Assignment 5: Propositional and First Order Logic

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Deadline: 03.11.2022, 23:59 hrs

Overview

This is a problem set for you to gain experience with propositional logic and First-Order Logic (FOL) by solving many small problems. Refer to the textbook (**Artificial Intelligence: A Modern Approach, 4rd ed. (Global edition)**) for reference, specially Chapters 7, 8 and 9.

1 Models and entailment in propositional logic

1.1 Validity and Soundness

a) Generate the vocabulary of the following argument.

b) Translate the argument into propositional logic statements.

c) Add a premise (P4) to make the conclusion of the argument valid.

P1 to P3 are the premises, C is the conclusion:

- (P1) If Peter's argument is valid and all the premises of Peter's argument are true, then Peter's argument is sound.
- (P2) If the premises of Peter's argument entail the conclusion of Peter's argument, then Peter's argument is valid.
- (P3) The premises of Peter's argument entail the conclusion of Peter's argument.
- (C) Peter's argument is sound.

1.2 Modelling

For each of the following statements, determine if they are satisfiable by building the complete model (truth table) and mark tautologies.

a) $(p \implies q) \implies ((p \implies r) \implies (q \implies r))$

b) $(p \lor (\neg q \implies r)) \implies (q \lor (\neg p \implies r))$

c) $(\neg(p \land (q \implies \neg r))) \implies ((p \implies q) \land (p \implies r))$

d) $(\neg(\neg p \implies (q \land r))) \implies (\neg(p \lor q) \land r)$

1.3 Modelling 2

Let ϕ be a sentence that contains three atomic constituents and let the truth conditions of ϕ be defined by the truth table below. Write a propositional logic statement that contains p, q, and r as constituents, and that is equivalent to ϕ.

p	q	r	ϕ
1	1	1	1
1	1	0	1
1	0	1	0
1	0	0	1
0	1	1	1
0	1	0	1
0	0	1	0
0	0	0	1

2 Resolution in propositional logic

2.1 Conjunctive Normal Form

Convert each of the following sentences to their Conjunctive Normal Form (CNF).

a) $p \iff q$

b) $\neg((p \implies q) \wedge r)$

c) $((p \vee q) \vee (r \wedge (\neg(q \implies r))))$

d) Is the solution to c really a CNF?

2.2 Inference in propositional logic

Use resolution to conclude r from the following statements.

a) $(p \implies q) \implies q$

b) $p \implies r$

c) $(r \implies s) \implies (\neg(s \implies q))$

3 Representation in First-Order Logic (FOL)

Consider the following baseball vocabulary:

1. $Pitcher(p)$ is a predicate where person p is a pitcher.
2. $flies(p_1, p_2)$ is a predicate where person p_1 flies[1] out to person p_2.
3. $Centerfielder(p)$ is a predicate where person p is a centerfielder.
4. $scores(p)$ is a predicate where person p scores.
5. $friend(p_1, p_2)$ is a predicate where person p_1 is the friend of person p_2 (but not vice versa).
6. $Robinson, Crabb, Samson, Jones$ are constants denoting persons.

[1] A flyout occurs when a batter hits the ball in the air, and a fielder catches it before it touches the ground.

Now look at the following translations of natural language into first order logic statements describing a baseball game. **Using the provided vocabulary, translate the conclusion of each of the following arguments into an FOL statement.**

a) **Argument A**

Only pitchers fly out to Robinson. Crabb scores only if Samson flies out to Robinson and Robinson is a centerfielder. Crabb scores.

Conclusion: *Samson is a pitcher.*

- $\forall x : [flies(x, Robinson) \implies Pitcher(x)]$
- $scores(Crabb) \implies (flies(Samson, Robinson) \land Centerfielder(Robinson))$
- $scores(Crabb)$

b) **Argument B**

No centerfielder who does not score has any friends. Robinson and Jones are both centerfielders. Any centerfielder who flies out to Jones does not score. Robinson flies out to Jones.

Conclusion: *Jones is not a friend of Robinson.*

- $\forall x : [((Centerfielder(x) \land \neg scores(x)) \implies \neg \exists y : [friend(y, x)]]$
- $Centerfielder(Robinson) \land Centerfielder(Jones)$
- $\forall x : [((Centerfielder(x) \land flies(x, Jones)) \implies \neg scores(x)]$
- $flies(Robinson, Jones)$

Square brackets [] are used to stress which parts of the statement belong under which quantifier.

4 Resolution in FOL

Using resolution, prove the conclusions from **Arguments A** and **B** from exercise 3.

Deliverables and recommendations

You must upload a **single** PDF containing the **typeset** equations, formulas and/or diagrams of your solutions.

- Include **natively digital** equations and diagrams. Do **not** upload scans or photos of hand-written solutions as these will be ignored by the TAs
- You can typeset your equations in Microsoft Word/Google Docs. If you're feeling adventurous, try LaTeX. Overleaf is a great place to start.