Neutral curve stability and the minima

Do interaction improve conditions for stability for determinant?

Reminder: We have following expression for neutral curves of det
with interactions:

$$b(q) = \frac{D_u D_v c q^4 + D_u a^2 c q^2 - D_v a q^2 + D_v c q^2 - a^3 + a^2 c}{2D_u a q^2 - 2D_v a q^2 + D_v c q^2 + a}$$

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And without interactions
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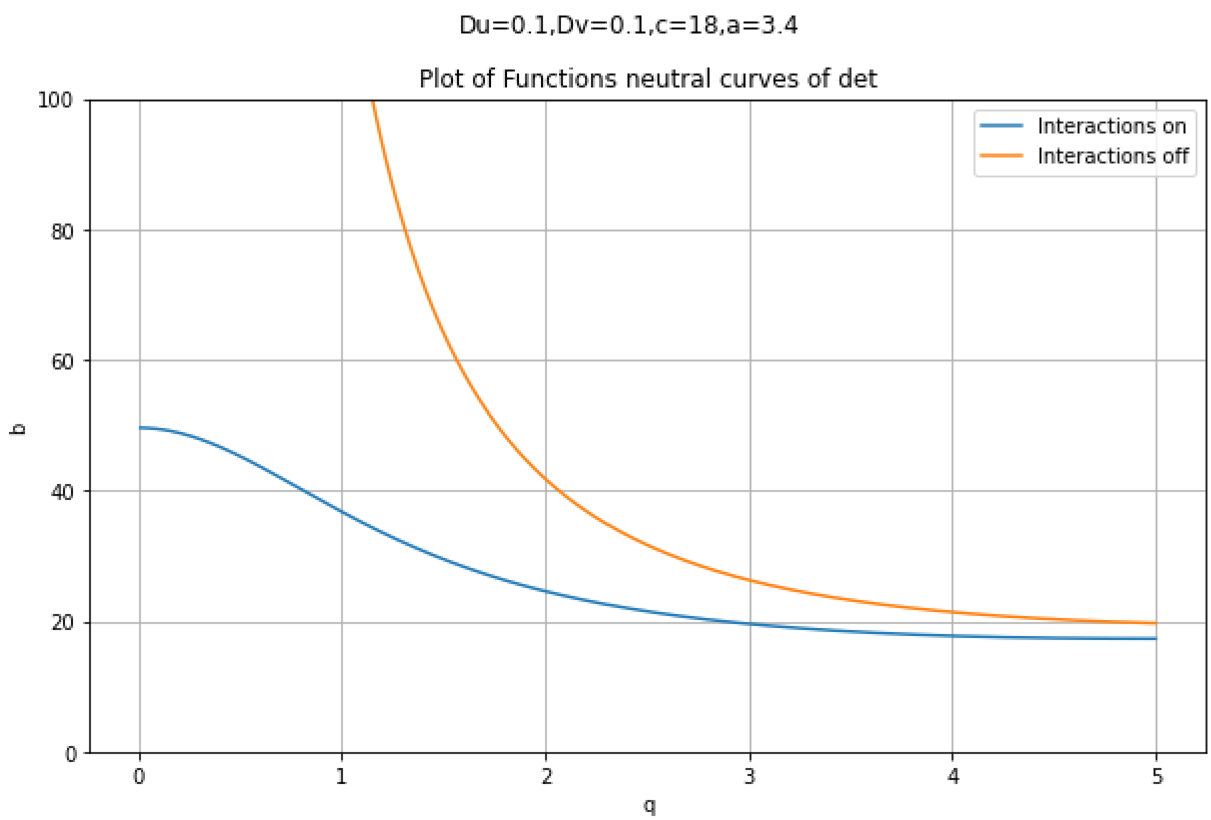
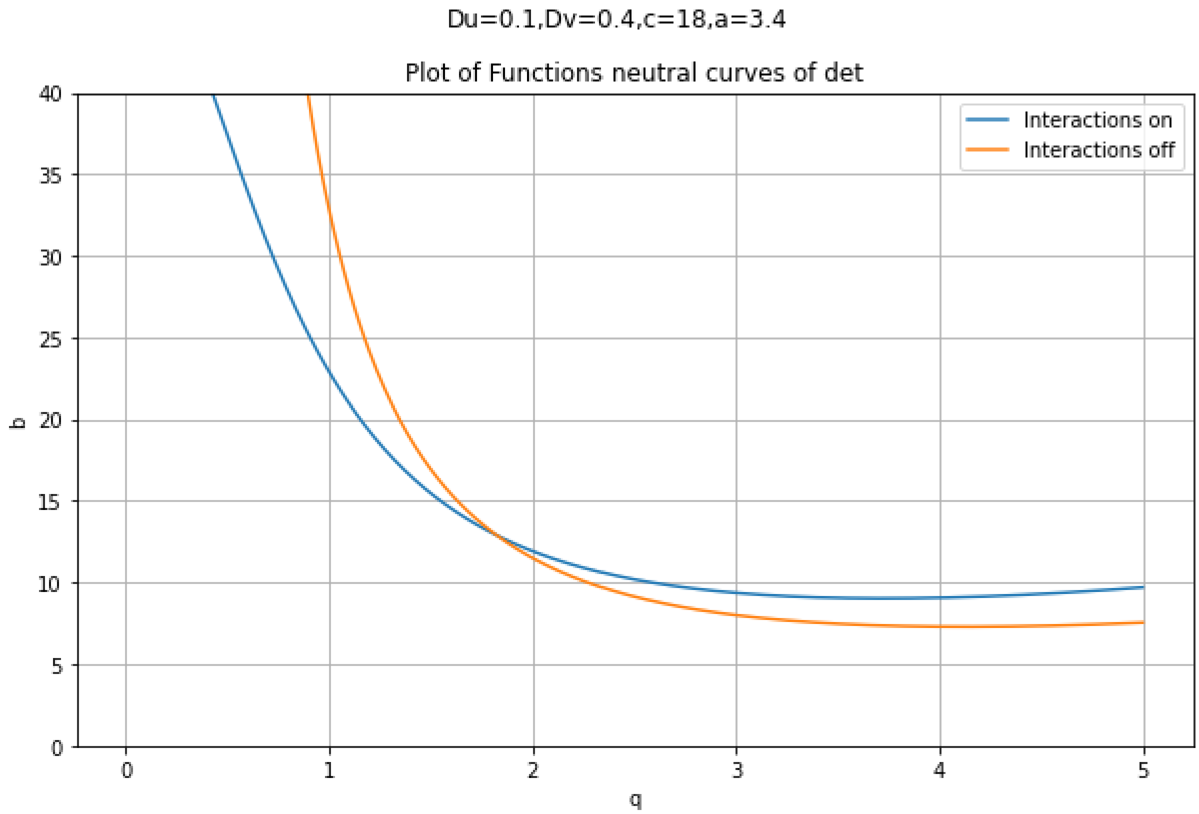
$$b(q) = D_u q^2 + \frac{D_u a^2 + D_v}{D_v} + \frac{a^2}{D_v q^2}$$

It doesn't seem like the interactions always improve stability. It can go either way, depending on the parameters.

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Plot of Functions neutral curves of det
Interactions on
Interactions off
b
q
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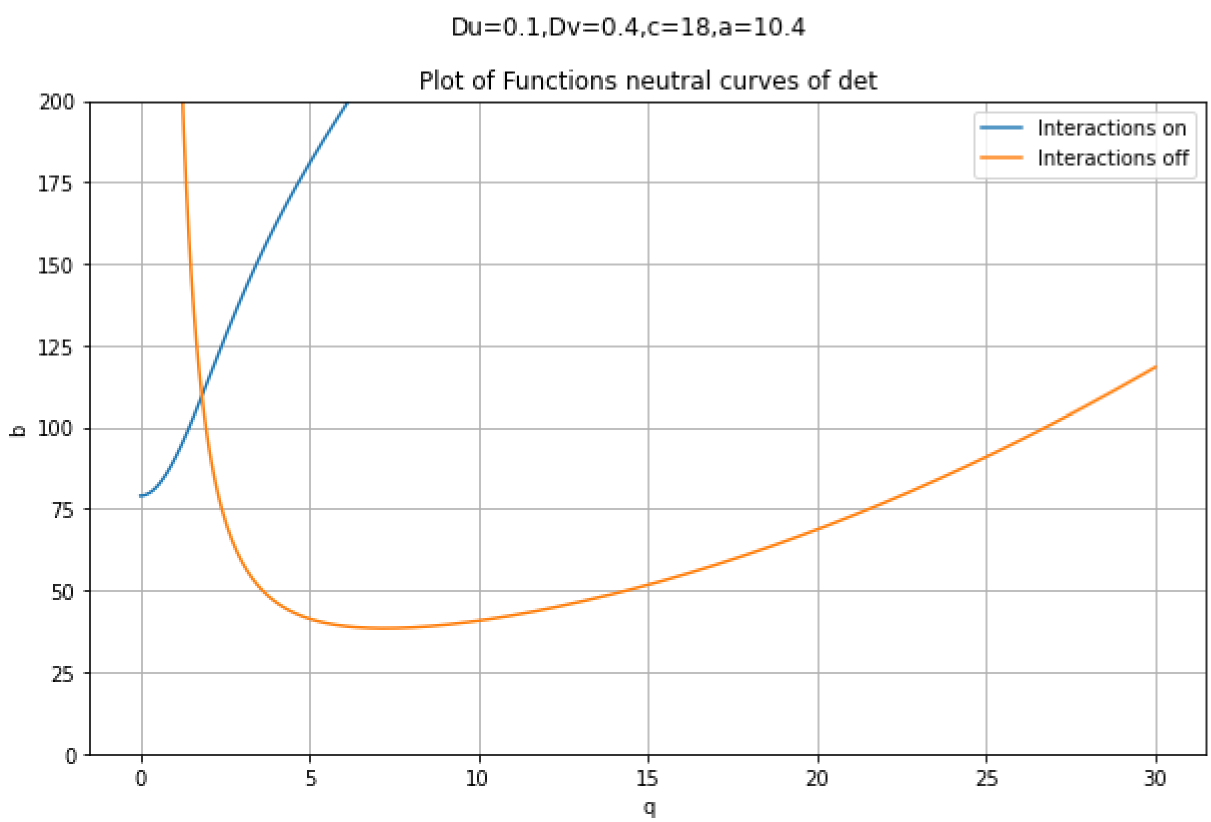
5
10
15
20
25
30
35
40
0
1
2
3
4
5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Plot of Functions neutral curves of det
Interactions on
Interactions off
b
q
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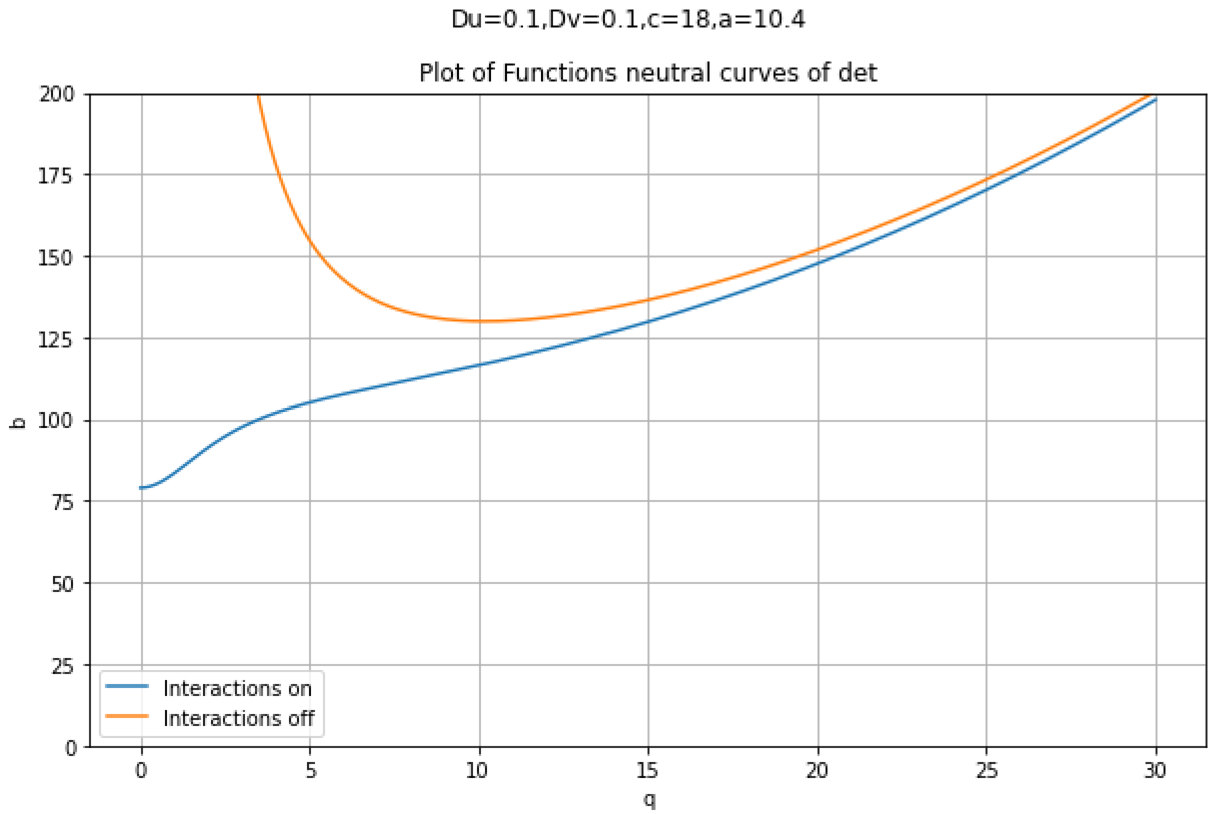
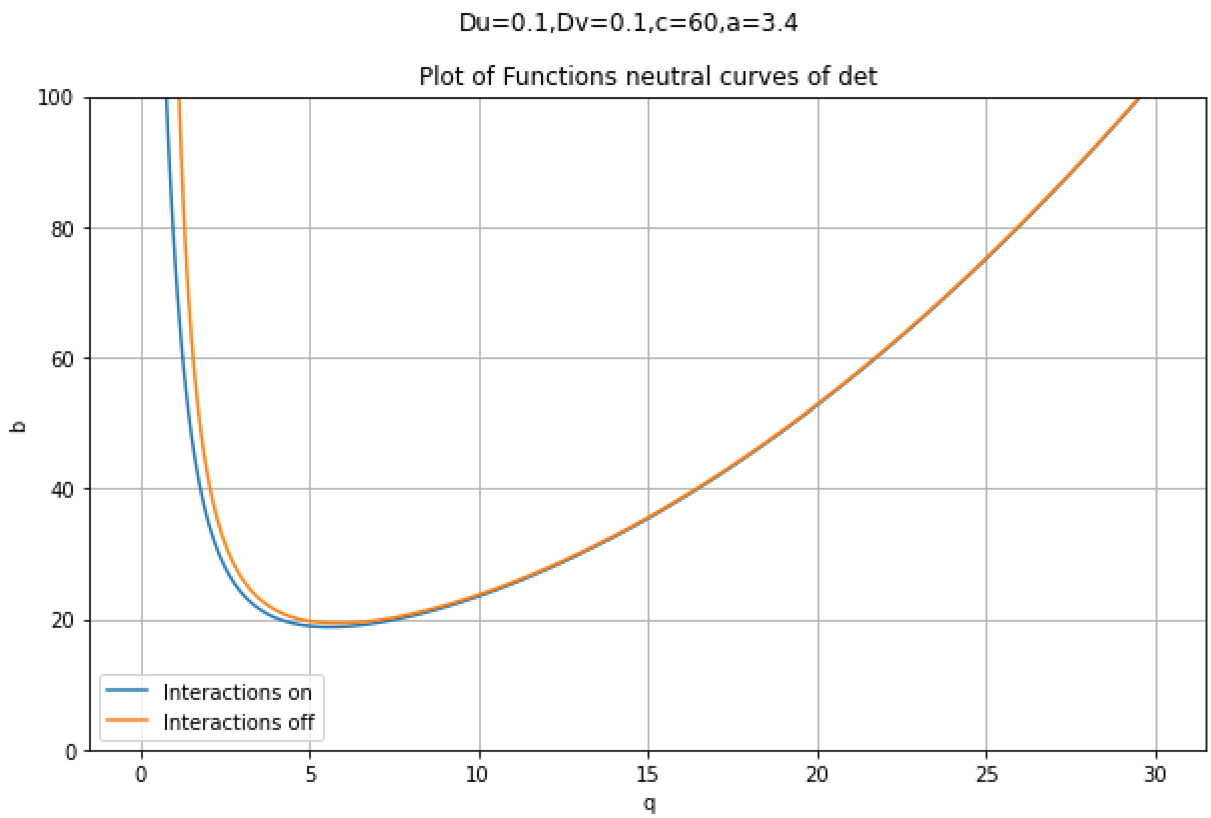
10
15
20
25
30





Minimum of neutral curve in case of one diffusion coefficient.

In case of identical diffusion coefficients the neutral curve for det has the following form

$$b = \frac{D^2cq^4 + Dq^2(a^2c - a + c) - a^3 + a^2c}{Dcq^2 + a}$$

We can rewrite it as

$$\begin{aligned} b(q) &= A_1L + A_2 + \frac{A_3}{L} \\ &= \frac{1}{c}L + \left(a^2 - \frac{3a}{c} + 1\right) + \frac{\frac{-2a^3c + a^2c^2 + 2a^2 - ac}{c}}{L} \\ &= \frac{L}{c} + a^2 - \frac{3a}{c} + 1 + \frac{-2a^3c + a^2c^2 + 2a^2 - ac}{cL} \\ &= \frac{L}{c} + (a^2 - \frac{3a}{c} + 1) + \frac{a(1-ac)(2a-c)}{cL}. \end{aligned},$$

where $L = Dcq^2 + a$

Then the minimum of the neutral curve is attained at $L^2 = -2a^3c + a^2c^2 + 2a^2 - ac$, because $\frac{db}{dq} = \frac{db}{dL}\frac{dL}{dq}$.

The $a(1-ac)(2a-c)$ changes signs at $\frac{1}{c}$ and $a = \frac{c}{2}$, so in between it is positive.

Then we get

$$b_T = b(q_{crit}) = 1 + a^2 + \frac{2\sqrt{a^2c^2 + 2a^2 - ac - 2a^3c} - 3a}{c}$$

Moreover, the neutral curve for tr seemed to be similar to an ellipse and it always had a minimum at $q = 0$. I'm gonna assume that it is in fact the minimum, in which case the minimum of $b_{\text{trace}}(q)$ is equal to $1 + a^2$.

So we're asking if we can have

$$b_T < 1 + a^2$$

This would mean

$$\frac{2\sqrt{a^2c^2 + 2a^2 - ac - 2a^3c} - 3a}{c} < 0 \iff 0 > 4a^2c^2 - 4ac - 8a^3c - a^2$$

If a is close to $\frac{c}{2}$ then this condition is satisfied. Unfortunately, $b \approx 1 + a^2$, so we would have

$$u + v = \frac{b}{a} + a = \frac{2}{c} + \frac{c}{2} + \frac{c}{2} \approx c + \frac{2}{c}$$

So it's not so easy to remain within bounds.