Homework 5, Problem 1

```
>> StdArray

Block Length:  5
Number of Data Symbols:  2
Number of Parity Symbols:  3
Code Rate:  0.4000

Generator Matrix
   g(1):  [1 0 1 0 1]
   g(2):  [0 1 0 1 1]

Parity-Check Matrix
   h(1):  [1 0 1 0 0]
   h(2):  [0 1 0 1 0]
   h(3):  [1 1 0 0 1]

Possible Correctable 2-error Patterns
   [1 1 0 0 0]
   [0 1 1 0 0]
   [1 0 0 1 0]
   [0 0 1 1 0]

Select e(6):  [1 1 0 0 0]

Minimum Distance:   3

Correctable Error Patterns
   1-error Patterns:   5 of   5
   2-error Patterns:   2 of  10
```

Standard Array

i	S(i)	e(i)	Coset of e(i)		
0	000	00000	10101	01011	11110
1	101	10000	00101	11011	01110
2	011	01000	11101	00011	10110
3	100	00100	10001	01111	11010
4	010	00010	10111	01001	11100
5	001	00001	10100	01010	11111
6	110	11000	01101	10011	00110
7	111	01100	11001	00111	10010

```
>> G

G =

     1     0     1     0     1
     0     1     0     1     1


>> H

H =

     1     0     1     0     0
     0     1     0     1     0
     1     1     0     0     1


>> mod(G * H', 2)

ans =

     0     0     0
     0     0     0
```

Homework 5, Problem 2

For this code $d_{min} = 11$

a) The maximum error-correcting capability is:

$$t = \left\lfloor \frac{d_{min}-1}{2} \right\rfloor = \left\lfloor \frac{11-1}{2} \right\rfloor = \underline{\underline{5}}$$

b) The maximum error-detection capability is:

$$e = d_{min} - 1 = 11 - 1 = \underline{\underline{10}}$$

c) $\alpha = 4$

$\beta = 5$

$\alpha + \beta = 4 + 5 = 9 \leq d_{min} - 1$ ✓

<u>Yes</u>, the code can simultaneously correct 4 errors and detect 5 errors.

d) $\alpha = 5$

$\beta = 6$

$\alpha + \beta = 5 + 6 = 11 > d_{min} - 1$ X

<u>No</u>, the code cannot simultaneously correct 5 errors and detect 6 errors.

Problem 5-3

The probability of a message error is

$$P_M = \sum_{j=t+1}^{n} \binom{n}{j} p^j (1-p)^{n-j}$$

$$= \sum_{j=3}^{24} \binom{24}{j} \cdot 0.001^j \cdot 0.999^{24-j}$$

$$= 1.9819 \times 10^{-6} + 1.0415 \times 10^{-8}$$

$$+ 4.1704 \times 10^{-11} + \ldots$$

$$= \underline{\underline{1.992 \times 10^{-6}}}$$

The probability of an individual bit error is

$$P_B = \frac{1}{n} \sum_{j=t+1}^{n} j \binom{n}{j} p^j (1-p)^{n-j}$$

$$= \frac{1}{24} \sum_{j=3}^{24} j \binom{24}{j} \cdot 0.001^j \cdot 0.999^{24-j}$$

$$= \frac{1}{24} \left(5.9458 \times 10^{-6} + 4.1662 \times 10^{-8} + \ldots \right)$$

$$= \underline{\underline{2.495 \times 10^{-7}}}$$

Problem 5-4

a) For an uncoded block of 92 bits the probability of <u>no</u> message error is:

$$\bar{P}_M^u = (1-p)^k$$

$$= 0.999^{92}$$

$$= 0.9121$$

The probability of a message error is then:

$$P_M^u = 1 - \bar{P}_M^u$$

$$= \underline{\underline{0.0879}}$$

b) For a coded (127,92) block with triple error correction the probability of a message error is:

$$P_M^c = \sum_{j=t+1}^{n} \binom{n}{j} p^j (1-p)^{n-j}$$

$$= \sum_{j=4}^{127} \binom{127}{j} \cdot 0.001^j \cdot 0.999^{127-j}$$

$$= \underline{\underline{9.368 \times 10^{-6}}}$$

A fairer way to compare uncoded and coded performance would be to calculate a new value for p that accounts for a necessarily faster coded symbol rate.

By trial and error an uncoded $p = 0.001$ corresponds to $\frac{E_b}{N_0} = 4.7747$.

So, $$\frac{E_s}{N_0} = r\frac{E_b}{N_0} = \frac{92}{127}(4.7747) = 3.4588$$

$$p^c = \frac{1}{2}\operatorname{erfc}\sqrt{\frac{E_s}{N_0}} = 0.004267$$

Now, $$P_M^c = \sum_{j=t+1}^{n} \binom{n}{j} p^j (1-p)^{n-j}$$

$$= \sum_{j=4}^{127} \binom{127}{j} p^j (1-p)^{127-j}$$

$$= 0.002259$$

$$= \underline{2.259 \times 10^{-3}}$$

This is a much larger message error rate than the value found above.

Problem 5-5

For the uncoded case:

$$\frac{E_b}{N_o} = 10^{\frac{10}{10}} = 10$$

$$p_u = \frac{1}{2}\,\mathrm{erfc}\sqrt{\frac{E_b}{N_o}} = 3.872\times 10^{-6}$$

$$\overline{P}_M^u = (1-p_u)^k = (1-3.872\times 10^{-6})^{12}$$

$$= 0.99995354$$

$$P_M^u = 1-\overline{P}_M^u$$

$$= \underline{\underline{4.646\times 10^{-5}}}$$

For the coded case with double error correction:

$$\frac{E_s}{N_o} = r\frac{E_b}{N_o} = \frac{k}{n}\frac{E_b}{N_o} = \frac{12}{24}(10) = 5$$

$$p_c = \frac{1}{2}\,\mathrm{erfc}\sqrt{\frac{E_s}{N_o}} = 7.827\times 10^{-4}$$

$$P_M^c = \sum_{j=t+1}^{n}\binom{n}{j}p_c^{\,j}(1-p_c)^{n-j}$$

$$= \underline{\underline{9.586\times 10^{-7}}}$$

<u>This is a ~50x improvement over uncoded.</u>