EENG 410 Homework #5

1. A (5,2) code has the following three parity equations:

   $p_1 = m_1$
   $p_2 = m_2$
   $p_3 = m_1 + m_2$

   a) Construct the generator and parity-check matrices for this code. Place the two information symbols first in each codeword followed by the three parity symbols. The codewords should therefore have the form: $\left[m_1\, m_2\, p_1\, p_2\, p_3\right]$.
   b) Verify that the rows of G and H are orthogonal.
   c) Construct the standard array for this code.

2. The minimum distance for a linear block code is 11.

   a) Determine the maximum error-correcting capability.
   b) Determine the maximum error-detecting capability.
   c) Can the code simultaneously correct 4 errors and detect 5?
   d) Can the code simultaneously correct 5 errors and detect 6?

3. Calculate the probabilities of a message error P_M and an individual bit error P_B for a 24-symbol sequence encoded with a (24,12) linear block code. Assume the code corrects all single and double symbol error patterns but no error patterns with more than two errors. The probability of a channel symbol error p is 10^{-3}.

4. Consider a (127,92) linear block code capable of triple symbol error corrections.

   a) Determine the probability of a message error for an *uncoded* block of 92 symbols if the channel symbol error probability p is 10^{-3}.
   b) Determine the probability of a message error for a *coded* block of 127 symbols if the channel symbol error probability p is 10^{-3}.

5. Calculate the improvement in the probability of a message error relative to an uncoded transmission for a (24,12) double error correcting linear block code. Assume the uncoded transmission has blocks of 12 symbols and $E_b/N_0 = 10$ dB. The channel waveforms are polar NRZ.

Note: Work Problems 3, 4 and 5 to four significant digits.