop, A \/ B -> ~ (~A /\ ~B).
Proof.
  intro h1.
  intro h2.
  intro h3.
  intro h4.
  destruct h4 as (h41 , h42).
  destruct h3 as [h31 | h32].
  - apply h41. assumption.
  - apply h42. assumption.
Qed.
```

```
2 goals
h1, h2 : Prop
h31 : h1
h41 : ~ h1
h42 : ~ h2
_____(1/2)
False
_____(2/2)
False
```

Messages ⁄   Errors ⁄   Jobs ⁄

```
Lemma ex2 : forall A B : Prop, A \/ B -> ~ (~A /\ ~B).
Proof.
  intro h1.
  intro h2.
  intro h3.
  intro h4.
  destruct h4 as (h41 , h42).
  destruct h3 as [h31 | h32].
  - apply h41. assumption.
  - apply h42. assumption.
Qed.
```

```
This subproof is complete, but there are some unfocused goals:
_____(1/1)
False
```

Messages ⁄   Errors ⁄   Jobs ⁄

🔒 *scratch*

```
Lemma ex2 : forall A B : Prop, A \/ B -> ~ (~A /\ ~B).
Proof.
  intro h1.
  intro h2.
  intro h3.
  intro h4.
  destruct h4 as (h41 , h42).
  destruct h3 as [h31 | h32].
  - apply h41. assumption.
  - apply h42. assumption.
Qed.
```

```
No more goals.
```

2.3   证明lemma3

2.3.1 源代码

```
1  Lemma ex3 : forall T (P : T -> Prop), (~ exists x, P x) -> forall x, ~ P x.
2  Proof.
3    intro h1.
4    intro h2.
5    intro h3.
6    intro h4.
7    intro h5.
8    apply h3.
9    exists h4.
10   apply h5.
11 Qed.
12
```

2.3.2 执行过程

scratch

```coq
Lemma ex3 : forall T (P : T -> Prop), (~ exists x, P x) -> forall x, ~ P x.
Proof.
  intro h1.
  intro h2.
  intro h3.
  intro h4.
  intro h5..
  apply h3.
  exists h4.
  apply h5.
Qed.
```

```
1 goal
h1 : Type
_____(1/1)
forall P : h1 -> Prop, ~ (exists x : h1, P x) -> forall x : h1, ~ P x
```

Messages ↗   Errors ↗   Jobs ↗

scratch

```coq
Lemma ex3 : forall T (P : T -> Prop), (~ exists x, P x) -> forall x, ~ P x.
Proof.
  intro h1.
  intro h2.
  intro h3.
  intro h4.
  intro h5..
  apply h3.
  exists h4.
  apply h5.
Qed.
```

```
1 goal
h1 : Type
h2 : h1 -> Prop
_____(1/1)
~ (exists x : h1, h2 x) -> forall x : h1, ~ h2 x
```

Messages ↗   Errors ↗   Jobs ↗

scratch

```coq
Lemma ex3 : forall T (P : T -> Prop), (~ exists x, P x) -> forall x, ~ P x.
Proof.
  intro h1.
  intro h2.
  intro h3.
  intro h4.
  intro h5..
  apply h3.
  exists h4.
  apply h5.
Qed.
```

```
1 goal
h1 : Type
h2 : h1 -> Prop
h3 : ~ (exists x : h1, h2 x)
_____(1/1)
forall x : h1, ~ h2 x
```

Messages ↗   Errors ↗   Jobs ↗

```
Lemma ex3 : forall T (P : T -> Prop), (~ exists x, P x) -> forall x, ~ P x.
Proof.
  intro h1.
  intro h2.
  intro h3.
  intro h4.
  intro h5.
  apply h3.
  exists h4.
  apply h5.
Qed.
```

```
1 goal
h1 : Type
h2 : h1 -> Prop
h3 : ~ (exists x : h1, h2 x)
h4 : h1
_____(1/1)
~ h2 h4
```

Messages ↗   Errors ↗   Jobs ↗

```
Lemma ex3 : forall T (P : T -> Prop), (~ exists x, P x) -> forall x, ~ P x.
Proof.
  intro h1.
  intro h2.
  intro h3.
  intro h4.
  intro h5.
  apply h3.
  exists h4.
  apply h5.
Qed.
```

```
1 goal
h1 : Type
h2 : h1 -> Prop
h3 : ~ (exists x : h1, h2 x)
h4 : h1
h5 : h2 h4
_____(1/1)
False
```

Messages ↗   Errors ↗   Jobs ↗

```
Lemma ex3 : forall T (P : T -> Prop), (~ exists x, P x) -> forall x, ~ P x.
Proof.
  intro h1.
  intro h2.
  intro h3.
  intro h4.
  intro h5.
  apply h3.
  exists h4.
  apply h5.
Qed.
```

```
1 goal
h1 : Type
h2 : h1 -> Prop
h3 : ~ (exists x : h1, h2 x)
h4 : h1
h5 : h2 h4
_____(1/1)
exists x : h1, h2 x
```

Messages ↗   Errors ↗   Jobs ↗

```
Lemma ex3 : forall T (P : T -> Prop), (~ exists x, P x) -> forall x, ~ P x.
Proof.
  intro h1.
  intro h2.
  intro h3.
  intro h4.
  intro h5.
  apply h3.
  exists h4.
  apply h5.
Qed.
```

```
1 goal
h1 : Type
h2 : h1 -> Prop
h3 : ~ (exists x : h1, h2 x)
h4 : h1
h5 : h2 h4
_____(1/1)
h2 h4
```

Messages Errors Jobs

```
Lemma ex3 : forall T (P : T -> Prop), (~ exists x, P x) -> forall x, ~ P x.
Proof.
  intro h1.
  intro h2.
  intro h3.
  intro h4.
  intro h5.
  apply h3.
  exists h4.
  apply h5.
Qed.
```

```
No more goals.
```

Messages Errors Jobs