We want to compute

$$\mu_u = \frac{\partial F}{\partial u}, \tag{1}$$

where

$$\begin{aligned} F =& u \ln \frac{u}{R} + v \ln \frac{v}{R} + c \ln \left(1 - \frac{u+v}{c}\right) && (2) \\ =& u \ln u - u \ln R + v \ln v - v \ln R + c \ln R - c \ln c && (3) \end{aligned}$$

We have

$$\begin{aligned} &\frac{\partial}{\partial u}(u \ln u) = \ln u + 1, \\ &\frac{\partial}{\partial u}(-u \ln R) = -\ln R + \frac{u}{R}, \text{ (the mistakes were here)} \\ &\frac{\partial}{\partial u}(v \ln v) = 0, \\ &\frac{\partial}{\partial u}(-v \ln R) = \frac{v}{R}, \\ &\frac{\partial}{\partial u}(c \ln R) = -\frac{c}{R}, \\ &\frac{\partial}{\partial u}(-c \ln c) = 0. \end{aligned}$$

So in the end we get

$$\mu_u = \ln u - \ln R = \ln u - \ln(c - u - v), \tag{4}$$

using

$$R = c - u - v. \tag{5}$$

Now we compute derivative We have

$$\partial_x \mu_u = \frac{u'}{u} - \frac{R'}{R} = \left(\frac{1}{u} + \frac{1}{R}\right) u' + \frac{1}{R} v' \tag{6}$$

So we would get

$$D_{uu} = (1 + \frac{u}{R}) = 1 + \frac{u}{c - u - v} \tag{7}$$

$$D_{uv} = \frac{u}{R} = \frac{u}{c - u - v} \tag{8}$$

So

$$\begin{aligned} \partial_x(u \partial_x \mu_u) &= u'' + \frac{u'R - R'u}{R^2}(u' + v') + \frac{u}{R}(u'' + v'') && (9) \\ &= u'' + \frac{u'}{R}(u' + v') + \frac{u}{R^2}(u' + v')^2 + \frac{u}{R}(u'' + v'') && (10) \\ &= u'' - \frac{u(RR'' - (R')^2) + R'u'R}{R^2} && (11) \\ &= u'' - \frac{u}{R}R'' + u\frac{(R')^2}{R^2} - \frac{R'u'}{R} && (12) \end{aligned}$$

Or finally

$$\partial_x(u\partial_x\mu_u) = u'' + \frac{u'}{(c-u-v)}(u'+v') \tag{13}$$

$$+ \frac{u}{(c-u-v)^2}(u'+v')^2 + \frac{u}{(c-u-v)}(u''+v'') \tag{14}$$

And from this we get the linearization $u \to u + \delta u$, by also assuming $u', v', u'', v'' = 0$ and ignoring higher order terms.

$$\delta\dot{u} = \delta u'' + \frac{u}{c-u-v}(\delta u'' + \delta v'') + \text{ reaction kinetics part}$$

If we introduce the coefficient in free energy we'll get

$$\delta\dot{u} = D_1\left(\delta u'' + \frac{u}{c-u-v}(\delta u'' + \delta v'')\right) + \text{ reaction kinetics part}$$

Or

$$\delta\dot{u} = D_1\left(D_{uu}\delta u'' + D_{uv}\delta v'' + \right)$$

ie. for $\delta w = (\delta u, \delta v)$

$$\delta\dot{w} = \begin{pmatrix} D_1(1 + \frac{u}{c-v-v}) & D_1\frac{u}{c-v-v} \\ D_2\frac{v}{c-u-v} & D_2(1 + \frac{v}{c-u-v}) \end{pmatrix}\delta w'' + \text{ reaction kinetics part}$$

So the end result is the same