

We want to compute

$$\mu_u = \frac{\partial F}{\partial u}, \quad (1)$$

where

$$F = u \ln \frac{u}{R} + v \ln \frac{v}{R} + c \ln \left( 1 - \frac{u+v}{c} \right) \quad (2)$$

$$= u \ln u - u \ln R + v \ln v - v \ln R + c \ln R - c \ln c \quad (3)$$

We have

$$\begin{aligned} \frac{\partial}{\partial u} (u \ln u) &= \ln u + 1, \\ \frac{\partial}{\partial u} (-u \ln R) &= -\ln R + \frac{u}{R}, \text{ (the mistakes were here)} \\ \frac{\partial}{\partial u} (v \ln v) &= 0, \\ \frac{\partial}{\partial u} (-v \ln R) &= \frac{v}{R}, \\ \frac{\partial}{\partial u} (c \ln R) &= -\frac{c}{R}, \\ \frac{\partial}{\partial u} (-c \ln c) &= 0. \end{aligned}$$

So in the end we get

$$\mu_u = \ln u - \ln R = \ln u - \ln(c - u - v), \quad (4)$$

using

$$R = c - u - v. \quad (5)$$

Now we compute derivative We have

$$\partial_x \mu_u = \frac{u'}{u} - \frac{R'}{R} = \left( \frac{1}{u} + \frac{1}{R} \right) u' + \frac{1}{R} v' \quad (6)$$

So we would get

$$D_{uu} = \left( 1 + \frac{u}{R} \right) = 1 + \frac{u}{c - u - v} \quad (7)$$

$$D_{uv} = \frac{u}{R} = \frac{u}{c - u - v} \quad (8)$$

So

$$\partial_x (u \partial_x \mu_u) = u'' + \frac{u'R - R'u}{R^2} (u' + v') + \frac{u}{R} (u'' + v'') \quad (9)$$

$$= u'' + \frac{u'}{R} (u' + v') + \frac{u}{R^2} (u' + v')^2 + \frac{u}{R} (u'' + v'') \quad (10)$$

$$= u'' - \frac{u(RR'' - (R')^2) + R'u'R}{R^2} \quad (11)$$

$$= u'' - \frac{u}{R} R'' + u \frac{(R')^2}{R^2} - \frac{R'u'}{R} \quad (12)$$

Or finally

$$\partial_x(u\partial_x\mu_u) = u'' + \frac{u'}{(c-u-v)}(u' + v') \quad (13)$$

$$+ \frac{u}{(c-u-v)^2}(u' + v')^2 + \frac{u}{(c-u-v)}(u'' + v'') \quad (14)$$

And from this we get the linearization  $u \rightarrow u + \delta u$ , by also assuming  $u', v', u'', v'' = 0$  and ignoring higher order terms.

$$\delta\dot{u} = \delta u'' + \frac{u}{c-u-v}(\delta u'' + \delta v'') + \text{reaction kinetics part}$$

If we introduce the coefficient in free energy we'll get

$$\delta\dot{u} = D_1 \left( \delta u'' + \frac{u}{c-u-v}(\delta u'' + \delta v'') \right) + \text{reaction kinetics part}$$

Or

$$\delta\dot{u} = D_1 (D_{uu}\delta u'' + D_{uv}\delta v'')$$

ie. for  $\delta w = (\delta u, \delta v)$

$$\delta\dot{w} = \begin{pmatrix} D_1(1 + \frac{u}{c-v-v}) & D_1\frac{u}{c-v-v} \\ D_2\frac{v}{c-u-v} & D_2(1 + \frac{v}{c-u-v}) \end{pmatrix} \delta w'' + \text{reaction kinetics part}$$

So the end result is the same