EENG 410 Homework #6

1. Consider a systematic block code with the parity-check equations:

   $p_1 = m_1 + m_2 + m_4$
   $p_2 = m_1 + m_3 + m_4$
   $p_3 = m_1 + m_2 + m_3$
   $p_4 = m_2 + m_3 + m_4$

   a) Construct the generator and parity-check matrices for this code. Place the four information symbols first in each codeword followed by the four parity symbols.
   b) How many errors can this code correct?
   c) Is [1 0 1 0 1 0 1 0] a valid codeword?
   d) Is [1 1 0 0 0 1 0 1] a valid codeword?

2. Determine the generator and parity-check matrices for a Rectangular(14,8) code. Make the code systematic with the message symbols first followed by four row parity symbols and finally two column parity symbols.

3. Is [1 1 1 1 0 0 0 1 0 0 0 1 0 1] a valid code word for the Rectangular(14,8) code in Problem 2 above? If not, what is the most likely corrected code word?

4. Is [1 1 0 0 1 1 1 0 0 0 1 1 0 0] a valid code word for the Rectangular(14,8) code in Problem 2 above? If not, what is the most likely corrected code word?

5. The Orthogonal(8,3) code can be generated by a table lookup using the H_3 matrix:

$$H_3 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

   Since this code is linear it can also be generated with a generator matrix G.

   a) If a codeword from a row of the H_3 matrix has the form $\left[p_1 p_2 p_3 p_4 p_5 m_1 m_2 m_3\right]$ determine the parity equations for p_1 through p_5.
   b) Using these parity equations determine the generator matrix G and the parity check matrix H for the Orthogonal(8,3) code. Make the code systematic with the parity symbols first followed by the message symbols.

c) Show that the rows of G and H are orthogonal.