BCH (15,7) Decoding Example

Suppose $r = [000100110010100]$.
Determine the most likely valid codeword.

- Calculate the syndromes over GF(16):

$$r(x) = x^3 + x^6 + x^7 + x^{10} + x^{12}$$

$$S_1 = r(\alpha) = \alpha^3 + \alpha^6 + \alpha^7 + \alpha^{10} + \alpha^{12} = \alpha^{10}$$

$$S_2 = r(\alpha^2) = \alpha^6 + \alpha^{12} + \alpha^{14} + \alpha^{20} + \alpha^{24} = \alpha^5$$

$$S_3 = r(\alpha^3) = \alpha^9 + \alpha^{18} + \alpha^{21} + \alpha^{30} + \alpha^{36} = \alpha^4$$

$$S_4 = r(\alpha^4) = \alpha^{12} + \alpha^{24} + \alpha^{28} + \alpha^{40} + \alpha^{48} = \alpha^{10}$$

- Determine the error locator polynomial:

$$\Lambda_0 = 1$$

$$\Lambda_1 = S_1 = \alpha^{10}$$

$$\Lambda_2 = \frac{S_3 + S_1^3}{S_1} = \frac{\alpha^4 + \alpha^{30}}{\alpha^{10}} = \alpha^5(\alpha^4 + \alpha^{30})$$

$$= \alpha^9 + \alpha^{35} = \alpha^6$$

So, $\Lambda(x) = 1 + \alpha^{10}x + \alpha^6 x^2$

- Find the roots of $\Lambda(X)$:

By an exhaustive Chien search the roots are:

$$\alpha^{11} = \frac{1}{X_1} \qquad X_1 = \alpha^4$$

$$\alpha^{13} = \frac{1}{X_2} \qquad X_2 = \alpha^2$$

- The exponents of X_1 and X_2 form the error polynomial:

$$e(X) = X^2 + X^4$$

- Form the most likely code polynomial:

$$\hat{c}(X) = r(X) + e(X)$$

$$= X^2 + X^3 + X^4 + X^6 + X^7 + X^{10} + X^{12}$$

So, $\hat{c} = [001110110010100]$