Code Rate

Suppose we have an information source producing information at a rate H' bits/second. If we use binary symbols (binits) to represent this source stream then we have a symbol rate:

$$R_B = \frac{H'}{\log 2} \text{ binits/second}$$

When we channel code we *purposefully* add extra *parity* symbols to the information stream to make $R \le C$ and to hopefully lower the error rate. For every k information symbols we form a block of n symbols with $n \ge k$. We demand even with the extra parity symbols added that the information arrive at the destination *at the same rate*:

$$k T_B = n T_S$$

where $T_B = \frac{1}{R_B}$, the original information symbol period and $T_S = \frac{1}{R_S}$, the period of a symbol in the new block of n symbols. Rearranging we define a code rate r:

$$r = \frac{k}{n} = \frac{T_S}{T_B} = \frac{R_B}{R_S}$$

The code rate is a dimensionless quantity and for binary systems $0 < r \le 1$. Using the code rate in signal-to-noise ratio calculations:

$$E_B = S T_B \text{ and } E_S = S T_S$$

$$\frac{E_B}{T_B} = S = \frac{E_S}{T_S}$$

$$E_S = \frac{T_S}{T_B} E_B = r E_B$$

$$\frac{E_S}{N_0} = r \frac{E_B}{N_0}$$

$$\left.\frac{E_S}{N_0}\right|_{\text{(dB)}} = \left.\frac{E_B}{N_0}\right|_{\text{(dB)}} + 10 \log_{10} r$$

To make fair comparisons between various channel coding schemes and code rates the *bit noise ratio* $\frac{E_B}{N_0}$ is used.

The price for demanding that the information rate be unchanged is *bandwidth expansion*. In general, system bandwidth is directly proportional to signal rate:

$$W \propto R$$

$$R_S = \frac{R_B}{r}$$

$$W_S = \frac{1}{r} W_B$$

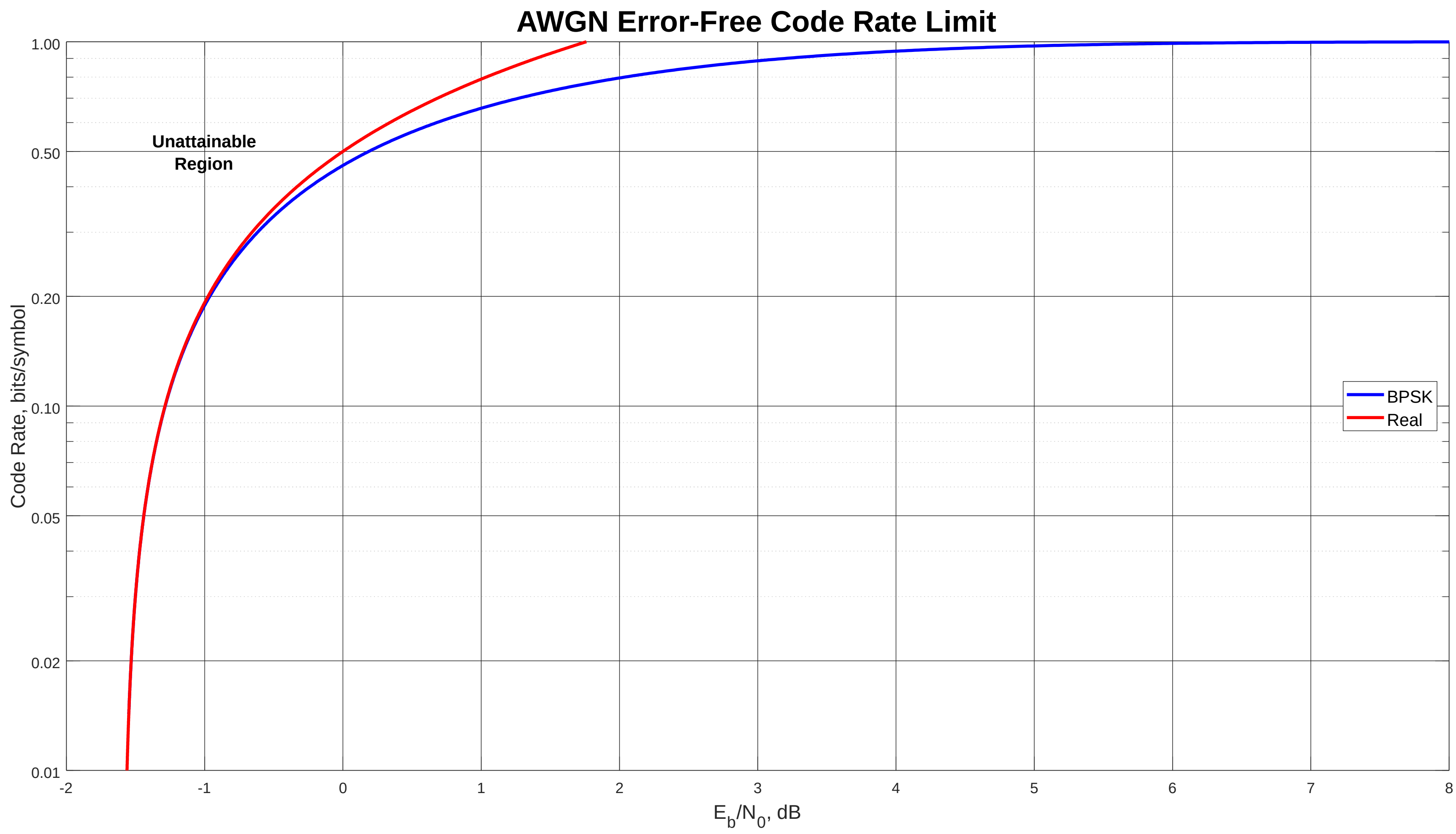
Example

Suppose we are working with a binary AWGN channel and a $\frac{E_B}{N_0}$ of 6 dB. We are interested in channel coding schemes with a code rate r of about 0.5.

Checking the capacity chart on the next page this is in the "attainable" region meaning that arbitrarily low error rates are possible.

Uncoded	2.4e-3
Hamming(8,4)	3.5e-3
Golay(24,12)	3.5e-4
Reed-Solomon(255,128)	1.7e-5
Reed-Solomon(511,256)	8.6e-6
DSN Concatenated	almost zero

The price for better error performance is larger block size n and greater coding/decoding complexity.



AWGN Error-Free Code Rate Limit
Unattainable Region
BPSK
Real
Code Rate, bits/symbol
Eb/N0, dB
1.00
0.50
0.20
0.10
0.05
0.02
0.01
-2
-1
0
1
2
3
4
5
6
7
8